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Vertex coloring edge-weighting of coronation by path graphs

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Abstract. In this paper, we study vertex coloring edge of corona graph. A k-edge weigting of graph G is mapping $w : (EG) \rightarrow \{1, 2, \cdots, k\}$. An edge-weighting w induces a vertex coloring $F_w : V(G) \rightarrow N$ defined by $f_w(v) = \sum_{v \in e} w(e)$. An edge-weighting w is *vertex coloring* if $f_w(u) \neq f_w(v)$ for any edge uv. Chang, et.al denoted by $\mu(G)$ the minimum k for which G has a vertex-coloring k-edge-weighting. In this paper, we obtain the lower bound of of vertex coloring edge-weighting of $P_n \odot H$ and we study the exact value of vertex coloring edge-weighting of path corona several graph.

1. Introduction

Let G be a nontrivial, finite, simple, undirected and connected graphs, with vertex set $V(G)$, edge set $E(G)$ and with no isolated vertex, for more detail definition of graph see [7, 4]. Chang, et.al in [3] introduced a k-edge weigting of graph G is mapping $w : (EG) \rightarrow \{1, 2, \cdots, k\}$. An edge-weighting w induces a vertex coloring $F_w : V(G) \rightarrow N$ defined by $f_w(v) = \sum_{v \in e} w(e)$. An edge-weighting w is *vertex coloring* if $f_w(u) \neq f_w(v)$ for any edge uv. Chang, et.al denoted by $\mu(G)$ the minimum k for which G has a vertex-coloring k-edge-weighting. The study of vertex coloring from an edge weighting see in [1], [3], [5], [6], [9], and [11]. Hongliang Lu, et.al in [9] obtained several simple sufficient conditions for graphs to be vertex-coloring 3 edge weighting. Dudek, et.al in [6] showed that deterniming whether a particular graph has a weighting of the edges that induces a proper vertex coloring is NP-complete. Chang, et.al in [3] proved $\mu(P_3), \mu(P_n), \mu(C_n), \mu(K_n)$ and $\mu(K_{m,n})$. Yezhow Wu, et.al in [11] found every 4 edge connected 4 colorable multigraph G admits a vertex coloring 3-edge weighting. Futhermore, Adawiyah, et.al in [1] discussed some unicyclic graphs and its vertex coloring edge-weighting and Dafik, et.al in [5] found vertex coloring edge-weighting of some wheel related of graphs.

Proposition 1 *[3] Let G be a connected graph, then we have*

- $\mu(P_3) = 1$ *and* $\mu(P_n) = 2$ *for* $n \geq 4$
- $\mu(C_n) = 2$ *for* $n \equiv 0(mod4)$ *and* $\mu(C_n) = 3$ *for* $n \neq 0(mod4)$
- $\mu(K_{m,n}) = 1$ *for* $m \neq n$ *and* $\mu(K_{m,n}) = 2$ *for* $m = n \geq 2$
- $\mu(W_n) = 2$ *for* $n \geq 4$

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- $\mu(F_n) = 2$ *for* $n \geq 4$

For any two graphs G and H. A coronation of G and H, denoted by $G \odot H$, is a connected graph which formed by taking n copies of graphs H_i, $1 \leq i \leq n$ of H and connecting i-th vertex of G to the vertices of H_i.

2. The Results

We will obtain the lower bound of vertex coloring edge weighting of path corona graph H and we study the exact value of vertex coloring edge-weighting of path corona several graphs, namely $\mu(P_n \odot P_m), \mu(P_n \odot S_m), \mu(P_n \odot F_m), \mu(P_n \odot C_m)$ and $\mu(P_n \odot W_m)$.

Lemma 1 *Let $P_n \odot H$ be corona graph of path graph P_n and graph H, order $n \geq 4$, then vertex coloring edge weighting of $P_n \odot H$ is $\mu(P_n \odot H) \geq \mu(H)$*

Proof: Let $P_n \odot H$ be corona graph of path graph P_n and graph H, order $n \geq 4$. Its graph have n subgraph H_i, $1 \leq i \leq n$. Thus, the edge-weighting of every H_i, $1 \leq i \leq n$ and the edge-weighting of P_n which can be use the edge weighting in H_i, The edge weighting of the edges between $u \in V(P_n)$ and $u \in V(H_i)$ which can be use the edge weighting of H_i such that we obtain that $\mu(P_n \odot H) \geq \mu(H)$.

Theorem 1 *Let $P_n \odot P_m$ be corona graph of path graph P_n and path graph P_m with $n, m \geq 4$, then vertex coloring edge weighting of graph $P_n \odot P_m$ is $\mu(P_n \odot P_m) = 2$.*

Proof: Let $P_n \odot P_m$ be corona graph with vertex set $V(P_n \odot P_m) = \{x_i, x_{ij}; 1 \leq i \leq n; 1 \leq j \leq m\}$ and edge set $E(P_n \odot P_m) = \{x_i x_{i+1}; 1 \leq i \leq n-1\} \cup \{x_i x_{ij}; 1 \leq i \leq n; 1 \leq j \leq m\} \cup \{x_{ij} x_{i(j+1)}; 1 \leq i \leq n; 1 \leq j \leq m-1\}$. The cardinality of vertices and edges, respectively are $|V(P_n \odot P_m)| = nm+n$ and $|E(P_n \odot P_m)| = 2nm-1$. We prove vertex coloring edge-weighting of $P_n \odot P_m$ for $n, m \geq 4$ is $\mu(P_n \odot P_m) = 2$.

We prove that lower bound of vertex coloring edge weighting of $P_n \odot P_m$ is $\mu(P_n \odot P_m) \geq 2$. Based Lemma 1 and Proposition that the lower bound of vertex coloring edge weighting of $P_n \odot P_m$ is $\mu(P_n \odot P_m) \geq \mu(P_m) = 2$. Thus, we have the lower bound of vertex coloring edge-weighting of $P_n \odot P_m$ is $\mu(P_n \odot P_m) \geq 2$.

Furthermore, we prove that the upper bound of vertex coloring edge-weighting of $P_n \odot P_m$ is $\mu(P_n \odot P_m) \leq 2$. We define the vertex coloring 2-edge-weighting of $P_n \odot P_m$ is function $w : E(P_n \odot P_m) \to \{1, 2\}$. The vertex coloring 2-edge weighting is

$$w(e) = \begin{cases} 1, & \text{if } e = x_i x_{i+1} \text{ for } i \equiv 1, 2 (\text{mod } 4),\ 1 \leq i \leq n-1 \\ 2, & \text{if } e = x_i x_{i+1} \text{ for } i \equiv 0, 3 (\text{mod } 4),\ 1 \leq i \leq n-1 \end{cases}$$

$$w(e) = \begin{cases} 1, & \text{if } e = x_i x_{ij} \text{ for } j \text{ odd or } j = m,\ 1 \leq j \leq m; 1 \leq i \leq n \\ 2, & \text{if } e = x_i x_{ij} \text{ for } j \text{ even},\ 1 \leq j \leq m; 1 \leq i \leq n \end{cases}$$

$$w(e) = 1, \text{ if } e = x_{ij} x_{i(j+1)} 1 \leq i \leq n; 1 \leq j \leq m$$

It is easy to see that the vertex coloring of $P_n \odot P_m$ are as follows

$$f_w(v) = \begin{cases} \frac{3(m-1)}{2} + 2, & \text{if } v = x_i \text{ for } m \text{ odd},\ i = 1 \\ \frac{3(m-1)}{2} + 3, & \text{if } v = x_i \text{ for } m \text{ odd},\ i = 2k, k \geq 1, 1 \leq i \leq n \\ \frac{3(m-1)}{2} + 4, & \text{if } v = x_i \text{ for } m \text{ odd},\ i = 2k+1, k \geq 1, 1 \leq i \leq n \\ \frac{3(m-2)}{2} + 3, & \text{if } v = x_i \text{ for } m \text{ even},\ i = 1 \\ \frac{3(m-2)}{2} + 4, & \text{if } v = x_i \text{ for } m \text{ even},\ i = 2k, k \geq 1, 1 \leq i \leq n \\ \frac{3(m-1)}{2} + 5, & \text{if } v = x_i \text{ for } m \text{ even},\ i = 2k+1, k \geq 1, 1 \leq i \leq n \end{cases}$$

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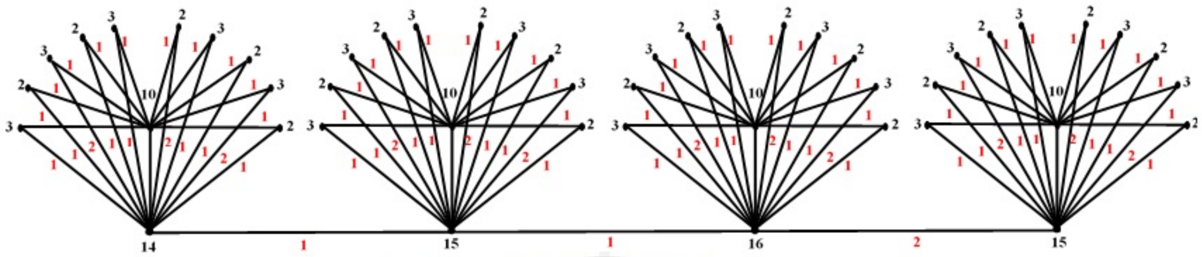

Figure 1. Example for the vertex coloring edge weighting of $P_4 \odot S_{10}$

$$f_w(v) = \begin{cases} 2, & \text{if } v = x_{ij} \text{ for } j = 1, m,\ 1 \leq i \leq n \\ 3, & \text{if } v = x_{ij} \text{ for } j \text{ even},\ 2 \leq j \leq m-1, 1 \leq i \leq n \\ 4, & \text{if } v = x_{ij} \text{ for } j \text{ odd},\ 2 \leq j \leq m-1, 1 \leq i \leq n \end{cases}$$

We get that $f_w(v)$ is vertex coloirng of $P_n \odot P_m$. Hence, the upper bound of vertex coloring edge-weighting of $P_n \odot P_m$ is $\mu(P_n \odot P_m) \leq 2$. Thus, we conclude that $\mu(P_n \odot P_m) = 2$.

Theorem 2 *Let $P_n \odot S_m$ be corona graph of path graph P_n and star graph S_m with $n, m \geq 4$, then vertex coloring edge weighting of graph $P_n \odot S_m$ is $\mu(P_n \odot S_m) = 2$.*

Proof: Let $P_n \odot S_m$ be corona graph with vertex set $V(P_n \odot S_m) = \{x_i, y_i, y_{ij}; 1 \leq i \leq n; 1 \leq j \leq m\}$ and edge set $E(P_n \odot S_m) = \{x_i x_{i+1}; 1 \leq i \leq n-1\} \cup \{x_i y_{ij}; 1 \leq i \leq n; 1 \leq j \leq m\} \cup \{x_i y_i; 1 \leq i \leq n\} \cup \{y_i y_{ij}; 1 \leq i \leq n; 1 \leq j \leq m\}$. The cardinality of vertices and edges, respectively are $|V(P_n \odot S_m)| = nm + 2n$ and $|E(P_n \odot S_m)| = 2nm + 2n - 1$. We prove vertex coloring edge-weighting of $P_n \odot S_m$ for $n, m \geq 4$ is $\mu(P_n \odot S_m) = 2$.

We prove that lower bound of vertex coloring edge weighting of $P_n \odot S_m$ is $\mu(P_n \odot S_m) \geq 2$. Based Lemma 1 and Proposition that the lower bound of vertex coloring edge weighting of $P_n \odot S_m$ is $\mu(P_n \odot S_m) \geq \mu(S_m) = 1$. However, we can not attain the sharpest lower bound. We assume that $\mu(P_n \odot S_m) < 2$, we have a vertex coloring 1-edge weighting. If the edges assigned the $w(e) = 1$ for $e \in E(P_n \odot S_m)$, then the vertices with $d(x_i) = m + 3$ for $2 \leq i \leq n-1$ have $f_w(x_i) = m + 3$. The vertices x_i and x_j for $2 \leq i, j \leq n-1$ and $j = i+1$ are adjacents and $d(x_i) = d(x_j) = m + 3$, then $f_w(x_i) = f_w(x_j) = m + 3$. It isn't satisfy the properties of vertex coloring, it is a contradiction. Thus, we have the lower bound of vertex coloring edge-weighting of F_n is $\mu(P_n \odot S_m) \geq 2$.
Furthermore, we prove that the upper bound of vertex coloring edge-weighting of $P_n \odot S_m$ is $\mu(P_n \odot S_m) \leq 2$. We define the vertex coloring 2-edge-weighting of $P_n \odot S_m$ is function $w: E(P_n \odot S_m) \rightarrow \{1, 2\}$. The vertex coloring 2-edge weighting is

$$w(e) = \begin{cases} 1, & \text{if } e = x_i x_{i+1} \text{ for } i \equiv 1, 2 (\text{mod } 4),\ 1 \leq i \leq n-1 \\ 2, & \text{if } e = x_i x_{i+1} \text{ for } i \equiv 0, 3 (\text{mod } 4),\ 1 \leq i \leq n-1 \end{cases}$$

$$w(e) = \begin{cases} 1, & \text{if } e = x_i y_{ij} \text{ for } j \text{ odd},\ 1 \leq j \leq m; 1 \leq i \leq n \\ 2, & \text{if } e = x_i y_{ij} \text{ for } j \text{ even},\ 1 \leq j \leq m; 1 \leq i \leq n \end{cases}$$

$$w(e) = 1, \text{ if } e = x_i y_i; 1 \leq i \leq n$$
$$w(e) = 1, \text{ if } e = y_i y_{ij}; 1 \leq i \leq n; 1 \leq j \leq m$$

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It is easy to see that the vertex coloring of $P_n \odot S_m$ are as follows

$$f_w(v) = \begin{cases} \frac{3(m-1)}{2}+3, & \text{if } v=x_i \text{ for } m \text{ odd}, i=1 \\ \frac{3(m-1)}{2}+4, & \text{if } v=x_i \text{ for } m \text{ odd}, i=2k, k\geq 1, 1\leq i\leq n \\ \frac{3(m-1)}{2}+5, & \text{if } v=x_i \text{ for } m \text{ odd}, i=2k+1, k\geq 1, 1\leq i\leq n \\ \frac{3(m-2)}{2}+5, & \text{if } v=x_i \text{ for } m \text{ even}, i=1 \\ \frac{3(m-2)}{2}+6, & \text{if } v=x_i \text{ for } m \text{ even}, i=2k, k\geq 1, 1\leq i\leq n \\ \frac{3(m-1)}{2}+7, & \text{if } v=x_i \text{ for } m \text{ even}, i=2k+1, k\geq 1, 1\leq i\leq n \end{cases}$$

$$f_w(v) = \begin{cases} 2, & \text{if } v=y_{ij} \text{ for } j \text{ odd}, 2\leq j\leq m, 1\leq i\leq n \\ 3, & \text{if } v=y_{ij} \text{ for } j \text{ even}, 2\leq j\leq m, 1\leq i\leq n \end{cases}$$

$$f_w(v) = m+1, \text{ if } v=y_i, 1\leq i\leq n$$

We get that $f_w(v)$ is vertex coloring of $P_n \odot S_m$. Hence, the upper bound of vertex coloring edge-weighting of $P_n \odot S_m$ is $\mu(P_n \odot S_m) \leq 2$. Thus, we conclude that $\mu(P_n \odot S_m) = 2$.

Theorem 3 *Let $P_n \odot F_m$ be corona graph of path graph P_n and fan graph F_m with $n, m \geq 4$, then vertex coloring edge weighting of graph $P_n \odot F_m$ is $\mu(P_n \odot F_m) = 2$.*

Proof: Let $P_n \odot F_m$ be corona graph with vertex set $V(P_n \odot F_m) = \{x_i, y_i, y_{ij}; 1\leq i\leq n; 1\leq j\leq m\}$ and edge set $E(P_n \odot F_m) = \{x_i x_{i+1}; 1\leq i\leq n-1\} \cup \{x_i y_{ij}; 1\leq i\leq n; 1\leq j\leq m\} \cup \{x_i y_i; 1\leq i\leq n\} \cup \{y_i y_{ij}; 1\leq i\leq n; 1\leq j\leq m\} \cup \{y_{ij} y_{i(j+1)}; 1\leq i\leq n; 1\leq j\leq m-1\}$. The cardinality of vertices and edges, respectively are $|V(P_n \odot F_m)| = nm+2n$ and $|E(P_n \odot F_m)| = 3nm+n-1$. We prove vertex coloring edge-weighting of $P_n \odot F_m$ for $n, m \geq 4$ is $\mu(P_n \odot F_m) = 2$.

We prove that lower bound of vertex coloring edge weighting of $P_n \odot F_m$ is $\mu(P_n \odot F_m) \geq 2$. Based Lemma 1 and Proposition that the lower bound of vertex coloring edge weighting of $P_n \odot F_m$ is $\mu(P_n \odot F_m) \geq \mu(F_m) = 2$. Thus, we have the lower bound of vertex coloring edge-weighting of $P_n \odot F_m$ is $\mu(P_n \odot F_m) \geq 2$.
Furthermore, we prove that the upper bound of vertex coloring edge-weighting of $P_n \odot F_m$ is $\mu(P_n \odot F_m) \leq 2$. We define the vertex coloring 2-edge-weighting of $P_n \odot F_m$ is function $w: E(P_n \odot F_m) \to \{1, 2\}$. The vertex coloring 2-edge weighting is

$$w(e) = \begin{cases} 1, & \text{if } e=x_i x_{i+1} \text{ for } i\equiv 1, 2 (\text{mod } 4),\ 1\leq i\leq n-1 \\ 2, & \text{if } e=x_i x_{i+1} \text{ for } i\equiv 0, 3 (\text{mod } 4),\ 1\leq i\leq n-1 \end{cases}$$

$$w(e) = \begin{cases} 1, & \text{if } e=x_i y_{ij} \text{ for } j \text{ odd}, 1\leq j\leq m; 1\leq i\leq n \\ 2, & \text{if } e=x_i y_{ij} \text{ for } j \text{ even}, 1\leq j\leq m; 1\leq i\leq n \end{cases}$$

$$w(e) = \begin{cases} 1, & \text{if } e=y_i y_{ij} \text{ for } j \text{ odd}, 1\leq j\leq m; 1\leq i\leq n \\ 2, & \text{if } e=y_i y_{ij} \text{ for } j \text{ even}, 1\leq j\leq m; 1\leq i\leq n \end{cases}$$

$$w(e) = 1, \text{ if } e=x_i y_i; 1\leq i\leq n$$
$$w(e) = 1, \text{ if } e=y_{ij} y_{i(j+1)}; 1\leq i\leq n; 1\leq j\leq m$$

It is easy to see that the vertex coloring of $P_n \odot F_m$ are as follows

$$f_w(v) = \begin{cases} \frac{3(m-1)}{2}+3, & \text{if } v=x_i \text{ for } m \text{ odd}, i=1 \\ \frac{3(m-1)}{2}+4, & \text{if } v=x_i \text{ for } m \text{ odd}, i=2k, k\geq 1, 1\leq i\leq n \\ \frac{3(m-1)}{2}+5, & \text{if } v=x_i \text{ for } m \text{ odd}, i=2k+1, k\geq 1, 1\leq i\leq n \\ \frac{3(m-2)}{2}+5, & \text{if } v=x_i \text{ for } m \text{ even}, i=1 \\ \frac{3(m-2)}{2}+6, & \text{if } v=x_i \text{ for } m \text{ even}, i=2k, k\geq 1, 1\leq i\leq n \\ \frac{3(m-1)}{2}+7, & \text{if } v=x_i \text{ for } m \text{ even}, i=2k+1, k\geq 1, 1\leq i\leq n \end{cases}$$

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$$f_w(v) = \begin{cases} \frac{3(m-1)}{2} + 2, & \text{if } v = y_i \text{ for } m \text{ odd, } 1 \le i \le n \\ \frac{3(m-2)}{2} + 4, & \text{if } v = y_i \text{ for } m \text{ even, } 1 \le i \le n \end{cases}$$

$$f_w(v) = \begin{cases} 3, & \text{if } v = y_{ij} \text{ for } j = 1, 1 \le i \le n \\ 3, & \text{if } v = y_{ij} \text{ for } j = m, m \text{ odd, } 1 \le i \le n \\ 5, & \text{if } v = y_{ij} \text{ for } j = m, m \text{ even, } 1 \le i \le n \\ 6, & \text{if } v = y_{ij} \text{ for } j \text{ even, } 2 \le j \le m-1; 1 \le i \le n \\ 4, & \text{if } v = y_{ij} \text{ for } j \text{ odd, } 2 \le j \le m-1; 1 \le i \le n \end{cases}$$

We get that $f_w(v)$ is vertex coloring of $P_n \odot F_m$. Hence, the upper bound of vertex coloring edge-weighting of $P_n \odot F_m$ is $\mu(P_n \odot F_m) \le 2$. Thus, we conclude that $\mu(P_n \odot F_m) = 2$.

Theorem 4 *Let $P_n \odot C_m$ be corona graph of path graph P_n and cycle graph C_m with $n, m \ge 3$, then vertex coloring edge weighting of graph $P_n \odot C_m$ is*

$$\mu_(G) = \begin{cases} 2, & \text{for } m \equiv 0 (mod\ 4) \\ 3, & \text{for } m \text{ else} \end{cases}$$

Proof: Let $P_n \odot C_m$ be corona graph with vertex set $V(P_n \odot C_m) = \{x_i, x_{ij}; 1 \le i \le n; 1 \le j \le m\}$ and edge set $E(P_n \odot C_m) = \{x_i x_{i+1}; 1 \le i \le n-1\} \cup \{x_{ij} x_{i(j+1)}; 1 \le i \le n; 1 \le j \le m-1\} \cup \{x_{i1} x_{im}; 1 \le i \le n\} \cup \{x_i x_{ij}; 1 \le i \le n; 1 \le j \le m\}$. The cardinality of vertices and edges, respectively are $|V(P_n \odot C_m)| = nm + n$ and $|E(P_n \odot C_m)| = 2nm + n - 1$. We prove vertex coloring edge-weighting of $P_n \odot C_m$ for $n, m \ge 4$ is $\mu(P_n \odot C_m) = 2$ for $m \equiv 0 (\text{mod } 4)$ and $\mu(P_n \odot C_m) = 3$ for m else.
Case 1: For $m \equiv 0(\text{mod}4)$, We prove that lower bound of vertex coloring edge weighting of $P_n \odot C_m$ is $\mu(P_n \odot C_m) \ge 2$. Based Lemma 1 and Proposition that the lower bound of vertex coloring edge weighting of $P_n \odot C_m$ is $\mu(P_n \odot C_m) \ge \mu(C_m) = 2$. Thus, we have the lower bound of vertex coloring edge-weighting of $P_n \odot C_m$ is $\mu(P_n \odot C_m) \ge 2$.
Furthermore, we prove that the upper bound of vertex coloring edge-weighting of $P_n \odot C_m$ is $\mu(P_n \odot C_m) \le 2$. We define the vertex coloring 2-edge-weighting of $P_n \odot C_m$ is function $w : E(P_n \odot C_m) \to \{1, 2\}$. The vertex coloring 2-edge weighting is

$$w(e) = \begin{cases} 1, & \text{if } e = x_i x_{i+1} \text{ for } i \equiv 1, 2 (\text{mod } 4),\ 1 \le i \le n-1 \\ 2, & \text{if } e = x_i x_{i+1} \text{ for } i \equiv 0, 3 (\text{mod } 4),\ 1 \le i \le n-1 \end{cases}$$

$$w(e) = \begin{cases} 1, & \text{if } e = x_i x_{ij} \text{ for } j \text{ odd, } 1 \le j \le m; 1 \le i \le n \\ 2, & \text{if } e = x_i x_{ij} \text{ for } j \text{ even, } 1 \le j \le m; 1 \le i \le n \end{cases}$$

$$w(e) = \begin{cases} 1, & \text{if } e = x_{ij} x_{i(j+1)} \text{ for } j \text{ odd, } 1 \le j \le m-1; 1 \le i \le n \\ 2, & \text{if } e = x_{ij} x_{i(j+1)} \text{ for } j \text{ even, } 1 \le j \le m-1; 1 \le i \le n \end{cases}$$

$$w(e) = 2, \text{ if } e = x_{i1} x_{im}; 1 \le i \le n$$

It is easy to see that the vertex coloring of $P_n \odot C_m$ are as follows

$$f_w(v) = \frac{3m}{2} + 1, \text{ if } v = x_i, i = 1$$
$$f_w(v) = \frac{3m}{2} + 2, \text{ if } v = x_i, i = 2k, k \ge 1, 1 \le i \le n$$
$$f_w(v) = \frac{3m}{2} + 3, \text{ if } v = x_i, i = 2k+1, k \ge 1, 1 \le i \le n$$

$$f_w(v) = \begin{cases} 4, & \text{if } v = x_{ij} \text{ for } j \text{ odd, } 1 \le j \le m; 1 \le i \le n \\ 5, & \text{if } v = x_{ij} \text{ for } j \text{ even, } 1 \le j \le m; 1 \le i \le n \end{cases}$$

We get that $f_w(v)$ is vertex coloring of $P_n \odot C_m$. Hence, the upper bound of vertex coloring edge-weighting of $P_n \odot C_m$ for $m \equiv 0 (\text{mod } 4)$ is $\mu(P_n \odot C_m) \le 2$. Thus, we conclude that

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$\mu(P_n \odot C_m) = 2$ for $m \equiv 0(\text{mod } 4)$.

Case 2: For $m \neq 0(\text{mod}4)$, We prove that lower bound of vertex coloring edge weighting of $P_n \odot C_m$ is $\mu(P_n \odot C_m) \geq 3$. Based Lemma 1 and Proposition that the lower bound of vertex coloring edge weighting of $P_n \odot C_m$ is $\mu(P_n \odot C_m) \geq \mu(C_m) = 3$. Thus, we have the lower bound of vertex coloring edge-weighting of $P_n \odot C_m$ is $\mu(P_n \odot C_m) \geq 3$.
Furthermore, we prove that the upper bound of vertex coloring edge-weighting of $P_n \odot C_m$ is $\mu(P_n \odot C_m) \leq 3$. We define the vertex coloring 2-edge-weighting of $P_n \odot C_m$ is function $w : E(P_n \odot C_m) \rightarrow \{1, 2, 3\}$. The vertex coloring 3-edge weighting is

$$w(e) = \begin{cases} 1, & \text{if } e = x_i x_{i+1} \text{ for } i \equiv 1, 2(\text{mod } 4), 1 \leq i \leq n-1 \\ 2, & \text{if } e = x_i x_{i+1} \text{ for } i \equiv 0, 3(\text{mod } 4), 1 \leq i \leq n-1 \end{cases}$$

$$w(e) = \begin{cases} 1, & \text{if } e = x_i x_{ij} \text{ for } i \equiv 1(\text{mod } 3), 1 \leq j \leq m; 1 \leq i \leq n \\ 2, & \text{if } e = x_i x_{ij} \text{ for } i \equiv 2(\text{mod } 3), 1 \leq j \leq m; 1 \leq i \leq n \\ 3, & \text{if } e = x_i x_{ij} \text{ for } i \equiv 0(\text{mod } 3), 1 \leq j \leq m; 1 \leq i \leq n \end{cases}$$

$$w(e) = \begin{cases} 1, & \text{if } e = x_{ij} x_{i(j+1)} \text{ for } j \text{ odd}, 1 \leq j \leq m-1; 1 \leq i \leq n \\ 2, & \text{if } e = x_{ij} x_{i(j+1)} \text{ for } j \text{ even}, 1 \leq j \leq m-1; 1 \leq i \leq n \end{cases}$$

$$w(e) = \begin{cases} 1, & \text{if } e = x_{i1} x_{im} \text{ for } j \text{ odd}, 1 \leq i \leq n \\ 2, & \text{if } e = x_{i1} x_{im} \text{ for } j \text{ even}, 1 \leq i \leq n \end{cases}$$

It is easy to see that the vertex coloring of $P_n \odot C_m$ are as follows

$$f_w(v) = \begin{cases} 2m + i, & \text{if } v = x_i \text{ for } m = 3k, k \geq 1, 1 \leq i \leq n \\ 2m - 1 + i, & \text{if } v = x_i \text{ for } m \neq 3k, k \geq 1, 1 \leq i \leq n \end{cases}$$

for m is odd

$$f_w(v) = \begin{cases} 3, & \text{if } v = x_{ij} \text{ for } j = 1, 1 \leq i \leq n \\ 5, & \text{if } v = x_{ij} \text{ for } j \equiv 2(\text{mod } 4), 1 \leq j \leq m; 1 \leq i \leq n \\ 6, & \text{if } v = x_{ij} \text{ for } j \equiv 3(\text{mod } 4), 1 \leq j \leq m; 1 \leq i \leq n \\ 4, & \text{if } v = x_{ij} \text{ for } j \equiv 0(\text{mod } 4), 1 \leq j \leq m; 1 \leq i \leq n \end{cases}$$

for m is even

$$f_w(v) = \begin{cases} 4, & \text{if } v = x_{ij} \text{ for } j \equiv 1(\text{mod } 3), 1 \leq j \leq m; 1 \leq i \leq n \\ 5, & \text{if } v = x_{ij} \text{ for } j \equiv 2(\text{mod } 3), 1 \leq j \leq m; 1 \leq i \leq n \\ 6, & \text{if } v = x_{ij} \text{ for } j \equiv 0(\text{mod } 3), 1 \leq j \leq m; 1 \leq i \leq n \end{cases}$$

We get that $f_w(v)$ is vertex coloring of $P_n \odot C_m$. Hence, the upper bound of vertex coloring edge-weighting of $P_n \odot C_m$ for m else is $\mu(P_n \odot C_m) \leq 3$. Thus, we conclude that $\mu(P_n \odot F_m) = 3$ for $m \neq 0(\text{mod}4)$. It conclude the proof.

Theorem 5 *Let $P_n \odot W_m$ be corona graph of path graph P_n and wheel graph W_m with $n, m \geq 4$, then vertex coloring edge weighting of graph $P_n \odot W_m$ is $\mu(P_n \odot W_m) = 2$.*

Proof: Let $P_n \odot W_m$ be corona graph with vertex set $V(P_n \odot W_m) = \{x_i, y_i, y_{ij}; 1 \leq i \leq n; 1 \leq j \leq m\}$ and edge set $E(P_n \odot W_m) = \{x_i x_{i+1}; 1 \leq i \leq n-1\} \cup \{x_i y_{ij}; 1 \leq i \leq n; 1 \leq j \leq m\} \cup \{x_i y_i; 1 \leq i \leq n\} \cup \{y_i y_{ij}; 1 \leq i \leq n; 1 \leq j \leq m\} \cup \{y_{ij} y_{i(j+1)}; 1 \leq i \leq n; 1 \leq j \leq m-1\} \cup \{y_{i1} y_{im}\}$. The cardinality of vertices and edges, respectively are $|V(P_n \odot W_m)| = nm + 2n$ and $|E(P_n \odot W_m)| = 3nm + n$. We prove vertex coloring edge-weighting of $P_n \odot W_m$ for $n, m \geq 4$ is $\mu(P_n \odot W_m) = 2$.

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We prove that lower bound of vertex coloring edge weighting of $P_n \odot W_m$ is $\mu(P_n \odot W_m) \geq 2$. Based Lemma 1 and Proposition that the lower bound of vertex coloring edge weighting of $P_n \odot W_m$ is $\mu(P_n \odot W_m) \geq \mu(C_m) = 2$. Thus, we have the lower bound of vertex coloring edge-weighting of $P_n \odot W_m$ is $\mu(P_n \odot W_m) \geq 2$.
Furthermore, we prove that the upper bound of vertex coloring edge-weighting of $P_n \odot W_m$ is $\mu(P_n \odot W_m) \leq 2$. We define the vertex coloring 2-edge-weighting of $P_n \odot W_m$ is function $w : E(P_n \odot W_m) \to \{1, 2\}$. The vertex coloring 2-edge weighting is

$$w(e) = \begin{cases} 1, & \text{if } e = x_i x_{i+1} \text{ for } i \equiv 1, 2 (\text{mod } 4),\ 1 \leq i \leq n-1 \\ 2, & \text{if } e = x_i x_{i+1} \text{ for } i \equiv 0, 3 (\text{mod } 4),\ 1 \leq i \leq n-1 \end{cases}$$

$$w(e) = \begin{cases} 1, & \text{if } e = x_i y_{ij} \text{ for } j \text{ odd},\ 1 \leq j \leq m; 1 \leq i \leq n \\ 2, & \text{if } e = x_i y_{ij} \text{ for } j \text{ even},\ 1 \leq j \leq m; 1 \leq i \leq n \end{cases}$$

$$w(e) = \begin{cases} 1, & \text{if } e = y_i y_{ij} \text{ for } j \text{ odd},\ 1 \leq j \leq m; 1 \leq i \leq n \\ 2, & \text{if } e = y_i y_{ij} \text{ for } j \text{ even},\ 1 \leq j \leq m; 1 \leq i \leq n \end{cases}$$

$$w(e) = \begin{cases} 1, & \text{if } e = y_{ij} y_{i(j+1} \text{ for } j \text{ odd},\ 1 \leq j \leq m-1; 1 \leq i \leq n \\ 2, & \text{if } e = y_{ij} y_{i(j+1} \text{ for } j \text{ even},\ 1 \leq j \leq m-1; 1 \leq i \leq n \end{cases}$$

$$w(e) = \begin{cases} 1, & \text{if } e = y_i y_{im} \text{ for } m \text{ odd},\ 1 \leq i \leq n \\ 2, & \text{if } e = y_i y_{im} \text{ for } m \text{ even},\ 1 \leq i \leq n \end{cases}$$

$$w(e) = 2, \text{ if } e = x_i y_i; 1 \leq i \leq n$$

It is easy to see that the vertex coloring of $P_n \odot W_m$ are as follows

$$f_w(v) = \begin{cases} 7 + i, & \text{if } v = x_i \text{ for } m = 3, 1 \leq i \leq n \\ \frac{3(m-2)}{2} + 5 + i, & \text{if } v = x_i \text{ for } m \text{ even},\ 1 \leq i \leq n \\ \frac{3(m-3)}{2} + 6 + i, & \text{if } v = x_i \text{ for } m \text{ odd},\ 1 \leq i \leq n \end{cases}$$

$$f_w(v) = \begin{cases} \frac{3(m-1)}{2} + 3, & \text{if } v = y_i \text{ for } m \text{ odd},\ 1 \leq i \leq n \\ \frac{3(m-2)}{2} + 5, & \text{if } v = y_i \text{ for } m \text{ even},\ 1 \leq i \leq n \end{cases}$$

$$f_w(v) = \begin{cases} 4, & \text{if } v = y_{ij} \text{ for } j = 1, m = 3, 1 \leq i \leq n \\ 5, & \text{if } v = y_{ij} \text{ for } j \text{ odd},\ 1 \leq j \leq m; 1 \leq i \leq n \\ 7, & \text{if } v = y_{ij} \text{ for } j \text{ even},\ 1 \leq j \leq m; 1 \leq i \leq n \end{cases}$$

We get that $f_w(v)$ is vertex coloring of $P_n \odot W_m$. Hence, the upper bound of vertex coloring edge-weighting of $P_n \odot W_m$ is $\mu(P_n \odot W_m) \leq 2$. Thus, we conclude that $\mu(P_n \odot W_m) = 2$.

3. Conclusion

In this paper we have given the lower bound of vertex coloring edge weighting of path corona graph H. We have concluded the exact value of vertex coloring edge-weighting of path corona several graphs, namely $\mu(P_n \odot P_m) = \mu(P_n \odot S_m) = \mu(P_n \odot F_m) = \mu(P_n \odot W_m) = 2$, but $\mu(P_n \odot C_m) = 2$ for $m \equiv 2(\text{mod } 4)$. Hence the following problem arises naturally.

Open Problem 1 *Determine lower and upper bound of the vertex coloring edge weighting of graph corona the others graph?*

Open Problem 2 *Determine lower and upper bound of the vertex coloring edge weighting of graph operation including cartesian, comb product and others?*

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