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Resolving Domination Numbers of Family of Tree Graph

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Abstract: *All graph in this paper are members of family of graph tree. Let G is a connected graph, for an ordered set W={$w_1,w_2,...,w_k$} of vertices and a vertex which is not element of W, then W is dominating set of graph G when the vertices that are not listed at W are vertices which are adjacent with W. The minimum cardinality of dominating set of graph G is called dominating numbers denoted $\gamma(G)$. If W and a vertex on graph G are connected each other, the metric representation of v which is element of W is the k-vector r(v|W)=(d(v,w_1), d(v,w_2),..., d(v,w_k)), where d(x,y) represents distance between x and y. Then, W is resolving dominating set of graph G if the distance of all vertices is different respect to W. The minimum cardinality of resolving dominating set is called resolving domination numbers denoted $\gamma_r(G)$. In this paper we found the exact values of resolving dominating for firecracker graph, caterpillar graph and banana tree graph.*

Keywords: Resolving Numbers, Domination Numbers, Resolving Domination Numbers, Family of Tree Graph.

1. INTRODUCTION (*Heading 1*)

Let G (V,E) be a connected graph, then the resolving dominating set is a set of vertices on graph G that are members of dominating sets and resolving sets. This concept firstly was introduced by Bringham, et. al [1]. It combines two different concepts, which are concept of dominance and resolver. Concept of resolver is developed from the basic of minimum metrik concept which was studied by Sater in 1975 [2]. W is an ordered set of element of vertices on graph G where $W=\{w_1,w_2,\ldots,w_k\}$, then W is dominating set of graph G when the vertices are not listed at W are vertices which is adjacent with W. The minimum cardinality of dominating set of graph G is called dominating numbers $\gamma(G)$. If W and a vertex on graph G are connected each other, the metric representation of v which is element of W is the $k-vector$ $r(v|W)=(d(v,w_1),d(v,w_2),\ldots,d(v,w_k))$, where $d(x,y)$ represents distance between x and y. Then, W is resolving dominating set of graph G if the distance of all vertices is different respect to W. The minimum cardinality of dominating resolving set is called resolving domination numbers denoted $\gamma_r(G)$ [1]. The studies of domination resolving numbers, [3], [4], [5].

For ilustration of placement of vertices which are element of domination resolving numbers is provided in Figure 1.

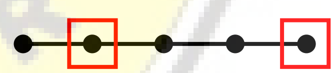

Figure 1: Vertices of resolving domination numbers of path, $\gamma_r(P_5)=2$

Bringham, et. al [1] have found propotition of this topic,

Propotition. For every graph G,

$$\max\{\gamma(G),\dim(G)\}\le\gamma_r(G)\le\gamma(G)+\dim(G)$$

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2. RESULT

Theorem 2.1. Resolving domination numbers of firecracker graph Fr_m^n for $n \geq 2$ and $m \geq 1$ is $\gamma_r(Fr_m^n) = nm$.

Proof. To prove that the resolving domination numbers of firecracker graph Fr_m^n, for $n \geq 2$ and $m \geq 1$ is $\gamma_r(Fr_m^n) = nm$, it needs to be proven using the lower bound : $\gamma_r(Fr_m^n) \geq nm$ and the upper bound : $\gamma_r(Fr_m^n) \leq nm$

First, we prove the lower bound of firecracker graph Fr_m^n, for $n \geq 2$ and $m \geq 1$ is $\gamma_r(Fr_m^n) \geq nm$. Assume that $\gamma_r(Fr_m^n) < nm$, we take $\gamma_r(Fr_m^n) = nm - 1$. Then we make possible placement of vertices of set W.

Possibility 1.

Let $W = \{y_i; 1 \leq i \leq n\} \cup \{z_j^i; 1 \leq i \leq n, 1 \leq j \leq m\}$

$- \{z_j^i; 1 \leq i \leq n, j = \{r\}, 1 \leq r \leq m\}$

$- \{z_j^i; i = \{q\}, 1 \leq i \leq n, j = \{s\}, 1 \leq s \leq m\}$

From the construction above, we know that all vertices are dominated by vertices of element W, but there are two vertices that has the same representation of W. It shows contradiction. So we got lower bound of resolving domination numbers of firecracker graph Fr_m^n, for $n \geq 2$ and $m \geq 1$ is $\gamma_r(Fr_m^n) \geq nm$.

Possibility 2.

Let $W = \{y_i; 1 \leq i \leq n\} - \{y_i; i = \{r\}, 1 \leq r \leq n\} \cup$
$\{z_j^i; 1 \leq i \leq n, 1 \leq j \leq m\} - \{z_j^i; 1 \leq i \leq n,$
$j = \{s\}, 1 \leq s \leq m\}$

From the construction above, we know that each vertex has different representation of W, but there are two vertices that are not dominated by W. It shows contradiction. So we got lower bound of resolving domination numbers of firecracker graph Fr_m^n, for $n \geq 2$ and $m \geq 1$ is $\gamma_r(Fr_m^n) \geq nm$.

Futhermore, we prove the upper bound of resolving dominating numbers of firecracker graph Fr_m^n, for $n \geq 2$ and $m \geq 1$ is $\gamma_r(Fr_m^n) \leq nm$. Let $W = \{y_i; 1 \leq i \leq n\} \cup \{z_j^i; 1 \leq i \leq n, 1 \leq j \leq m\}$ $- \{z_j^i; 1 \leq i \leq n, j = \{r\}, 1 \leq r \leq m\}$, so we got representation of vertices :

$$r(z_j^i \mid W) = (\underbrace{2i-k, \underbrace{2i-k+1, \ldots, 2i-k+1}_{m-1}, 1}_{i-1}, \underbrace{2, \ldots, 2}_{m-1}, \underbrace{h-i+3, \underbrace{h-i+4, \ldots h-i+4}_{m-1}}_{n-i})$$

$$for: 1 \leq i \leq n, 1 \leq k \leq i-1, i+1 \leq h \leq n$$

$$r(x_i \mid W) = (\underbrace{i-k+1, \underbrace{i-k+2, \ldots, i-k+2}_{m-1}, 1}_{i-1}, \underbrace{2, \ldots, 2}_{m-1}, \underbrace{h-i+1, \underbrace{h-i+2, \ldots h-i+2}_{m-1}}_{n-i})$$

$$for: 1 \leq i \leq n, 1 \leq k \leq i-1, i+1 \leq h \leq n$$

From the construction above, we know that each vertex has different representation of W and dominated. So we got upper bound of resolving domination numbers of firecracker graph Fr_m^n, for $n \geq 2$ and $m \geq 1$ is $\gamma_r(Fr_m^n) \leq nm$.

So, from those upper bound and lower bound we can conclude that the resolving domination numbers of firecracker graph Fr_m^n, for $n \geq 2$ and $m \geq 1$ is $\gamma_r(Fr_m^n) = nm$.

Theorema 2.2. Resolving domination numbers of caterpillar graph Ct_m^n, for $n \geq 1$ and $m \geq 2$ is $\gamma_r(Ct_m^n) = nm$, it needs to be proven using the lower bound : $\gamma_r(Ct_m^n) \geq nm$ and the upper bound: $\gamma_r(Ct_m^n) \leq nm$.

First, we prove the lower bound of aterpillar graph Ct_m^n, for $n \geq 1$ and $m \geq 2$ is $\gamma_r(Ct_m^n) \geq nm$. Assume that $\gamma_r(Ct_m^n) < nm$, we take $\gamma_r(Ct_m^n) = nm - 1$. Then we make possible placement of vertices of set W.

Possibility 1.

Let $W = \{x_i; 1 \leq i \leq n\} \cup \{y_j^i; 1 \leq i \leq n, 1 \leq j \leq m\}$

$- \{y_j^i; 1 \leq i \leq m, j = \{r\}, 1 \leq r \leq m\}$

$$-\left\{y_j^i; i=\{q\}, 1\le q\le m, j=\{s\}, 1\le s\le m\right\}$$

From the construction above, we know that all vertices are dominated by vertices of element W, but there are two vertices that has the same representation of W. It shows contradiction. So we got lower bound of resolving domination numbers of caterpillar graph Ct_m^n, for $n\ge 1$ and $m\ge 2$ is $\gamma_r(Ct_m^n)\ge nm$.

Possibility 2.

Let $W=\{x_i;1\le i\le n\}-\{y_i;i=\{r\},1\le r\le n\}\cup$
$\{y_j^i;1\le i\le n,1\le j\le m\}-\{y_j^i;,1\le i\le n,$
$j=\{s\},1\le s\le m\}$

From the construction above, we know that each vertex has different representation of W, but there are two vertices that are not dominated by W. It shows contradiction. So we got lower bound of resolving domination numbers of caterpillar graph Ct_m^n, for $n\ge 1$ and $m\ge 2$ is $\gamma_r(Ct_m^n)\ge nm$.

Futhermore, we prove the upper bound of resolving dominating numbers of caterpillar graph Ct_m^n, for $n\ge 1$ and $m\ge 2$ is $\gamma_r(Ct_m^n)\le nm$. Let $W=\left\{x_i;1\le i\le n\right\}\cup\left\{y_j^i;1\le i\le n,1\le j\le m\right\}$ $-\left\{y_j^i;1\le i\le n, j=\{r\},1\le r\le m\right\}$, so we got representation of vertices :

$$r\left(y_j^i\,|\,W\right)=(\underbrace{i-k+1,\underbrace{i-k+2,\ldots,i-k+2}_{m-1}}_{i-1},1,\underbrace{2,\ldots,2}_{m-1},$$
$$\underbrace{h+1,\underbrace{h+2,\ldots h+2}_{m-1}}_{n-i})$$
$$for: 1\le i\le n, 1\le k\le i-1, 1\le h\le n-i$$

From the construction above, we know that each vertex has different representation of W and dominated. So we got upper bound of resolving domination numbers of caterpillar graph Ct_m^n, for $n\ge 1$ and $m\ge 2$ is $\gamma_r(Ct_m^n)\le nm$.

So, from those upper bound and lower bound we can conclude that the resolving domination numbers of firecracker graph Ct_m^n, for $n\ge 1$ and $m\ge 2$ is $\gamma_r(Ct_m^n)=nm$.

Theorema 2.3. Resolving domination numbers of banana tree graph B_m^n, for $n\ge 2$ and $m\ge 1$ is $\gamma_r(B_m^n)=nm$, it needs to be proven using the lower bound : $\gamma_r(B_m^n)\ge nm$ and the upper bound: $\gamma_r(B_m^n)\le nm$.

First, we prove the lower bound of aterpillar graph B_m^n, for $n\ge 2$ and $m\ge 1$ is $\gamma_r(B_m^n)\ge nm$. Assume that $\gamma_r(B_m^n)< nm$, we take $\gamma_r(B_m^n)=nm-1$. Then we make possible placement of vertices of set W.

Possibility 1.

Let $W=\left\{y_i;1\le i\le n\right\}\cup\left\{z_j^i;1\le i\le n,1\le j\le m\right\}$
$-\left\{z_j^i;1\le i\le n, j=\{r\},1\le r\le m\right\}$
$-\left\{z_j^i;i=q,1\le q\le n, j=\{s\},1\le s\le m\right\}$

From the construction above, we know that all vertices are dominated by vertices of element W, but there are two vertices that has the same representation of W. It shows contradiction. So we got lower bound of resolving domination numbers of banana tree graph B_m^n, for $n\ge 2$ and $m\ge 1$ is $\gamma_r(B_m^n)\ge nm$.

Possibility 2.

Let $W=\{x_i;1\le i\le n\}-\{y_i;i=\{r\},1\le r\le n\}\cup$
$\{y_j^i;1\le i\le n,1\le j\le m\}-\{y_j^i;1\le i\le n,$
$j=\{s\};1\le s\le m\}$

From the construction above, we know that each vertex has different representation of W, but there are two vertices that are not dominated by W. It shows contradiction. So we got lower bound of resolving domination numbers of caterpillar graph B_m^n, for $n\ge 2$ and $m\ge 1$ is $\gamma_r(B_m^n)\ge nm$.

Futhermore, we prove the upper bound of resolving dominating numbers of firecracker graph B_m^n, for $n\ge 2$ and $m\ge 1$ is $\gamma_r(B_m^n)\le nm$. Let $W=\left\{y_i;1\le i\le n\right\}\cup\left\{z_j^i;1\le i\le n,1\le j\le m\right\}$ $-\left\{z_j^i;1\le i\le n, j=\{r\},1\le r\le m\right\}$, so we got representation of vertices :

$$r\left(z_j^i \mid W\right) = (\underbrace{3,\ldots,3}_{i-1},1,\underbrace{3,\ldots 3}_{n-1},\underbrace{4,\ldots,4}_{(i-1)(m-1)},\underbrace{2,\ldots,2}_{m-1},\underbrace{4,\ldots,4}_{(n-i)(m-1)})$$

$$for : 1 \le i \le n$$

$$r\left(x_i \mid W\right) = (\underbrace{1,\ldots,1}_{n},\underbrace{2,\ldots,2}_{n(m-1)})$$

From the construction above, we know that each vertex has different representation of W and dominated. So we got upper bound of resolving domination numbers of banana tree graph B_m^n, for $n \ge 2$ and $m \ge 1$ is $\gamma_r(B_m^n) \le nm$.

So, from those upper bound and lower bound we can conclude that the resolving domination numbers of banana tree graph B_m^n, for $n \ge 2$ and $m \ge 1$ is $\gamma_r(B_m^n) = nm$.

3. CONCLUSION

In this paper we have been studied about resolving domination numbers of family of tree graph. We have been concluded the exact value of firecracker graph Fr_m^n, for $n \ge 2$ and $m \ge 1$ is $\gamma_r(Fr_m^n) = nm$, the resolving domination numbers of firecracker graph Ct_m^n, for $n \ge 1$ and $m \ge 2$ is $\gamma_r(Ct_m^n) = nm$, and the resolving domination numbers of banana tree graph B_m^n, for $n \ge 2$ and $m \ge 1$ is $\gamma_r(B_m^n) = nm$.

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