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ICSAS 2018

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Surakarta, Indonesia
12 May 2018

Editors
A. Suparmi and Dewanta Arya Nugraha

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Preface: International Conference on Science and Applied Science (ICSAS) 2018

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Preface: International Conference on Science and Applied Science (ICSAS) 2018

International Conference on Science and Applied Science (ICSAS) 2018 was held at the Solo Paragon Hotel, Surakarta, Indonesia on 12 May 2018. The ICSAS 2018 conference is aimed to bring together scholars, leading researchers and experts from diverse backgrounds and applications areas in Science. Special emphasis is placed on promoting interaction between the science theoretical, experimental, and education sciences, engineering so that a high level exchange in new and emerging areas within Mathematics, Chemistry, Physics and Biology, all areas of sciences and applied mathematics and sciences is achieved.

In ICSAS 2018, there are eight parallel sessions and four keynote speakers. It is an honour to present this volume of AIP Conference Proceedings and we deeply thank the authors for their enthusiastic and high-grade contribution. From the review results, there are 166 papers which will be published in AIP Conference Proceedings We would like to express our sincere gratitude to all in the Programming Committee who have reviewed the papers and developed a very interesting Conference Program, as well as thanking the invited and plenary speakers. Finally, we would like to thank the conference chairman, the members of the steering committee, the organizing committee, the organizing secretariat and the financial support from the Sebelas Maret University that allowed ICSAS 2018 to be a success.

The Editors

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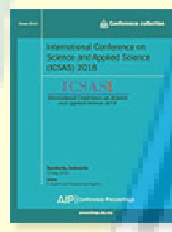

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The lower bound of the *r*-dynamic chromatic number of corona product by wheel graphs

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Abstract. The dynamic coloring of a graph G is proper coloring such that every vertex of G with degree has at least two neighbors that are colored differently. A generalization of the dynamic coloring was also introduced by Montgomery in [12], the generalized concept is called *r*-dynamic *k*-coloring. An *r*-dynamic coloring of a graph G is a proper coloring c of the vertices such that $|c(N(v)| \geq \min\{r, d(v)\}$, for each $v \in V(G)$. The *r*-dynamic chromatic number of a graph G, denoted $\chi_r(G)$ is the smallest k such that c is an *r*-dynamic k coloring of G. We will find the lower bound of the *r*-dynamic chromatic number of graphs corona wheel graph and some new results the exact value of *r*-dynamic chromatic number of corona graphs. In this paper, we study the lower bound of $\chi_r(H \odot W_m), \chi_r(W_n \odot H)$ and we also prove the exact value of *r*-dynamic chromatic number of some graphs.

INTRODUCTION

Let G be a graph. We denote the vertex set and the edge set of G, by $V(G)$ and $E(G)$, respectively. Thus for a graph G, $\Delta(G)$, $\delta(G)$ denote the maximum degree and the minimum degree of G, respectively. For $v \in V(G)$, let $N(v)$ denote the set of vertices adjacent to v in G, and $d(v) = |N(v)|$. Vertices in $N(v)$ are neighbors of v. The dynamic coloring of a graph G is proper coloring such that every vertex of G with degree has at least two neighbors that are colored differently. A generalization of the dynamic coloring was also introduced by Montgomery in [12], the generalized concept is called r-dynamic k-coloring. The chromatic number of a graph G, denoted $\chi_r(G)$ is the smallest k such that c is an *r*-dynamic k coloring of G. The *r*-dynamic chromatic number has been studied by several authors, for instance in [2], [3], [4], [5] and [9]. The following observations are immediate from the definition of *r*-dynamic coloring introduced by Montgomery [12] and the definition of corona product.

Observation 1 Let $\Delta(G)$ be the maximum degree of graph G. It holds $\chi_r(G) \geq \min\{r, \Delta(G)\} + 1$

Observation 2 Let $G = G_1 \odot G_2$ be corona product, then we have $\delta(G) = \delta(G_2) + 1$ and $\Delta(G) = \Delta(G_1) + |V(G_2)|$, where $\Delta(G)$ is maximum degree of G and $\delta(G)$ is minimum degree of G.

Observation 3 Let graph W_n, be the *r*-dynamic chromatic number:

$$\chi_{r=1,2,3}(W_n) = \begin{cases} 3, & \text{for } n \text{ even} \\ 4, & \text{for } n \text{ odd} \end{cases}$$

$$\chi_{r=4,5}(W_n) = \begin{cases} r+1, & \text{for } n = 3k, k \geq 1 \\ 6, & \text{for } n = 5 \\ 5, & \text{for } n \text{ otherwise} \end{cases}$$

$$\chi_r(W_n) = \begin{cases} r+1, & \text{for } 6 \leq r \leq n+1 \\ n+2, & \text{for } r \geq n+2 \end{cases}$$

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Alishahi, in [4] introduced an upper bound for the dynamic list chromatic number of regular graphs. Gao, *et.al* in [8] given some upper bounds for $\chi_2(G)$ - $\chi(G)$ of $K_{1,4}$-free graphs and graphs without even cycles. The corona product of G and H, denoted by $G\odot H$, is a connected graph obtained by taking a number of vertices $|V(G)|$ copy of H, and making the i^{th} of $V(G)$ adjacent to every vertex of the i^{th} copy of $V(H)$ introduced Furmanczyk in [7]. Furthermore, Ramya in [15] found acyclic coloring and star coloring of corona graphs. Pathinathan *et.al* in [14] found chromatic number of corona with subdivision vertex and subdivision edge for graph path and cycle. Kristiana *et.al* in [10] has initiated to study the coronation of path and several graphs and in [11] studied r-dynamic chromatic number of coronation by complete graph.

THE RESULTS

In the following, we will find the lower bound of the *r*-dynamic chromatic number of graphs corona wheel graph and some new results the exact value of $\chi_r(G)$.

Lemma 1 Let $G = H\odot W_m$ be a corona product of graph H and wheel graph W_m, the lower bound of *r*-dynamic chromatic number is:

$$\chi_{r=1,2,3}(G) \geq \begin{cases} 4, & \text{for } m \text{ even} \\ 5, & \text{for } m \text{ odd} \end{cases}$$

$$\chi_{r=4,5}(G) \geq \begin{cases} r+1, & \text{for } m = 3k, k \geq 1 \\ 7, & \text{for } m = 5 \\ 6, & \text{for } n \text{ otherwise} \end{cases}$$

$$\chi_r(G) \geq \begin{cases} r+1, & \text{for } 6 \leq r \leq \Delta(H)+m+1 \\ \Delta(H)+m+2, & \text{for } r \geq \Delta(H)+m+2 \end{cases}$$

Proof: Let W_m be a wheel graph with vertex set, $V(W_m) = \{x, x_i;\ 1 \leq i \leq m\}$ and the order $|V(W_m)| = m+1$.

For case r = 1, 2, 3 and m even, choose r = 3, based on Observation 1, $\chi_r(G) \geq \min\{3; \Delta(G)\} + 1 = 3 + 1 = 4$ then $\chi_r(G) \geq 4$. For m odd is obtained $\chi_r(G) \geq 5$ since wheel graph must have different colors, $c(x_{m-1}) \neq c(x_m) \neq c(x_1)$, hence add one color from m even. For case r = 4, 5, choose m = 5, r = 5. The 5-dynamic coloring must has 6 colors, for each vertex in W_5. Choose $u \in H$, based defined of corona product, $u \in H \odot W_m$ and based defined of *r*-dynamic coloring, $c(u) \neq c(x) \neq c(x_i)$. Hence, $\chi_5(G) \geq 7$. For $r \geq \Delta(H)+m+2$, based Observation 1, we get $\chi_r(G) \geq \min\{r, \Delta(G)\} + 1 = \min\{r, \Delta(H) + |V(W_m)|\} + 1 = \Delta(H) + m + 2$. The proof is complete.

By Observation 3, Lemma 1 can be a proposition below.

Proposition 1 $\chi_r(H\odot W_m) = \chi_r(Wm) + 1$.

Theorem 1 Let star graph S_n and wheel graph W_m. For $n,m \geq 3$, the *r*-dynamic chromatic number is:

$$\chi_{r=1,2,3}(H\odot W_m) = \begin{cases} 4, & \text{for } m \text{ even} \\ 5, & \text{for } m \text{ odd} \end{cases}$$

$$\chi_{r=4,5}(H\odot W_m) = \begin{cases} r+1, & \text{for } m = 3k, k \geq 1 \\ 7, & \text{for } m = 5 \\ 6, & \text{for } n \text{ otherwise} \end{cases}$$

$$\chi_r(H\odot W_m) = \begin{cases} r+1, & \text{for } 6 \leq r \leq n+m+1 \\ n+m+2, & \text{for } r \geq n+m+2 \end{cases}$$

Proof: The graph $S_n\odot W_m$ have vertex set, $V(S_n\odot W_m) = \{x\} \cup \{x_i; 1 \leq i \leq n\} \cup \{y_{ij}; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{y_i;\ 1 \leq i \leq n\} \cup \{p\} \cup \{p_j; 1 \leq j \leq m\}$ and $|V(S_n\odot W_m)| = nm + m + 2n + 2$. The edge set is $E(S_n\odot W_m) = \{xx_i; 1 \leq i \leq n\} \cup \{xp\} \cup \{x_i y_i;\ 1 \leq i \leq n\} \cup \{x_i y_{ij}; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{y_i y_{ij}; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{y_{ij} y_{i(j+1)}; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{p_j p_{j+1};\ 1 \leq j \leq m\} \cup \{pp_j;\ 1 \leq j \leq m\} \cup \{y_{i1} y_{im};\ 1 \leq i \leq n\} \cup \{p_1 p_m\}$ and $|E(S_n\odot W_m)| = 3mn + 2m + 2n + 1$. Thus, $\Delta(S_n\odot W_m) = m + n + 1$.

We define four cases, namely for $\chi_{r=1,2,3}(S_n\odot W_m), \chi_{r=4,5}(S_n\odot W_m), \chi_{6\leq r\leq m+n+1}(S_n\odot W_m)$ and $\chi_{r\geq m+n+2}(S_n\odot W_m)$.

Case 1: For *r* = 1; 2; 3

Sub Case 1: For *m* even

Based on Lemma 1, the lower bound $\chi_r(S_n\odot W_m) \geq 4$. The upper bound for $\chi_r(S_n\odot W_m)$ are by explicit construction. We define $c_1: V(S_n\odot W_m) \to \{1,2, \ldots k\}$ where n, $m \geq 3$, by following:

$$c_1(x) = 1$$
$$c_1(x_i) = 2, 1 \leq i \leq n$$
$$c_1(y_i) = 1, 1 \leq i \leq n$$

$$c_1(p) = 2$$
$$c_1(y_{ij}) = \begin{cases} 3, & \text{for } j \text{ is odd}, 1 \le i \le n, 1 \le j \le m \\ 4, & \text{for } j \text{ is even}, 1 \le i \le n, 1 \le j \le m \end{cases}$$
$$c_1(p_j) = \begin{cases} 3, & \text{for } j \text{ odd}, 1 \le j \le m \\ 4, & \text{for } j \text{ even}, 1 \le j \le m \end{cases}$$

A map $c_1: V(S_n \odot W_m) \to \{1,2,3,4\}$ gives the upper bound $\chi_{r=1,2,3}(S_n \odot W_m) \le 4$, m even. Hence, $\chi_{r=1,2,3}(S_n \odot W_m) = 4$.

Sub Case 2: For *m* odd

Based on Lemma 1, for the lower bound $\chi_r(S_n \odot W_m) \ge 5$. The upper bound $\chi_r(S_n \odot W_m)$ are by explicit construction. We define $c_2: V(S_n \odot W_m) \to \{1,2, \dots k\}$ where $n, m \ge 3$, by following:

$$c_2(x) = 1$$
$$c_2(x_i) = 2, 1 \le i \le n$$
$$c_2(y_i) = 1, 1 \le i \le n$$
$$c_2(p) = 2$$
$$c_2(y_{ij}) = \begin{cases} 3, & \text{for } j \text{ is odd}, 1 \le i \le n, 1 \le j \le m-1 \\ 4, & \text{for } j \text{ is even}, 1 \le i \le n, 1 \le j \le m-1 \\ 5, & \text{for } j = m, 1 \le i \le n \end{cases}$$
$$c_2(p_j) = \begin{cases} 3, & \text{for } j \text{ is odd}, 1 \le j \le m-1 \\ 4, & \text{for } j \text{ is even}, 1 \le j \le m-1 \\ 5, & \text{for } j = m \end{cases}$$

A map $c_2: V(S_n \odot W_m) \to \{1,2, \dots ,5\}$ gives the upper bound $\chi_{r=1,2,3}(S_n \odot W_m) \le 5$, m odd. Hence, $\chi_{r=1,2,3}(S_n \odot W_m) = 5$.

Case 2: For *r* = 4; 5

Sub Case 1: For *m* = 5

Based on Lemma 1, for the lower bound $\chi_r(S_n \odot W_m) \ge 7$. The upper bound for $\chi_r(S_n \odot W_m)$ are by explicit construction. We define $c_3: V(S_n \odot W_m) \to \{1,2, \dots k\}$ where $n, m \ge 3$, by following:

$$c_3(x) = 1$$
$$c_3(x_i) = 2, 1 \le i \le n$$
$$c_3(y_i) = 1, 1 \le i \le n$$
$$c_3(p) = 2$$
$$c_3(y_{ij}) = 2 + j, 1 \le i \le n, 1 \le j \le m$$
$$c_3(p_j) = 2 + j, 1 \le j \le n$$

A map $c_3: V(S_n \odot W_m) \to \{1,2, \dots ,2+j\}$ gives the upper bound for $\chi_{r=4,5}(S_n \odot W_m) \le 2 + m = 7$ for $m = 5$. Hence, $\chi_{r=4,5}(S_n \odot W_m) = 7$.

Sub Case 2: For *m* = 3*k*, *k* ≥ 1

Based on Lemma 1, for the lower bound $\chi_r(S_n \odot W_m) \ge r + 1$. The upper bound for $\chi_r(S_n \odot W_m)$ are by explicit construction. We define $c_4: V(S_n \odot W_m) \to \{1,2, \dots k\}$ where $n, m \ge 3$, by following:

$$c_4(x) = 1$$
$$c_4(x_i) = 2, 1 \le i \le n$$
$$c_4(y_i) = 1, 1 \le i \le n$$
$$c_4(p) = 2$$
$$c_4(p_j) = \begin{cases} 3, & \text{for } j \equiv 1 (mod\ 3), 1 \le j \le m \\ 4, & \text{for } j \equiv 2 (mod\ 3), 1 \le j \le m \\ 5, & \text{for } j \equiv 0 (mod\ 3), 1 \le j \le m \end{cases}$$

$$c_4(y_{ij}) = \begin{cases} 3, & \text{for } j \equiv 1(mod\ 3), 1 \le j \le m, 1 \le i \le n, r = 4 \\ 4, & \text{for } j \equiv 2(mod\ 3), 1 \le j \le m, 1 \le i \le n, r = 4 \\ 5, & \text{for } j \equiv 0(mod\ 3), 1 \le j \le m, 1 \le i \le n, r = 4 \end{cases}$$

$$c_4(y_{ij}) = \begin{cases} 4, & \text{for } j \equiv 1(mod\ 3), 1 \le j \le m, 1 \le i \le n, r = 5 \\ 5, & \text{for } j \equiv 2(mod\ 3), 1 \le j \le m, 1 \le i \le n, r = 5 \\ 6, & \text{for } j \equiv 0(mod\ 3), 1 \le j \le m, 1 \le i \le n, r = 5 \end{cases}$$

A map $c_4: V(S_n \odot W_m) \to \{1,2, \dots r+1\}$ gives the upper bound for $\chi_{r=4,5}(S_n \odot W_m) \le r+1$ for $m = 3k$, $k \ge 1$. Hence, $\chi_{r=4,5}(S_n \odot W_m) = r+1$.

Sub Case 1: For *m* otherwise

Based on Lemma 1, for the lower bound $\chi_r(S_n \odot W_m) \ge 6$. The upper bound for $\chi_r(S_n \odot W_m)$ are by explicit construction. We define $c_5: V(S_n \odot W_m) \to \{1,2, \dots k\}$ where $n, m \ge 3$, by following:

$$c_5(x) = 1$$
$$c_5(x_i) = 2, 1 \le i \le n$$
$$c_5(y_i) = 1, 1 \le i \le n$$
$$c_5(p) = 2$$

$$c_5(y_{ij}) = \begin{cases} 3, & \text{for } j \equiv 1(mod\ 4), 1 \le j \le m, 1 \le i \le n \\ 4, & \text{for } j \equiv 2(mod\ 4), 1 \le j \le m, 1 \le i \le n \\ 5, & \text{for } j \equiv 3(mod\ 4), 1 \le j \le m, 1 \le i \le n \\ 6, & for\ j \equiv 0(mod\ 4), 1 \le j \le m, 1 \le i \le n \end{cases}$$

$$c_5(p_j) = \begin{cases} 3, & \text{for } j \equiv 1(mod\ 4), 1 \le j \le m \\ 4, & \text{for } j \equiv 2(mod\ 4), 1 \le j \le m \\ 5, & \text{for } j \equiv 3(mod\ 4), 1 \le j \le m \\ 6, & for\ j \equiv 0(mod\ 4), 1 \le j \le m \end{cases}$$

A map $c_5: V(S_n \odot W_m) \to \{1,2, \dots, 6\}$ gives the upper bound for $\chi_{r=4,5}(S_n \odot W_m) \le 6$ for m otherwise. It gives $\chi_{r=4,5}(S_n \odot W_m) = 6$.

Case 3: For $6 \le r \le m + n + 1$

Based on Lemma 1, for the lower bound $\chi_{6 \le r \le m+n+1}(S_n \odot W_m) \ge r+1$. The upper bound *r*-dynamic chromatic number $G = S_n \odot W_m$ are by explicit construction.

$n = 3$; $m = 3$; $r = 6$ so that $\chi_{r=6}(S_3 \odot W_3) = 7$
$n = 3$; $m = 3$; $r = 7$ so that $\chi_{r=7}(S_3 \odot W_3) = 8$
$n = 3$; $m = 4$; $r = 6$ so that $\chi_{r=6}(S_3 \odot W_4) = 7$
$n = 3$; $m = 4$; $r = 7$ so that $\chi_{r=7}(S_3 \odot W_4) = 8$
$n = 3$; $m = 5$; $r = 7$ so that $\chi_{r=7}(S_3 \odot W_5) = 8$
$n = 3$; $m = 5$; $r = 8$ so that $\chi_{r=8}(S_3 \odot W_5) = 9$
$n = 3$; $m = 5$; $r = 9$ so that $\chi_{r=6}(S_3 \odot W_5) = 10$

It gives the upper bound for $\chi_{6 \le r \le m+n+1}(S_n \odot W_m) \le r+1$. Hence, $\chi_{6 \le r \le m+n+1}(S_n \odot W_m) = r+1$.

Case 4: For $r \ge m + n + 2$

Based on Lemma 1, for the lower bound $\chi_{r \ge m+n+2}(S_n \odot W_m) \ge m+n+2$. The upper bound for $\chi_r(S_n \odot W_m)$ are by explicit construction. We define $c_7: V(S_n \odot W_m) \to \{1,2, \dots, k\}$ where $n, m \ge 3$, by following:

$$c_7(x) = 1$$
$$c_7(x_i) = 1 + i, 1 \le i \le n$$
$$c_7(y_i) = 1 + n, 1 \le i \le n$$
$$c_7(p) = 1 + n$$
$$c_7(y_{ij}) = 2 + n + j, 1 \le i \le n, 1 \le j \le m$$
$$c_7(p_j) = 2 + n + j, 1 \le j \le m$$

A map $c_7: V(S_n \odot W_m) \to \{1,2, \dots, 2+n+m\}$ gives the upper bound for $\chi_{r \ge m+n+2}(S_n \odot W_m) \le m+n+2$. Hence, $\_\chi_{r \ge m+n+2}(S_n \odot W_m) = m+n+2$.

The proof is complete.

Theorem 2 Let $G = W_n \odot W_m$ be a corona product of W_n and W_m. For $n, m \ge 3$, the *r*-dynamic chromatic number is:

$$\chi_{r=1,2,3}(G) = \begin{cases} 4, & \text{for } m \text{ even} \\ 5, & \text{for } m \text{ odd} \end{cases}$$

$$\chi_{r=4,5}(G) = \begin{cases} r+1, & \text{for } m = 3k, k \geq 1 \\ 7, & \text{for } m = 5 \\ 6, & \text{for } n \text{ otherwise} \end{cases}$$

$$\chi_r(G) = \begin{cases} r+1, & \text{for } 6 \leq r \leq n+m+1 \\ n+m+2, & \text{for } r \geq n+m+2 \end{cases}$$

Proof: The graph $W_n \odot W_m$ have vertex set, $V(W_n \odot W_m) = \{x\} \cup \{x_i; 1 \leq i \leq n\} \cup \{y_{ij}; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{y_i; 1 \leq i \leq n\} \cup \{p\} \cup \{p_j; 1 \leq j \leq m\}$ and $|V(W_n \odot W_m)| = nm + m + 2n + 2$. The edge set is $E(W_n \odot W_m) = \{xx_i; 1 \leq i \leq n\} \cup \{x_i x_{i+1}; 1 \leq i \leq n-1\} \cup \{x_1 x_n\} \cup \{xp\} \cup \{x_i y_i; 1 \leq i \leq n\} \cup \{x_i y_{ij}; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{y_i y_{ij}; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{y_{ij} y_{i(j+1)}; 1 \leq i \leq n, 1 \leq j \leq m-1\} \cup \{p_j p_{j+1}; 1 \leq j \leq m-1\} \cup \{pp_j; 1 \leq j \leq m\} \cup \{y_{i1} y_{im}; 1 \leq i \leq n\} \cup \{p_1 p_m\}$ and $|E(W_n \odot W_m)| = 3mn + 3m + 3n$. Thus, $\Delta(W_n \odot W_m) = m + n + 1$.
We define four cases, namely for $\chi_{r=1,2,3}(W_n \odot W_m), \chi_{r=4,5}(W_n \odot W_m), \chi_{6 \leq r \leq m+n+1}(W_n \odot W_m)$ and $\chi_{r \geq m+n+1}(W_n \odot W_m)$. To prove the $\chi_r(W_n \odot W_m)$ is the same as the proof of $\chi_r(S_n \odot W_m)$.

Case 1: For *r* = 1; 2; 3, *m* even

Based on Lemma 1, for the lower bound $\chi_r(W_n \odot W_m) \geq 4$. The upper bound $\chi_r(W_n \odot W_m)$ are by explicit construction. We defined $c_8: V(W_n \odot W_m) \to \{1,2,\ldots,k\}$ where $n, m \geq 3$, by following:

$$c_8(x) = 1$$
$$c_8(y_i) = 1, 1 \leq i \leq n$$
$$c_8(p) = 2$$
$$c_8(x_i) = \begin{cases} 2, & \text{for } i \equiv 1 (mod\ 3), 1 \leq i \leq n \\ 3, & \text{for } i \equiv 2 (mod\ 3), 1 \leq i \leq n \\ 4, & \text{for } i \equiv 0 (mod\ 3), 1 \leq i \leq n \end{cases}$$
$$c_8(y_{ij}) = \begin{cases} 2, & \text{for } j \text{ odd}, i \text{ even } 1, \leq i \leq n, 1 \leq j \leq m \\ 3, & \text{for } j \text{ odd}, i \text{ odd}, 1 \leq i \leq n, 1 \leq j \leq m \\ 4, & \text{for } j \text{ even}, 1 \leq i \leq n, 1 \leq j \leq m \end{cases}$$
$$c_8(p_j) = \begin{cases} 3, & \text{for } j \text{ odd}, 1 \leq j \leq m \\ 4, & \text{for } j \text{ even}, 1 \leq j \leq m \end{cases}$$

A map $c_8: V(W_n \odot W_m) \to \{1,2,3,4\}$ gives the upper bound for $\chi_{r=1,2,3}(W_n \odot W_m) \leq 4$ for m is even. Hence, $\chi_{r=1,2,3}(W_n \odot W_m) = 4$.

Case 2: For *r* = 4, 5 and *m* = 3*k*, *k* ≥ 1

Based on Lemma 1, for the lower bound $\chi_r(W_n \odot W_m) \geq r + 1$. The upper bound $\chi_r(W_n \odot W_m)$ are by explicit construction. We defined $c_9: V(W_n \odot W_m) \to \{1,2,\ldots,k\}$ where $n, m \geq 3$, by following:

$$c_9(x) = 1$$
$$c_9(x_i) = 2, 1 \leq i \leq n$$
$$c_9(y_i) = 1, 1 \leq i \leq n$$
$$c_9(p) = 2$$
$$c_9(p_j) = \begin{cases} 3, & \text{for } j \equiv 1 (mod\ 3), 1 \leq j \leq m \\ 4, & \text{for } j \equiv 2 (mod\ 3), 1 \leq j \leq m \\ 5, & \text{for } j \equiv 0 (mod\ 3), 1 \leq j \leq m \end{cases}$$
$$c_9(y_{ij}) = \begin{cases} 3, & \text{for } j \equiv 1 (mod\ 3), 1 \leq j \leq m, 1 \leq i \leq n, r = 4 \\ 4, & \text{for } j \equiv 2 (mod\ 3), 1 \leq j \leq m, 1 \leq i \leq n, r = 4 \\ 5, & \text{for } j \equiv 0 (mod\ 3), 1 \leq j \leq m, 1 \leq i \leq n, r = 4 \end{cases}$$
$$c_9(y_{ij}) = \begin{cases} 4, & \text{for } j \equiv 1 (mod\ 3), 1 \leq j \leq m, 1 \leq i \leq n, r = 5 \\ 5, & \text{for } j \equiv 2 (mod\ 3), 1 \leq j \leq m, 1 \leq i \leq n, r = 5 \\ 6, & \text{for } j \equiv 0 (mod\ 3), 1 \leq j \leq m, 1 \leq i \leq n, r = 5 \end{cases}$$

A map $c_9: V(W_n \odot W_m) \rightarrow \{1,2,\dots,r+1\}$ gives the upper bound for $\chi_{r=4,5}(W_n \odot W_m) \leq r+1$, $m = 3k$, $k \geq 1$. Hence, $\chi_{r=4,5}(W_n \odot W_m) = r+1$.

Case 3: For $r \geq m + n + 2$

Based on Lemma 1, for the lower bound $\chi_{r \geq m+n+2}(W_n \odot W_m) \geq m+n+2$. The upper bound $\chi_r(W_n \odot W_m)$ are by explicit construction. We defined $c_{10}: V(W_n \odot W_m) \rightarrow \{1,2,\dots,k\}$ where n, $m \geq 3$, by following:

$$c_{10}(x) = 1$$
$$c_{10}(x_i) = 1+i, 1 \leq i \leq n$$
$$c_{10}(y_i) = 1+n, 1 \leq i \leq n$$
$$c_{10}(p) = 1+n$$
$$c_{10}(y_{ij}) = 2+n+j, 1 \leq i \leq n, 1 \leq j \leq m$$
$$c_{10}(p_j) = 2+n+j, 1 \leq j \leq m$$

A map $c_{10}: V(W_n \odot W_m) \rightarrow \{1,2,\dots,2+n+m\}$ gives the upper bound for $\chi_{r \geq m+n+2}(W_n \odot W_m) \leq m+n+2$. Hence $\chi_{r \geq m+n+2}(W_n \odot W_m) = m+n+2$.
The proof is complete.

Lemma 2 Let $G = W_n \odot H$ be a corona product of wheel graph W_n and graph $H \neq K_m$, C_m, the lower bound of r-dynamic chromatic number is:

$$\chi_{r=1,2}(G) \geq \begin{cases} 3, & \text{for } n \text{ even} \\ 4, & \text{for } n \text{ odd} \end{cases}$$
$$\chi_r(G) \geq \begin{cases} r+1, & \text{for } 3 \leq r \leq n + |V(H)| + 1 \\ n + |V(H)| + 2, & \text{for } r \geq n + |V(H)| + 2 \end{cases}$$

Proof: Vertex set of wheel graph, W_n is $V(W_n) = \{x, x_i; 1 \leq i \leq n\}$. Choose $x \in W_n$, based defined of corona product, $x \in W_n \odot H$. For $r = 2$, $|c(N(x))| \geq \min\{r, d(x)\} = 2$ and added the color itself so that $\chi_{r=2}(W_n \odot H) \geq 3$ for n even. Based on Observation 2, $\Delta(G = W_n \odot H) = n + |V(H)| + 1$. For $r \geq n + |V(H)| + 2$, based on Observation 1 and choose $r = n + |V(H)| + 2$, we get $\chi_r(W_n \odot H) \geq \min\{r, \Delta(W_n \odot H)\} + 1 = n + |V(H)| + 2$. The proof is complete.

By Observation 3, Lemma 2 can be a proposition below.

Proposition 2 $\chi_{r=1,2}(W_n \odot H) = \chi_{r=1,2}(W_n)$.

CONCLUSION

We have found the lower bound of the r-dynamic chromatic number of corona product by wheel graph and some new results the exact value of r-dynamic chromatic number of corona graphs. We obtain the r-dynamic chromatic number of $\chi_r(S_n \odot W_m)$ and $\chi_r(W_n \odot W_m)$ for n, $m \geq 3$.

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The lower bound of the *r*-dynamic chromatic number of corona product by wheel graphs

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Abstract. The dynamic coloring of a graph G is proper coloring such that every vertex of G with degree has at least two neighbors that are colored differently. A generalization of the dynamic coloring was also introduced by Montgomery in [12], the generalized concept is called r-dynamic k-coloring. An r-dynamic coloring of a graph G is a proper coloring c of the vertices such that $|c(N(v)| \geq \min\{r, d(v)\}$, for each $v \in V(G)$. The r-dynamic chromatic number of a graph G, denoted $\chi_r(G)$ is the smallest k such that c is an r-dynamic k coloring of G. We will find the lower bound of the r-dynamic chromatic number of graphs corona wheel graph and some new results the exact value of r-dynamic chromatic number of corona graphs. In this paper, we study the lower bound of $\chi_r(H \odot W_m), \chi_r(W_n \odot H)$ and we also prove the exact value of r-dynamic chromatic number of some graphs.

INTRODUCTION

Let G be a graph. We denote the vertex set and the edge set of G, by $V(G)$ and $E(G)$, respectively. Thus for a graph G, $\Delta(G)$, $\delta(G)$ denote the maximum degree and the minimum degree of G, respectively. For $v \in V(G)$, let $N(v)$ denote the set of vertices adjacent to v in G, and $d(v) = |N(v)|$. Vertices in $N(v)$ are neighbors of v. The dynamic coloring of a graph G is proper coloring such that every vertex of G with degree has at least two neighbors that are colored differently. A generalization of the dynamic coloring was also introduced by Montgomery in [12], the generalized concept is called r-dynamic k-coloring. The chromatic number of a graph G, denoted $\chi_r(G)$ is the smallest k such that c is an r-dynamic k coloring of G. The r-dynamic chromatic number has been studied by several authors, for instance in [2], [3], [4], [5] and [9]. The following observations are immediate from the definition of r-dynamic coloring introduced by Montgomery [12] and the definition of corona product.

Observation 1 Let $\Delta(G)$ be the maximum degree of graph G. It holds $\chi_r(G) \geq \min\{r, \Delta(G)\} + 1$

Observation 2 Let $G = G_1 \odot G_2$ be corona product, then we have $\delta(G) = \delta(G_2) + 1$ and $\Delta(G) = \Delta(G_1) + |V(G_2)|$, where $\Delta(G)$ is maximum degree of G and $\delta(G)$ is minimum degree of G.

Observation 3 Let graph W_n, be the r-dynamic chromatic number:

$$\chi_{r=1,2,3}(W_n) = \begin{cases} 3, & \text{for } n \text{ even} \\ 4, & \text{for } n \text{ odd} \end{cases}$$

$$\chi_{r=4,5}(W_n) = \begin{cases} r+1, & \text{for } n = 3k, k \geq 1 \\ 6, & \text{for } n = 5 \\ 5, & \text{for } n \text{ otherwise} \end{cases}$$

$$\chi_r(W_n) = \begin{cases} r+1, & \text{for } 6 \leq r \leq n+1 \\ n+2, & \text{for } r \geq n+2 \end{cases}$$

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Alishahi, in [4] introduced an upper bound for the dynamic list chromatic number of regular graphs. Gao, et al in [8] given some upper bounds for $\chi_2(G)$ - $\chi(G)$ of $K_{1,4}$-free graphs and graphs without even cycles. The corona product of G and H, denoted by $G \odot H$, is a connected graph obtained by taking a number of vertices $|V(G)|$ copy of H, and making the i^{th} of $V(G)$ adjacent to every vertex of the i^{th} copy of $V(H)$ introduced Furmanczyk in [7]. Furthermore, Ramya in [15] found acyclic coloring and star coloring of corona graphs. Pathinathan *et.al* in [14] found chromatic number of corona with subdivision vertex and subdivision edge for graph path and cycle. Kristiana *et.al* in [10] has initiated to study the coronation of path and several graphs and in [11] studied r-dynamic chromatic number of coronation by complete graph.

THE RESULTS

In the following, we will find the lower bound of the r-dynamic chromatic number of graphs corona wheel graph and some new results the exact value of $\chi_r(G)$.

Lemma 1 Let $G = H \odot W_m$ be a corona product of graph H and wheel graph W_m, the lower bound of r-dynamic chromatic number is:

$$\chi_{r=1,2,3}(G) \geq \begin{cases} 4, & \text{for } m \text{ even} \\ 5, & \text{for } m \text{ odd} \end{cases}$$

$$\chi_{r=4,5}(G) \geq \begin{cases} r+1, & \text{for } m = 3k, k \geq 1 \\ 7, & \text{for } m = 5 \\ 6, & \text{for } n \text{ otherwise} \end{cases}$$

$$\chi_r(G) \geq \begin{cases} r+1, & \text{for } 6 \leq r \leq \Delta(H) + m + 1 \\ \Delta(H) + m + 2, & \text{for } r \geq \Delta(H) + m + 2 \end{cases}$$

Proof: Let W_m be a wheel graph with vertex set, $V(W_m) = \{x, x_i; 1 \leq i \leq m\}$ and the order $|V(W_m)| = m+1$. For case $r = 1, 2, 3$ and m even, choose $r = 3$, based on Observation 1, $\chi_r(G) \geq \min\{3; \Delta(G)\} + 1 = 3 + 1 = 4$ then $\chi_r(G) \geq 4$. For m odd is obtained $\chi_r(G) \geq 5$ since wheel graph must have different colors, $c(x_{m-1}) \neq c(x_m) \neq c(x_1)$, hence add one color from m even. For case $r = 4, 5$, choose $m = 5$, $r = 5$. The 5-dynamic coloring must has 6 colors, for each vertex in W_5. Choose $u \in H$, based defined of corona product, $u \in H \odot W_m$ and based defined of r-dynamic coloring, $c(u) \neq c(x) \neq c(x_i)$. Hence, $\chi_5(G) \geq 7$. For $r \geq \Delta(H) + m + 2$, based Observation 1, we get $\chi_r(G) \geq \min\{r, \Delta(G)\} + 1 = \min\{r, \Delta(H) + |V(W_m)|\} + 1 = \Delta(H) + m + 2$. The proof is complete.

By Observation 3, Lemma 1 can be a proposition below.

Proposition 1 $\chi_r(H \odot W_m) = \chi_r(Wm) + 1$.

Theorem 1 Let star graph S_n and wheel graph W_m. For $n, m \geq 3$, the r-dynamic chromatic number is:

$$\chi_{r=1,2,3}(H \odot W_m) = \begin{cases} 4, & \text{for } m \text{ even} \\ 5, & \text{for } m \text{ odd} \end{cases}$$

$$\chi_{r=4,5}(H \odot W_m) = \begin{cases} r+1, & \text{for } m = 3k, k \geq 1 \\ 7, & \text{for } m = 5 \\ 6, & \text{for } n \text{ otherwise} \end{cases}$$

$$\chi_r(H \odot W_m) = \begin{cases} r+1, & \text{for } 6 \leq r \leq n + m + 1 \\ n + m + 2, & \text{for } r \geq n + m + 2 \end{cases}$$

Proof: The graph $S_n \odot W_m$ have vertex set, $V(S_n \odot W_m) = \{x\} \cup \{x_i; 1 \leq i \leq n\} \cup \{y_{ij}; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{y_i; 1 \leq i \leq n\} \cup \{p\} \cup \{p_j; 1 \leq j \leq m$ and $|V(S_n \odot W_m)| = nm + m + 2n + 2$. The edge set is $E(S_n \odot W_m) = \{xx_i; 1 \leq i \leq n\} \cup \{xp\} \cup \{x_i y_i; 1 \leq i \leq n\} \cup \{x_i y_{ij}; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{y_i y_{ij}; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{y_{ij} y_{i(j+1)}; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{p_j p_{j+1}; 1 \leq j \leq m\} \cup \{pp_j; 1 \leq j \leq m\} \cup \{y_{i1} y_{im}; 1 \leq i \leq n\} \cup \{p_1 p_m\}$ and $|E(S_n \odot W_m)| = 3mn + 2m + 2n + 1$. Thus, $\Delta(S_n \odot W_m) = m + n + 1$.

We define four cases, namely for $\chi_{r=1,2,3}(S_n \odot W_m), \chi_{r=4,5}(S_n \odot W_m), \chi_{6 \leq r \leq m+n+1}(S_n \odot W_m)$ and $\chi_{r \geq m+n+2}(S_n \odot W_m)$.

Case 1: For $r = 1; 2; 3$

Sub Case 1: For m even

Based on Lemma 1, the lower bound $\chi_r(S_n \odot W_m) \geq 4$. The upper bound for $\chi_r(S_n \odot W_m)$ are by explicit construction. We define $c_1: V(S_n \odot W_m) \to \{1,2, \ldots k\}$ where n, $m \geq 3$, by following:

$$c_1(x) = 1$$
$$c_1(x_i) = 2, 1 \leq i \leq n$$
$$c_1(y_i) = 1, 1 \leq i \leq n$$

$$c_1(p) = 2$$
$$c_1(y_{ij}) = \begin{cases} 3, & \text{for } j \text{ is odd}, 1 \le i \le n, 1 \le j \le m \\ 4, & \text{for } j \text{ is even}, 1 \le i \le n, 1 \le j \le m \end{cases}$$
$$c_1(p_j) = \begin{cases} 3, & \text{for } j \text{ odd}, 1 \le j \le m \\ 4, & \text{for } j \text{ even}, 1 \le j \le m \end{cases}$$

A map $c_1: V(S_n \odot W_m) \to \{1,2,3,4\}$ gives the upper bound $\chi_{r=1,2,3}(S_n \odot W_m) \le 4$, m even. Hence, $\chi_{r=1,2,3}(S_n \odot W_m) = 4$.

Sub Case 2: For *m* odd

Based on Lemma 1, for the lower bound $\chi_r(S_n \odot W_m) \ge 5$. The upper bound $\chi_r(S_n \odot W_m)$ are by explicit construction. We define $c_2: V(S_n \odot W_m) \to \{1,2,\dots k\}$ where $n, m \ge 3$, by following:

$$c_2(x) = 1$$
$$c_2(x_i) = 2, 1 \le i \le n$$
$$c_2(y_i) = 1, 1 \le i \le n$$
$$c_2(p) = 2$$
$$c_2(y_{ij}) = \begin{cases} 3, & \text{for } j \text{ is odd}, 1 \le i \le n, 1 \le j \le m-1 \\ 4, & \text{for } j \text{ is even}, 1 \le i \le n, 1 \le j \le m-1 \\ 5, & \text{for } j = m, 1 \le i \le n \end{cases}$$
$$c_2(p_j) = \begin{cases} 3, & \text{for } j \text{ is odd}, 1 \le j \le m-1 \\ 4, & \text{for } j \text{ is even}, 1 \le j \le m-1 \\ 5, & \text{for } j = m \end{cases}$$

A map $c_2: V(S_n \odot W_m) \to \{1,2,\dots,5\}$ gives the upper bound $\chi_{r=1,2,3}(S_n \odot W_m) \le 5$, m odd. Hence, $\chi_{r=1,2,3}(S_n \odot W_m) = 5$.

Case 3: For *r* = 4; 5

Sub Case 1: For *m* = 5

Based on Lemma 1, for the lower bound $\chi_r(S_n \odot W_m) \ge 7$. The upper bound for $\chi_r(S_n \odot W_m)$ are by explicit construction. We define $c_3: V(S_n \odot W_m) \to \{1,2,\dots k\}$ where $n, m \ge 3$, by following:

$$c_3(x) = 1$$
$$c_3(x_i) = 2, 1 \le i \le n$$
$$c_3(y_i) = 1, 1 \le i \le n$$
$$c_3(p) = 2$$
$$c_3(y_{ij}) = 2 + j, 1 \le i \le n, 1 \le j \le m$$
$$c_3(p_j) = 2 + j, 1 \le j \le n$$

A map $c_3: V(S_n \odot W_m) \to \{1,2,\dots,2+j\}$ gives the upper bound for $\chi_{r=4,5}(S_n \odot W_m) \le 2 + m = 7$ for $m = 5$. Hence, $\chi_{r=4,5}(S_n \odot W_m) = 7$.

Sub Case 2: For *m* = 3*k*, *k* ≥ 1

Based on Lemma 1, for the lower bound $\chi_r(S_n \odot W_m) \ge r + 1$. The upper bound for $\chi_r(S_n \odot W_m)$ are by explicit construction. We define $c_4: V(S_n \odot W_m) \to \{1,2,\dots k\}$ where $n, m \ge 3$, by following:

$$c_4(x) = 1$$
$$c_4(x_i) = 2, 1 \le i \le n$$
$$c_4(y_i) = 1, 1 \le i \le n$$
$$c_4(p) = 2$$
$$c_4(p_j) = \begin{cases} 3, & \text{for } j \equiv 1 (mod\, 3), 1 \le j \le m \\ 4, & \text{for } j \equiv 2 (mod\, 3), 1 \le j \le m \\ 5, & \text{for } j \equiv 0 (mod\, 3), 1 \le j \le m \end{cases}$$

$$c_4(y_{ij}) = \begin{cases} 3, & \text{for } j \equiv 1(mod\ 3), 1 \le j \le m, 1 \le i \le n, r = 4 \\ 4, & \text{for } j \equiv 2(mod\ 3), 1 \le j \le m, 1 \le i \le n, r = 4 \\ 5, & \text{for } j \equiv 0(mod\ 3), 1 \le j \le m, 1 \le i \le n, r = 4 \end{cases}$$

$$c_4(y_{ij}) = \begin{cases} 4, & \text{for } j \equiv 1(mod\ 3), 1 \le j \le m, 1 \le i \le n, r = 5 \\ 5, & \text{for } j \equiv 2(mod\ 3), 1 \le j \le m, 1 \le i \le n, r = 5 \\ 6, & \text{for } j \equiv 0(mod\ 3), 1 \le j \le m, 1 \le i \le n, r = 5 \end{cases}$$

A map $c_4: V(S_n \odot W_m) \to \{1,2, \ldots r+1\}$ gives the upper bound for $\chi_{r=4,5}(S_n \odot W_m) \le r+1$ for $m = 3k,\ \ k \ge 1$. Hence $\chi_{r=4,5}(S_n \odot W_m) = r+1$.

Sub Case 1: For *m* otherwise

Based on Lemma 1, for the lower bound $\chi_r(S_n \odot W_m) \ge 6$. The upper bound for $\chi_r(S_n \odot W_m)$ are by explicit construction. We define $c_5: V(S_n \odot W_m) \to \{1,2, \ldots k\}$ where $n,\ m \ge 3$, by following:

$$c_5(x) = 1$$
$$c_5(x_i) = 2, 1 \le i \le n$$
$$c_5(y_i) = 1, 1 \le i \le n$$
$$c_5(p) = 2$$

$$c_5(y_{ij}) = \begin{cases} 3, & \text{for } j \equiv 1(mod\ 4), 1 \le j \le m, 1 \le i \le n \\ 4, & \text{for } j \equiv 2(mod\ 4), 1 \le j \le m, 1 \le i \le n \\ 5, & \text{for } j \equiv 3(mod\ 4), 1 \le j \le m, 1 \le i \le n \\ 6, & \text{for } j \equiv 0(mod\ 4), 1 \le j \le m, 1 \le i \le n \end{cases}$$

$$c_5(p_j) = \begin{cases} 3, & \text{for } j \equiv 1(mod\ 4), 1 \le j \le m \\ 4, & \text{for } j \equiv 2(mod\ 4), 1 \le j \le m \\ 5, & \text{for } j \equiv 3(mod\ 4), 1 \le j \le m \\ 6, & \text{for } j \equiv 0(mod\ 4), 1 \le j \le m \end{cases}$$

A map $c_5: V(S_n \odot W_m) \to \{1,2, \ldots, 6\}$ gives the upper bound for $\chi_{r=4,5}(S_n \odot W_m) \le 6$ for *m* otherwise. It gives $\chi_{r=4,5}(S_n \odot W_m) = 6$.

Case 3: For $6 \le r \le m+n+1$

Based on Lemma 1, for the lower bound $\chi_{6 \le r \le m+n+1}(S_n \odot W_m) \ge r+1$. The upper bound *r*-dynamic chromatic number $G = S_n \odot W_m$ are by explicit construction.

$n = 3$; $m = 3$; $r = 6$ so that $\chi_{r=6}(S_3 \odot W_3) = 7$
$n = 3$; $m = 3$; $r = 7$ so that $\chi_{r=7}(S_3 \odot W_3) = 8$
$n = 3$; $m = 4$; $r = 6$ so that $\chi_{r=6}(S_3 \odot W_4) = 7$
$n = 3$; $m = 4$; $r = 7$ so that $\chi_{r=7}(S_3 \odot W_4) = 8$
$n = 3$; $m = 5$; $r = 7$ so that $\chi_{r=7}(S_3 \odot W_5) = 8$
$n = 3$; $m = 5$; $r = 8$ so that $\chi_{r=8}(S_3 \odot W_5) = 9$
$n = 3$; $m = 5$; $r = 9$ so that $\chi_{r=6}(S_3 \odot W_5) = 10$

It gives the upper bound for $\chi_{6 \le r \le m+n+1}(S_n \odot W_m) \le r+1$. Hence, $\chi_{6 \le r \le m+n+1}(S_n \odot W_m) = r+1$.

Case 4: For $r \ge m+n+2$

Based on Lemma 1, for the lower bound $\chi_{r \ge m+n+2}(S_n \odot W_m) \ge m+n+2$. The upper bound for $\chi_r(S_n \odot W_m)$ are by explicit construction. We define $c_7: V(S_n \odot W_m) \to \{1,2, \ldots, k\}$ where $n,\ m \ge 3$, by following:

$$c_7(x) = 1$$
$$c_7(x_i) = 1 + i, 1 \le i \le n$$
$$c_7(y_i) = 1 + n, 1 \le i \le n$$
$$c_7(p) = 1 + n$$
$$c_7(y_{ij}) = 2 + n + j, 1 \le i \le n, 1 \le j \le m$$
$$c_7(p_j) = 2 + n + j, 1 \le j \le m$$

A map $c_7: V(S_n \odot W_m) \to \{1,2, \ldots, 2+n+m\}$ gives the upper bound for $\chi_{r \ge m+n+2}(S_n \odot W_m) \le m+n+2$. Hence, $\chi_{r \ge m+n+2}(S_n \odot W_m) = m+n+2$.

The proof is complete.

Theorem 2 Let $G = W_n \odot W_m$ be a corona product of W_n and W_m. For $n,\ m \ge 3$, the *r*-dynamic chromatic number is:

$$\chi_{r=1,2,3}(G)=\begin{cases}4, & \text{for } m \text{ even}\\ 5, & \text{for } m \text{ odd}\end{cases}$$
$$\chi_{r=4,5}(G)=\begin{cases}r+1, & \text{for } m=3k, k\geq 1\\ 7, & \text{for } m=5\\ 6, & \text{for } n \text{ otherwise}\end{cases}$$
$$\chi_r(G)=\begin{cases}r+1, & \text{for } 6\leq r\leq n+m+1\\ n+m+2, & \text{for } r\geq n+m+2\end{cases}$$

Proof: The graph $W_n \odot W_m$ have vertex set, $V(W_n \odot W_m)=\{x\}\cup\{x_i; 1\leq i\leq n\}\cup\{y_{ij}; 1\leq i\leq n, 1\leq j\leq m\}\cup\{y_i; 1\leq i\leq n\}\cup\{p\}\cup\{p_j; 1\leq j\leq m\}$ and $|V(W_n \odot W_m)|=nm+m+2n+2$. The edge set is $E(W_n \odot W_m)=\{xx_i; 1\leq i\leq n\}\cup\{x_i x_{i+1}; 1\leq i\leq n-1\}\cup\{x_1 x_n\}\cup\{xp\}\cup\{x_i y_i; 1\leq i\leq n\}\cup\{x_i y_{ij}; 1\leq i\leq n, 1\leq j\leq m\}\cup\{y_i y_{ij}; 1\leq i\leq n, 1\leq j\leq m\}\cup\{y_{ij} y_{i(j+1)}; 1\leq i\leq n, 1\leq j\leq m-1\}\cup\{p_j p_{j+1}; 1\leq j\leq m-1\}\cup\{pp_j; 1\leq j\leq m\}\cup\{y_{i1} y_{im}; 1\leq i\leq n\}\cup\{p_1 p_m\}$ and $|E(W_n \odot W_m)|=3mn+3m+3n$. Thus, $\Delta(W_n \odot W_m)=m+n+1$.
We define four cases, namely for $\chi_{r=1,2,3}(W_n \odot W_m), \chi_{r=4,5}(W_n \odot W_m), \chi_{6\leq r\leq m+n+1}(W_n \odot W_m)$ and $\chi_{r\geq m+n+1}(W_n \odot W_m)$. To prove the $\chi_r(W_n \odot W_m)$ is the same as the proof of $\chi_r(S_n \odot W_m)$.

Case 1: For $r = 1; 2; 3$, m even

Based on Lemma 1, for the lower bound $\chi_r(W_n \odot W_m)\geq 4$. The upper bound $\chi_r(W_n \odot W_m)$ are by explicit construction. We defined $c_8: V(W_n \odot W_m)\to\{1,2,\ldots,k\}$ where $n, m\geq 3$, by following:

$$c_8(x)=1$$
$$c_8(y_i)=1, 1\leq i\leq n$$
$$c_8(p)=2$$
$$c_8(x_i)=\begin{cases}2, & \text{for } i\equiv 1(mod\ 3), 1\leq i\leq n\\ 3, & \text{for } i\equiv 2(mod\ 3), 1\leq i\leq n\\ 4, & \text{for } i\equiv 0(mod\ 3), 1\leq i\leq n\end{cases}$$
$$c_8(y_{ij})=\begin{cases}2, & \text{for } j \text{ odd}, i \text{ even } 1,\leq i\leq n, 1\leq j\leq m\\ 3, & \text{for } j \text{ odd}, i \text{ odd}, 1\leq i\leq n, 1\leq j\leq m\\ 4, & \text{for } j \text{ even}, 1\leq i\leq n, 1\leq j\leq m\end{cases}$$
$$c_8(p_j)=\begin{cases}3, & \text{for } j \text{ odd}, 1\leq j\leq m\\ 4, & \text{for } j \text{ even}, 1\leq j\leq m\end{cases}$$

A map $c_8: V(W_n \odot W_m)\to\{1,2,3,4\}$ gives the upper bound for $\chi_{r=1,2,3}(W_n \odot W_m)\leq 4$ for m is even. Hence, $\chi_{r=1,2,3}(W_n \odot W_m)=4$.

Case 2: For $r = 4, 5$ and $m = 3k$, $k\geq 1$

Based on Lemma 1, for the lower bound $\chi_r(W_n \odot W_m)\geq r+1$. The upper bound $\chi_r(W_n \odot W_m)$ are by explicit construction. We defined $c_9: V(W_n \odot W_m)\to\{1,2,\ldots,k\}$ where $n, m\geq 3$, by following:

$$c_9(x)=1$$
$$c_9(x_i)=2, 1\leq i\leq n$$
$$c_9(y_i)=1, 1\leq i\leq n$$
$$c_9(p)=2$$
$$c_9(p_j)=\begin{cases}3, & \text{for } j\equiv 1(mod\ 3), 1\leq j\leq m\\ 4, & \text{for } j\equiv 2(mod\ 3), 1\leq j\leq m\\ 5, & \text{for } j\equiv 0(mod\ 3), 1\leq j\leq m\end{cases}$$
$$c_9(y_{ij})=\begin{cases}3, & \text{for } j\equiv 1(mod\ 3), 1\leq j\leq m, 1\leq i\leq n, r=4\\ 4, & \text{for } j\equiv 2(mod\ 3), 1\leq j\leq m, 1\leq i\leq n, r=4\\ 5, & \text{for } j\equiv 0(mod\ 3), 1\leq j\leq m, 1\leq i\leq n, r=4\end{cases}$$
$$c_9(y_{ij})=\begin{cases}4, & \text{for } j\equiv 1(mod\ 3), 1\leq j\leq m, 1\leq i\leq n, r=5\\ 5, & \text{for } j\equiv 2(mod\ 3), 1\leq j\leq m, 1\leq i\leq n, r=5\\ 6, & \text{for } j\equiv 0(mod\ 3), 1\leq j\leq m, 1\leq i\leq n, r=5\end{cases}$$

A map $c_9: V(W_n \odot W_m) \to \{1,2,\dots,r+1\}$ gives the upper bound for $\chi_{r=4,5}(W_n \odot W_m) \le r+1$, $m = 3k$, $k \ge 1$.
Hence, $\chi_{r=4,5}(W_n \odot W_m) = r+1$.

Case 3: For $r \ge m+n+2$

Based on Lemma 1, for the lower bound $\chi_{r \ge m+n+2}(W_n \odot W_m) \ge m+n+2$. The upper bound $\chi_r(W_n \odot W_m)$ are by explicit construction. We defined $c_{10}: V(W_n \odot W_m) \to \{1,2,\dots,k\}$ where $n, m \ge 3$, by following:

$$c_{10}(x) = 1$$
$$c_{10}(x_i) = 1 + i, 1 \le i \le n$$
$$c_{10}(y_i) = 1 + n, 1 \le i \le n$$
$$c_{10}(p) = 1 + n$$
$$c_{10}(y_{ij}) = 2 + n + j, 1 \le i \le n, 1 \le j \le m$$
$$c_{10}(p_j) = 2 + n + j, 1 \le j \le m$$

A map $c_{10}: V(W_n \odot W_m) \to \{1,2,\dots,2+n+m\}$ gives the upper bound for $\chi_{r \ge m+n+2}(W_n \odot W_m) \le m+n+2$.
Hence $\chi_{r \ge m+n+2}(W_n \odot W_m) = m+n+2$.
The proof is complete.

Lemma 2 Let $G = W_n \odot H$ be a corona product of wheel graph W_n and graph $H \ne K_m$, C_m, the lower bound of r-dynamic chromatic number is:

$$\chi_{r=1,2}(G) \ge \begin{cases} 3, & \text{for } n \text{ even} \\ 4, & \text{for } n \text{ odd} \end{cases}$$
$$\chi_r(G) \ge \begin{cases} r+1, & \text{for } 3 \le r \le n + |V(H)| + 1 \\ n + |V(H)| + 2, & \text{for } r \ge n + |V(H)| + 2 \end{cases}$$

Proof: Vertex set of wheel graph, W_n is $V(W_n) = \{x, x_i; 1 \le i \le n\}$. Choose $x \in W_n$, based defined of corona product, $x \in W_n \odot H$. For $r = 2$, $|c(N(x))| \ge \min\{r, d(x)\} = 2$ and add the color itself so that $\chi_{r=2}(W_n \odot H) \ge 3$ for n even. Based on Observation 2, $\Delta(G = W_n \odot H) = n + |V(H)| + 1$. For $r \ge n + |V(H)| + 2$, based on Observation 1 and choose $r = n + |V(H)| + 2$, we get $\chi_r(W_n \odot H) \ge \min\{r, \Delta(W_n \odot H)\} + 1 = n + |V(H)| + 2$. The proof is complete.

By Observation 3, Lemma 2 can be a proposition below.

Proposition 2 $\chi_{r=1,2}(W_n \odot H) = \chi_{r=1,2}(W_n)$.

CONCLUSION

We have found the lower bound of the r-dynamic chromatic number of corona product by wheel graph and some new results the exact value of r-dynamic chromatic number of corona graphs. We obtain the r-dynamic chromatic number of $\chi_r(S_n \odot W_m)$ and $\chi_r(W_n \odot W_m)$ for $n, m \ge 3$.

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