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The 1st International Conference of Combinatorics, Graph Theory, and Network Topology

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The First International Conference on Combinatorics, Graph Theory and Network Topology (ICCGANT)

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Preface

It is with my great pleasure and honor to organize the First International Conference on Combinatorics, Graph Theory and Network Topology which is held from 25-26 November 2017 in the University of Jember, East Java, Indonesia and present a conference proceeding index by Scopus. It is the first international conference organized by CGANT Research Group University of Jember in cooperation with Indonesian Combinatorics Society (INACOBMS). The conference is held to welcome participants from many countries, with broad and diverse research interests of mathematics especially combinatorical study. The mission is to become an annual international forum in the future, where, civil society organization and representative, research students, academics and researchers, scholars, scientist, teachers and practitioners from all over the world could meet in and exchange an idea to share and to discuss theoretical and practical knowledge about mathematics and its applications. The aim of the first conference is to present and discuss the latest research that contributes to the sharing of new theoretical, methodological and empirical knowledge and a better understanding in the area mathematics, application of mathematics as well as mathematics education.

The themes of this conference are as follows: (1) Connection of distance to other graph properties, (2) Degree/diameter problem, (3) Distance-transitive and distance-regular graphs, (4) Metric dimension and related parameters, (5) Cages and eccentric graphs, (6) Cycles and factors in graphs, (7) Large graphs and digraphs, (8) Spectral Techniques in graph theory, (9) Ramsey numbers, (10) Dimensions of graphs, (11) Communication networks, (12) Coding theory, (13) Cryptography, (14) Rainbow connection, (15) Graph labelings and coloring, (16). Applications of graph theory

The topics are not limited to the above themes but they also include the mathematical application research of interest in general including mathematics education, such as:(1) Applied Mathematics and Modelling, (2) Applied Physics: Mathematical Physics, Biological Physics, Chemistry Physics,(3) Applied Engineering: Mathematical Engineering, Mechanical engineering, Informatics Engineering, Civil Engineering,(4) Statistics and Its Application,(5) Pure Mathematics (Analysis, Algebra and Geometry),(6) Mathematics Education, (7) Literacy of Mathematics,(8) The Use of ICT Based Media In Mathematics Teaching and Learning,(9) Technological, Pedagogical, Content Knowledge for Teaching Mathematics, (10) Students Higher Order Thinking Skill of Mathematics, (11) Contextual Teaching and Realistic Mathematics, (12) Science, Technology, Engineering, and Mathematics Approach, (13) Local Wisdom Based

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Education: Ethnomathematics, (14) Showcase of Teaching and Learning of Mathematics, (16) The 21st Century Skills: The Integration of 4C Skill in Teaching Math.

The participants of this ICCGANT 2017 conference were 200 people consisting research students, academics and researchers, scholars, scientist, teachers and practitioners from many countries. The selected papers to be publish of Journal of Physics: Conference Series are 80 papers. On behalf of the organizing committee, finally we gratefully acknowledge the support from the University of Jember of this conference. We would also like to extend our thanks to all lovely participants who are joining this unforgettable and valuable event.

Prof. Drs. Dafik, M.Sc., Ph.D.

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The Committees of The First International Conference on Combinatorics, Graph Theory and Network Topology (ICCGANT)

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The Committees of The First International Conference on Combinatorics, Graph Theory and Network Topology (ICCGANT)

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Peer review statement

All papers published in this volume of *Journal of Physics: Conference Series* have been peer reviewed through processes administered by the proceedings Editors. Reviews were conducted by expert referees to the professional and scientific standards expected of a proceedings journal published by IOP Publishing.

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On the r-dynamic chromatic number of the corronation by complete graph

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On the r-dynamic chromatic number of the corronation by complete graph

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Abstract. In this paper we will study the r-dynamic chromatic number of the coronation by complete graph. A proper k-coloring of graph G such that the neighbors of any vertex v receive at least $\min\{r, d(v)\}$ different colors. The r-dynamic chromatic number, $\chi_r(G)$ is the minimum k such that graph G has an r-dynamic k-coloring. We will obtain lower bound of the r-dynamic chromatic number of $\chi_r(K_n \odot H)$, and $\chi_r(H \odot K_m)$ We also study the exact value of the r-dynamic chromatic number of $\chi_r(K_n \odot S_m), \chi_r(K_n \odot F_m), \chi_r(S_n \odot K_m), \chi_r(F_n \odot K_m)$ and $\chi_r(K_n \odot K_m)$ for $m, n \geq 3$.

1. Introduction

Let G be graph. Graphs in this paper are simple and finite. We denote the vertex set of G by $V(G)$ and the number of vertices of G is called the order of G. Thus for graph G, $\Delta(G)$ and $\delta(G)$ denote the maximum degree and minimum degree. For every $v \in V(G)$, $d(v), N(v), c(v)$ denote the degree of v, the neighbor set of v and the color of v. A proper vertex coloring of G by k colors is a function $c : V(G) \rightarrow \{1, 2, \cdots, k\}$ with this property: if $u, v \in V(G)$ are adjancent, then $c(u)$ and $c(v)$ are different. The r-dynamic coloring of a graph G, introduced by Montgomery [8] defined a proper k-coloring of graph G such that the neighbors of any vertex v receive at least $\min\{r, d(v)\}$ different colors. The r-dynamic chromatic number, $\chi_r(G)$ is the minimum k such that graph G has an r-dynamic k-coloring. The r-dynamic chromatic number has been studied by several authors, for instance in [1], [2], [3], [4], [6], [7], [9], [10], [11] and [12]. The following observations are useful for our study, proposed by Montgomery [8].

Observation 1.1 *Let $\Delta(G)$ be the maximum degree of graph G. It holds $\chi_r(G) \geq \min\{\Delta(G), r\} + 1$.*

Observation 1.2 *Let $G = K_n$ then $\chi_r(G) = n$.*

Observation 1.3 *Let $G = G_1 \odot G_2$ be corona graph, then we have $\delta(G) = \delta(G_2) + 1$ and $\Delta(G) = \Delta(G_1) + |V(G_2)|$, where $\Delta(G)$ is maximum degree of G and $\delta(G)$ is minimum degree of G.*

Furmanczyk [5], denoted by $G \odot H$, is a connected graph obtained by taking a number of vertices $|V(G)|$ copy of H, and making the i^{th} of $V(G)$ adjacent to every vertex of the i^{th} copy

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of $V(H)$. There have been many results already found, Pathinathan, et.al. in [11] find the b-chromatic number on corona graph of subdivision vertex path with path, corona graph of any graph with path, cycle, and complete graph. Ramya in [12] discuss about the acyclic coloring of corona of $C_n \odot K_{1,3}, P_n \odot K_2$ and star coloring of corona of $P_n \odot K_2$.

The Results

We will obtain lower bound of the r-dynamic chromatic number of $\chi_r(K_n \odot H)$, and $\chi_r(H \odot K_m)$. We alsa find the exact value of the r-dynamic chromatic number of $\chi_r(K_n \odot S_m), \chi_r(K_n \odot F_m), \chi_r(S_n \odot K_m), \chi_r(F_n \odot K_m)$ and $\chi_r(K_n \odot K_m)$ for $m, n \geq 3$.There are seven theorems found in this study. We will show the complete results in the followings theorem.

Lemma 1.1 *Let $G = K_n \odot H$ be a corona graph of K_n and graph $H \neq C_m, W_m$, for $n \geq 4$, the lower bound of the r-dynamic chromatic number is:*

$$\chi_r(G) \geq \begin{cases} \chi_r(K_n), & 1 \leq r \leq n-1 \\ r+1, & n \leq r \leq n+|V(H)|-1 \\ n+|V(H)|, & r \geq n+|V(H)| \end{cases}$$

Proof. Based defined of corona graph, $V(K_n \odot H) = V(K_n) \cup \bigcup_{i=1}^{n} V(H_i)$ and the maximum degree of $G = K_n \odot H$ is $\Delta(K_n \odot H) = \Delta(K_n) + |V(H)| = (n-1) + |V(H)|$. For $1 \leq r \leq n-1$, based Observation 1.1 and chosen $r = n-1$, we get $\chi_r(G = K_n \odot H) \geq \min\{\ r, \Delta(G)\} + 1 = \min\{\ n-1, n-1+|V(H)|\} + 1 = (n-1)+1 = n$. From Observation 1.2, $\chi(K_n) = n$ so proven $\chi_r(G = K_n \odot H) \geq \chi_r(K_n)$ for $1 \leq r \leq m$. For $r \geq n+|V(H)|$, based Observation 1.1 and chosen $r = n+|V(H)|$, we get $\chi_r(G = K_n \odot H) \geq \min\{\ r, \Delta(G)\} + 1 = \min\{\ n+|V(H)|, n+|V(H)|-1\} + 1 = (n+|V(H)|-1)+1 = n+|V(H)|$. It concludes the proof. □

Theorem 1.1 *Let $G = K_n \odot S_m$ be a corona graph of K_n and S_m, for $n \geq 4, m \geq 3$, the r-dynamic chromatic number is:*

$$\chi_r(G) = \begin{cases} n, & 1 \leq r \leq n-1 \\ r+1, & n \leq r \leq m+n \\ m+n+1, & r \geq\ m+n+1 \end{cases}$$

Proof. The graph $G = K_n \odot S_m$ is a connected graph with vertex set $V(K_n \odot S_m) = \{x_i; 1 \leq i \leq n\} \cup \{y_i; 1 \leq i \leq n\} \cup \{y_{ij}; 1 \leq i \leq n; 1 \leq j \leq m\}$. The order of this graph is $p = |V(K_n \odot S_m)| = mn + 2n$. Thus the maximum degree of graph $G = K_n \odot S_m$ is $\Delta(K_n \odot S_m) = m+n$ and the minimum degree of graph $G = K_n \odot S_m$ is $\delta(K_n \odot S_m) = 2$. To find the exact value of r-dynamic chromatic number of $G = K_n \odot S_m$, we define two cases, namely for $\chi_{1 \leq r \leq n-1}(K_n \odot S_m)$ and $\chi_{r \geq m+n+1}(K_n \odot S_m)$.
Case 1. For $1 \leq r \leq n-1$, based on Lemma 1.1 and Observation 1.2, $\chi_r(K_n) = n$ then we have the lower bound of the $r-$dynamic chromatic number of $G = K_n \odot S_m$ is $\chi_{1 \leq r \leq n-1}(K_n \odot S_m) \geq \chi_r(K_n) = n$. We will show that the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot S_m$ is $\chi_{1 \leq r \leq n-1}(K_n \odot S_m) \leq \chi_r(K_n)$, define a map of the r-dynamic coloring of $G = K_n \odot S_m$ is $c_1 : V(K_n \odot S_m) \rightarrow \{1, 2, \ldots, k\}$ where $n \geq 4, m \geq 3$, by the following:

$$c_1(x_i) = i,\ 1 \leq i \leq n$$

$$c_1(y_i) = \begin{cases} 1, & \text{for } i = n \\ i+1, & \text{for } 1 \leq i \leq n-1 \end{cases}$$

$$c_1(y_{ij}) = \begin{cases} i+2, & \text{for } j \equiv 1(mod\ 3), 1 \le i \le n-2 \\ 1, & \text{for } j \equiv 1(mod\ 3),\ i = n-1 \\ 2, & \text{for } j \equiv 1(mod\ 3),\ i = n \end{cases}$$

$$c_1(y_{ij}) = \begin{cases} i+3, & \text{for } j \equiv 2(mod\ 3), 1 \le i \le n-3 \\ 1, & \text{for } j \equiv 2(mod\ 3), i = n-2 \\ 2, & \text{for } j \equiv 2(mod\ 3), i = n-1 \\ 3, & \text{for } j \equiv 2(mod\ 3), i = n \end{cases}$$

$$c_1(y_{ij}) = \begin{cases} i-1, & \text{for } j \equiv 0(mod\ 3), 2 \le i \le n \\ n, & \text{for } j \equiv 0(mod\ 3), i = 1 \end{cases}$$

It easy to see that c_1 is a map $c_1 : V(K_n \odot S_m) \rightarrow \{1, 2, \cdots n\}$. It is clearly that c_1 gives the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot S_m$ is $\chi_{1 \le r \le n-1}(K_n \odot S_m) \le n$. It concludes that $r-$dynamic chromatic number of $G = K_n \odot S_m$ is $\chi_{1 \le r \le n-1}(K_n \odot S_m) = n$.
Case 2. For $r \ge m+n+1$, based on Lemma 1.1 with $|V(S_m)| = m+1$ then we have the lower bound of the $r-$dynamic chromatic number of $G = K_n \odot S_m$ is $\chi_{r \ge m+n+1}(K_n \odot S_m) \ge n + |V(S_m) = n + (m+1) = m+n+1$. We will show that the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot S_m$ is $\chi_{r \ge m+n+1}(K_n \odot S_m) \le m+n+1$, define a map of the r-dynamic coloring of $G = K_n \odot S_m$ is $c_2 : V(K_n \odot S_m) \rightarrow \{1, 2, \ldots, k\}$ where $n \ge 4, m \ge 3$, by the following:

$$c_2(x_i) = i, 1 \le i \le n$$
$$c_2(y_i) = n+i, 1 \le i \le n$$
$$c_2(x_{ij}) = n+j+1, 1 \le i \le n, 1 \le j \le m$$

It easy to see that c_2 is a map $c_2 : V(K_n \odot S_m) \rightarrow \{1, 2, \cdots n+m+1\}$. It is clearly that c_2 gives the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot S_m$ is $\chi_{r \ge m+n+1}(K_n \odot S_m) \le m+n+1$. It concludes that $r-$dynamic chromatic number of $G = K_n \odot S_m$ is $\chi_{r \ge m+n+1}(K_n \odot S_m) = m+n+1$. It concludes the proof. □

Theorem 1.2 *Let $G = K_n \odot F_m$ be a corona graph of K_n and F_m, for $n \ge 4, m \ge 3$, the r-dynamic chromatic number is:*

$$\chi_r(G) = \begin{cases} n, & \text{for } 1 \le r \le n-1 \\ r+1, & \text{for } n \le r \le m+n \\ m+n+1, & \text{for } r \ge m+n+1 \end{cases}$$

Proof. The graph $G = K_n \odot F_m$ is a connected graph with vertex set $V(K_n \odot F_m) = \{x_i; 1 \le i \le n\} \cup \{y_i; 1 \le i \le n\} \cup \{x_{ij}; 1 \le i \le n; 1 \le j \le m\}$. The order of this graph is $p = |V(K_n \odot F_m)| = mn + 2n$. Thus the maximum degree of graph $G = K_n \odot F_m$ is $\Delta(K_n \odot F_m) = m+n$ and the minimum degree of graph $G = K_n \odot F_m$ is $\delta(K_n \odot F_m) = 3$. To find the exact value of r-dynamic chromatic number of $G = K_n \odot F_m$, we define two cases, namely for $\chi_{1 \le r \le n-1}(K_n \odot F_m)$ and $\chi_{r \ge m+n+1}(K_n \odot F_m)$.
Case 1. For $1 \le r \le n-1$, based on Lemma 1.1 and Observation 1.2, $\chi_r(K_n) = n$ then we have the lower bound of the $r-$dynamic chromatic number of $G = K_n \odot F_m$ is $\chi_{1 \le r \le n-1}(K_n \odot F_m) \ge \chi_r(K_n) = n$. We will show that the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot F_m$ is $\chi_{1 \le r \le n-1}(K_n \odot F_m) \le \chi_r(K_n)$, define a map of the r-dynamic coloring of $G = K_n \odot F_m$ is $c_3 : V(K_n \odot F_m) \rightarrow \{1, 2, \ldots, k\}$ where $n \ge 4, m \ge 3$, by the following:

$$c_3(x_i) = i, 1 \le i \le n$$

$$c_3(y_i) = \begin{cases} 1, & \text{for } i = n \\ i+1, & \text{for } 1 \le i \le n-1 \end{cases}$$

$$c_3(y_{ij}) = \begin{cases} i+2, & \text{for } j \text{ odd}, 1 \le j \le m, 1 \le i \le n-2 \\ 1, & \text{for } j \text{ odd}, 1 \le j \le m, i = n-1 \\ 2, & \text{for } j \text{ odd}, 1 \le j \le m, i = n \\ i-1, & \text{for } j \text{ even}, 1 \le j \le m, 2 \le i \le n \\ n, & \text{for } j \text{ even}, 1 \le j \le m, i = 1 \end{cases}$$

It easy to see that c_3 is a map $c_3 : V(K_n \odot F_m) \to \{1, 2, \cdots n\}$. It is clearly that c_3 gives the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot F_m$ is $\chi_{1 \le r \le n-1}(K_n \odot F_m) \le n$. It concludes that $r-$dynamic chromatic number of $G = K_n \odot F_m$ is $\chi_{1 \le r \le n-1}(K_n \odot F_m) = n$.
Case 2. For $r \ge m + n + 1$, based on Lemma 1.1 with $|V(F_m)| = m + 1$ then we have the lower bound of the $r-$dynamic chromatic number of $G = K_n \odot F_m$ is $\chi_{r \ge m+n+1}(K_n \odot F_m) \ge n + |V(F_m)| = n + (m + 1)$. We will show that the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot F_m$ is $\chi_{r \ge m+n+1}(K_n \odot F_m) \le m + n + 1$, define a map of the r-dynamic coloring of $G = K_n \odot F_m$ is $c_4 : V(K_n \odot F_m) \to \{1, 2, \ldots, k\}$ where $n \ge 4, m \ge 3$, by the following:

$$c_4(x_i) = i, 1 \le i \le n$$
$$c_4(y_i) = n + m + 1, 1 \le i \le n$$
$$c_4(x_{ij}) = j + i, 1 \le i \le n, 1 \le j \le m$$

It easy to see that c_4 is a map $c_4 : V(K_n \odot F_m) \to \{1, 2, \cdots m + n + 1\}$. It is clearly that c_4 gives the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot F_m$ is $\chi_{r \ge m+n+1}(K_n \odot F_m) \le n+m+1$. It concludes that $r-$dynamic chromatic number of $G = K_n \odot F_m$ is $\chi_{r \ge m+n+1}(K_n \odot F_m) = m + n + 1$. It concludes the proof. □

Lemma 1.2 *Let $G = H \odot K_m$ be a corona graph of $H \ne C_n, W_n$ and graph K_m, for $m \ge 4$, the lower bound of the r-dynamic chromatic number is:*

$$\chi_r(G) \ge \begin{cases} \chi_r(K_m) + 1, & 1 \le r \le m \\ r + 1, & m + 1 \le r \le m + \Delta(H) \\ \Delta(H) + m + 1, & r \ge m + \Delta(H) + 1 \end{cases}$$

Proof. Based defined of corona graph, $V(H \odot K_m) = V(H) \cup \bigcup_{i=1}^{n} V((K_m)_i)$ and the maximum degree of $G = H \odot K_m$ is $\Delta(H \odot K_m) = \Delta(H) + |V(K_m)| = \Delta(H) + m$. For $1 \le r \le m$, based Observation 1.1 and choosen $r = m$, we get $\chi_r(G = H \odot K_m) \ge \min\{ r, \Delta(G)\} + 1 = \min\{ m, \Delta(H) + m\} + 1 = m + 1$. From Observation 1.2, $\chi(K_m) = m$ so proven $\chi_r(G = H \odot K_m) \ge \chi_r(K_m) + 1$ for $1 \le r \le m$. For $r \ge m + \Delta(H) + 1$, based Observation 1.1 and choosen $r = m+\Delta(H)+1$, we get $\chi_r(G = H \odot K_m) \ge \min\{ r, \Delta(G)\}+1 = \min\{ m + \Delta(H) + 1, \Delta(H) + m\} + 1 = m + \Delta(H) + 1$. It concludes the proof. □

Theorem 1.3 *Let $G = S_n \odot K_m$ be a corona graph of S_n and K_m, for $n \ge 3, m \ge 4$, the r-dynamic chromatic number is:*

$$\chi_r(G) = \begin{cases} r + 1, & n \le r \le m + n \\ m + n + 1, & r \ge \ m + n + 1 \end{cases}$$

Proof. The graph $g = S_n \odot K_m$ is a connected graph with vertex set $V(S_n \odot K_m) = \{x\} \cup \{x_i; 1 \le i \le n\} \cup \{y_j; 1 \le i \le m\} \cup \{x_{ij}; 1 \le i \le n; 1 \le j \le m\}$. The order of this graph is $p = |V(S_n \odot K_m)| = mn + m + n + 1$. Thus the maximum degree of graph $G = S_n \odot K_m$ is $\Delta(S_n \odot K_m) = m + n$ and the minimum degree of graph $G = S_n \odot K_m$ is $\delta(S_n \odot K_m) = m$.

To find the exact value of r-dynamic chromatic number of $G = S_n \odot K_m$, we define two cases, namely for $\chi_{1\leq r\leq m}(S_n \odot K_m)$ and $\chi_{r\geq m+n+1}(S_n \odot K_m)$.
Case 1. For $1 \leq r \leq n-1$, based on Lemma 1.2 and Observation 1.2, $\chi_r(K_m) = m$ then we have the lower bound of the $r-$dynamic chromatic number of $G = S_n \odot K_m$ is $\chi_{1\leq r\leq m}(S_n \odot K_m) \geq \chi_r(K_m)+1 = m+1$. We will show that the upper bound of the $r-$dynamic chromatic number of $G = S_n \odot K_m$ is $\chi_{1\leq r\leq m}(S_n \odot K_m) \leq m+1$, define a map of the r-dynamic coloring of $G = S_n \odot K_m$ is $c_5 : V(S_n \odot K_m) \to \{1, 2, \ldots, k\}$ where $n \geq 3, m \geq 4$, by the following:

$$c_5(x) = 1$$
$$c_5(x_i) = 2, 1 \leq i \leq n$$

$$c_5(x_{ij}) = \begin{cases} 1, & \text{for } j = 1, 1 \leq i \leq n \\ 1+j, & \text{for } 2 \leq j \leq m, 1 \leq i \leq n \end{cases}$$

$$c_5(y_j) = \begin{cases} 2, & \text{for } j = 1 \\ 1+j, & \text{for } 2 \leq j \leq m \end{cases}$$

It easy to see that c_5 is a map $c_5 : V(S_n \odot K_m) \to \{1, 2, \cdots m+1\}$. It is clearly that c_5 gives the upper bound of the $r-$dynamic chromatic number of $G = S_n \odot K_m$ is $\chi_{1\leq r\leq m}(S_n \odot K_m) \leq m+1$. It concludes that $r-$dynamic chromatic number of $G = S_n \odot K_m$ is $\chi_{1\leq r\leq m}(S_n \odot K_m) = m+1$.
Case 2. For $r \geq r \geq n+m+1$, based on Lemma 1.2 with $\Delta(S_n) = n$ then we have the lower bound of the $r-$dynamic chromatic number of $G = S_n \odot K_m$ is $\chi_{r\geq n+m+1}(S_n \odot K_m) \geq \Delta(S_n) + m + 1 = n + m + 1$. We will show that the upper bound of the $r-$dynamic chromatic number of $G = S_n \odot K_m$ is $\chi_{r\geq r\geq n+m+1}(S_n \odot K_m) \leq m+n+1$, define a map of the r-dynamic coloring of $G = S_n \odot K_m$ is $c_6 : V(S_n \odot K_m) \to \{1, 2, \ldots, k\}$ where $n \geq 3, m \geq 4$, by the following:

$$c_6(x) = 1$$
$$c_6(x_i) = 1+i, 1 \leq i \leq n$$
$$c_6(x_{ij}) = 1+n+j, 1 \leq i \leq n, 1 \leq j \leq m$$
$$c_6(y_j) = 1+n+j, 1 \leq j \leq m$$

It easy to see that c_6 is a map $c_6 : V(S_n \odot K_m) \to \{1, 2, \cdots m+n+1\}$. It is clearly that c_6 gives the upper bound of the $r-$dynamic chromatic number of $G = S_n \odot K_m$ is $\chi_{r\geq m+n+1}(S_n \odot K_m) \leq m+n+1$. It concludes that $r-$dynamic chromatic number of $G = S_n \odot K_m$ is $\chi_{r\geq m+n+1}(S_n \odot K_m) = m+n+1$. It concludes the proof. □

Theorem 1.4 *Let $G = F_n \odot K_m$ be a corona graph of F_n and K_m, for $n \geq 3, m \geq 4$, the r-dynamic chromatic number is:*

$$\chi_r(G) = \begin{cases} m+1, & 1 \leq r \leq m \\ r+1, & n \leq r \leq m+n \\ m+n+1, & r \geq m+n+1 \end{cases}$$

Proof. The graph $G = F_n \odot K_m$ is a connected graph with vertex set $V(S_n \odot K_m) = \{x\} \cup \{x_i; 1 \leq i \leq n\} \cup \{y_j; 1 \leq i \leq m\} \cup \{x_{ij}; 1 \leq i \leq n; 1 \leq j \leq m\}$. The order of this graph is $p = |V(F_n \odot K_m)| = mn+m+n+1$. Thus the maximum degree of graph $G = F_n \odot K_m$ is $\Delta(F_n \odot K_m) = m+n$ and the minimum degree of graph $G = F_n \odot K_m$ is $\delta(F_n \odot K_m) = m$. To find the exact value of r-dynamic chromatic number of $G = F_n \odot K_m$, we define two cases, namely for $\chi_{1\leq r\leq m}(F_n \odot K_m)$ and $\chi_{r\geq m+n+1}(F_n \odot K_m)$.
Case 1. For $1 \leq r \leq m$, based on Lemma 1.2 with $\chi_r(K_m) = m$ then we have the lower bound of the $r-$dynamic chromatic number of $F_n \odot K_m$ is $\chi_{1\leq r\leq m}(F_n \odot K_m) \geq \chi_r(K_m) + 1 = m+1$.

We will show that the upper bound of the $r-$dynamic chromatic number of $G = F_n \odot K_m$ is $\chi_{1\le r\le m}(F_n \odot K_m) \le m+1$, define a map of the r-dynamic coloring of $G = F_n \odot K_m$ is $c_7 : V(F_n \odot K_m) \to \{1, 2, \ldots, k\}$ where $n \ge 3, m \ge 4$, by the following:

$$c_7(x) = 1$$

$$c_7(x_i) = \begin{cases} 2, & \text{for } i \text{ odd}, 1 \le i \le n \\ 3, & \text{for } i \text{ even}, 1 \le i \le n \end{cases}$$

$$c_7(x_i) = \begin{cases} 1, & \text{for } j = 1, 1 \le i \le n \\ 2, & \text{for } j = 2, i \text{ even}, 1 \le i \le n \\ 3, & \text{for } j = 2, i \text{ odd}, 1 \le i \le n \\ 1+j, & \text{for } 3 \le j \le m, 1 \le i \le n \end{cases}$$

$$c_7(y_j) = j+1, 2 \le i \le m$$

It easy to see that c_7 is a map $c_7 : V(F_n \odot K_m) \to \{1, 2, \cdots m+1\}$. It is clearly that c_7 gives the upper bound of the $r-$dynamic chromatic number of $G = F_n \odot K_m$ is $\chi_{1\le r\le m}(F_n \odot K_m) \le m+1$. It concludes that $r-$dynamic chromatic number of $G = F_n \odot K_m$ is $\chi_{1\le r\le m}(F_n \odot K_m) = m+1$.

Case 2. For $r \ge m+n+1$, based on Lemma 1.2 with $\Delta(F_n) = n+1$ then we have the lower bound of the $r-$dynamic chromatic number of $G = F_n \odot K_m$ is $\chi_{r\ge m+n+1}(F_n \odot K_m) \ge \Delta(F_n) + m + 1 = n + m + 2$. We will show that the upper bound of the $r-$dynamic chromatic number of $G = F_n \odot K_m$ is $\chi_{r\ge m+n+1}(F_n \odot K_m) \le m+n+1$, define a map of the r-dynamic coloring of $G = F_n \odot K_m$ is $c_8 : V(F_n \odot K_m) \to \{1, 2, \ldots, k\}$ where $n \ge 3, m \ge 4$, by the following:

$$c_8(x) = 1$$
$$c_8(x_i) = 1+i, 1 \le i \le n$$
$$c_8(x_{ij}) = 1+n+j, 1 \le i \le n, 1 \le j \le m$$
$$c_8(y_j) = 1+n+j, 1 \le j \le m$$

It easy to see that c_8 is a map $c_8 : V(F_n \odot K_m) \to \{1, 2, \cdots m+n+1\}$. It is clearly that c_6 gives the upper bound of the $r-$dynamic chromatic number of $G = F_n \odot K_m$ is $\chi_{r\ge m+n+1}(F_n \odot K_m) \le m+n+1$. It concludes that $r-$dynamic chromatic number of $G = F_n \odot K_m$ is $\chi_{r\ge m+n+1}(F_n \odot K_m) = m+n+1$. It concludes the proof. □

Theorem 1.5 *Let $G = K_n \odot K_m$ be a corona graph of K_n and K_m, for $m, n \ge 4$, the r-dynamic chromatic number is:*

i. For $n \le m$

$$\chi_r(G) = \begin{cases} m+1, & 1 \le r \le m \\ r+1, & m+1 \le r \le m+n-1 \\ m+n, & r \ge m+n \end{cases}$$

ii. For $n > m$

$$\chi_r(G) = \begin{cases} n, & 1 \le r \le n-1 \\ r+1, & n \le r \le m+n-1 \\ m+n, & r \ge m+n \end{cases}$$

Proof. The graph $G = K_n \odot K_m$ is a connected graph with vertex set $V(K_n \odot K_m) = \{x_i; 1 \le i \le n\} \cup \{x_{ij}; 1 \le i \le n\ 1 \le j \le m\}$. The order of this graph is $p = |V(K_n \odot K_m)| = n(m+1)$. Thus the maximum degree of graph $G = K_n \odot K_m$ is $\Delta(K_n \odot K_m) = m+n-1$ and the minimum degree of graph $G = K_n \odot K_m$ is $\delta(K_n \odot K_m) = m$. To find the exact value of r-dynamic chromatic number of $K_n \odot K_m$, we define two cases, namely for $\chi_{1\le r\le m}(K_n \odot K_m)$ and $\chi_{r\ge m+n}(K_n \odot K_m)$.

Case 1. We take for $n \leq m$. For $1 \leq r \leq m$, based on Lemma 1.2 and Observation 1.2, $\chi_r(K_m) = m$ then we have the lower bound of the $r-$dynamic chromatic number of $G = K_n \odot K_m$ is $\chi_{1\leq r\leq m}(K_n \odot K_m) \geq \chi_r(K_m) + 1 = m + 1$. We will show that the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot K_m$ is $\chi_{1\leq r\leq m}(K_n \odot K_m) \leq m + 1$, define a map of the r-dynamic coloring of $G = K_n \odot K_m$ is $c_9 : V(K_n \odot K_m) \to \{1, 2, \ldots, k\}$ where $m, n \geq 4$, by the following:

$$c_9(x_i) = i, 1 \leq i \leq n$$

$$c_1(x_{ij}) = \begin{cases} i+1, & \text{for } j = 1, 1 \leq i \leq n-1 \\ 1, & \text{for } j = 1, i = n \\ j+1, & \text{for } 1 \leq i \leq n, 2 \leq j \leq m \end{cases}$$

It easy to see that c_9 is a map $c_9 : V(K_n \odot K_m) \to \{1, 2, \cdots m+1\}$. It is clearly that c_9 gives the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot K_m$ is $\chi_{1\leq r\leq m}(K_n \odot K_m) \leq m+1$. It concludes that $r-$dynamic chromatic number of $G = K_n \odot K_m$ is $\chi_{1\leq r\leq m}(K_n \odot K_m) = m+1$.
Case 2. For $r \geq m+n$, based on Lemma 1.1 with $\Delta(K_n) = n-1$ then we have the lower bound of the $r-$dynamic chromatic number of $G = K_n \odot K_m$ is $\chi_{r\geq m+n}(K_n \odot K_m) \geq \Delta(K_n) + m + 1 = (n-1) + m + 1 = m + n$. We will show that the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot K_m$ is $\chi_{r\geq m+n}(K_n \odot K_m) \leq m+n$, define a map of the r-dynamic coloring of $G = K_n \odot K_m$ is $c_{10} : V(K_n \odot K_m) \to \{1, 2, \ldots, k\}$ where $m, n \geq 4$, by the following:

$$c_{10}(x_i) = i, 1 \leq i \leq n$$
$$c_{10}(x_{ij}) = i + j, 1 \leq i \leq n, 1 \leq j \leq m$$

It easy to see that c_{10} is a map $c_{10} : V(K_n \odot K_m) \to \{1, 2, \cdots n + m\}$. It is clearly that c_{10} gives the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot K_m$ is $\chi_{r\geq m+n}(K_n \odot K_m) \leq m + n$. It concludes that $r-$dynamic chromatic number of $G = K_n \odot K_m$ is $\chi_{r\geq m+n}(K_n \odot K_m) = m + n$. It concludes the proof. □

Conclusion

We have found the lower bound of $\chi_r(K_n \odot H)$ and $\chi_r(H \odot K_m)$ where $H \neq C_m, W_m$. We also find the exact value some r-dynamic chromatic number of corronation by complete graphs, namely $\chi_r(K_n \odot S_m), \chi_r(K_n \odot F_m), \chi_r(S_n \odot K_m), \chi_r(F_n \odot K_m)$ and $\chi_r(K_n \odot K_m)$. We got $\chi_r(K_n \odot S_m) = \chi_r(K_n \odot F_m)$ and $\chi_r(S_n \odot K_m) = \chi_r(F_n \odot K_m)$. For the characterization of the lower bound of $\chi_r(G \odot H)$ for any connected graphs G and H, we have not found any result yet, thus we propose the following open problem.

Open Problem 1.1 *Given the any connected graphs G and H. Determine the sharp lower bound of $\chi_r(G \odot H)$.*

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On the r-dynamic chromatic number of the corronation by complete graph

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Abstract. In this paper we will study the r-dynamic chromatic number of the coronation by complete graph. A proper k-coloring of graph G such that the neighbors of any vertex v receive at least $\min\{r, d(v)\}$ different colors. The r-dynamic chromatic number, $\chi_r(G)$ is the minimum k such that graph G has an r-dynamic k-coloring. We will obtain lower bound of the r-dynamic chromatic number of $\chi_r(K_n \odot H)$, and $\chi_r(H \odot K_m)$ We also study the exact value of the r-dynamic chromatic number of $\chi_r(K_n \odot S_m), \chi_r(K_n \odot F_m), \chi_r(S_n \odot K_m), \chi_r(F_n \odot K_m)$ and $\chi_r(K_n \odot K_m)$ for $m, n \geq 3$.

1. Introduction

Let G be graph. Graphs in this paper are simple and finite. We denote the vertex set of G by $V(G)$ and the number of vertices of G is called the order of G. Thus for graph G, $\Delta(G)$ and $\delta(G)$ denote the maximum degree and minimum degree. For every $v \in V(G)$, $d(v), N(v), c(v)$ denote the degree of v, the neighbor set of v and the color of v. A proper vertex coloring of G by k colors is a function $c : V(G) \to \{1, 2, \cdots, k\}$ with this property: if $u, v \in V(G)$ are adjancent, then $c(u)$ and $c(v)$ are different. The r-dynamic coloring of a graph G, introduced by Montgomery [8] defined a proper k-coloring of graph G such that the neighbors of any vertex v receive at least $\min\{r, d(v)\}$ different colors. The r-dynamic chromatic number, $\chi_r(G)$ is the minimum k such that graph G has an r-dynamic k-coloring. The r-dynamic chromatic number has been studied by several authors, for instance in [1], [2], [3], [4], [6], [7], [9], [10], [11] and [12]. The following observations are useful for our study, proposed by Montgomery [8].

Observation 1.1 *Let $\Delta(G)$ be the maximum degree of graph G. It holds* $\chi_r(G) \geq \min\{\Delta(G), r\} + 1$.

Observation 1.2 *Let $G = K_n$ then $\chi_r(G) = n$.*

Observation 1.3 *Let $G = G_1 \odot G_2$ be corona graph, then we have $\delta(G) = \delta(G_2) + 1$ and $\Delta(G) = \Delta(G_1) + |V(G_2)|$, where $\Delta(G)$ is maximum degree of G and $\delta(G)$ is minimum degree of G.*

Furmanczyk [5], denoted by $G \odot H$, is a connected graph obtained by taking a number of vertices $|V(G)|$ copy of H, and making the i^{th} of $V(G)$ adjacent to every vertex of the i^{th} copy of $V(H)$. There have been many results already found, Pathinathan, et.al. in [11] find the b-chromatic number on corona graph of subdivision vertex path with path, corona graph of any graph with path, cycle, and complete graph. Ramya in [12] discuss about the acyclic coloring of corona of $C_n \odot K_{1,3}, P_n \odot K_2$ and star coloring of corona of $P_n \odot K_2$.

The Results

We will obtain lower bound of the r-dynamic chromatic number of $\chi_r(K_n \odot H)$, and $\chi_r(H \odot K_m)$. We alsa find the exact value of the r-dynamic chromatic number of $\chi_r(K_n \odot S_m), \chi_r(K_n \odot F_m), \chi_r(S_n \odot K_m), \chi_r(F_n \odot K_m)$ and $\chi_r(K_n \odot K_m)$ for $m, n \geq 3$.There are seven theorems found in this study. We will show the complete results in the followings theorem.

Lemma 1.1 *Let $G = K_n \odot H$ be a corona graph of K_n and graph $H \neq C_m, W_m$, for $n \geq 4$, the lower bound of the r-dynamic chromatic number is:*

$$\chi_r(G) \geq \begin{cases} \chi_r(K_n), & 1 \leq r \leq n-1 \\ r+1, & n \leq r \leq n+|V(H)|-1 \\ n+|V(H)|, & r \geq n+|V(H)| \end{cases}$$

Proof. Based defined of corona graph, $V(K_n \odot H) = V(K_n) \cup \bigcup_{i=1}^{n} V(H_i)$ and the maximum degree of $G = K_n \odot H$ is $\Delta(K_n \odot H) = \Delta(K_n) + |V(H)| = (n-1) + |V(H)|$. For $1 \leq r \leq n-1$, based Observation 1.1 and chosen $r = n-1$, we get $\chi_r(G = K_n \odot H) \geq \min\{\ r, \Delta(G)\} + 1 = \min\{\ n-1, n-1+|V(H)|\} + 1 = (n-1)+1 = n$. From Observation 1.2, $\chi(K_n) = n$ so proven $\chi_r(G = K_n \odot H) \geq \chi_r(K_n)$ for $1 \leq r \leq m$. For $r \geq n+|V(H)|$, based Observation 1.1 and chosen $r = n+|V(H)|$, we get $\chi_r(G = K_n \odot H) \geq \min\{\ r, \Delta(G)\} + 1 = \min\{\ n+|V(H)|, n+|V(H)|-1\} + 1 = (n+|V(H)|-1)+1 = n+|V(H)|$. It concludes the proof. □

Theorem 1.1 *Let $G = K_n \odot S_m$ be a corona graph of K_n and S_m, for $n \geq 4, m \geq 3$, the r-dynamic chromatic number is:*

$$\chi_r(G) = \begin{cases} n, & 1 \leq r \leq n-1 \\ r+1, & n \leq r \leq m+n \\ m+n+1, & r \geq \ m+n+1 \end{cases}$$

Proof. The graph $G = K_n \odot S_m$ is a connected graph with vertex set $V(K_n \odot S_m) = \{x_i; 1 \leq i \leq n\} \cup \{y_i; 1 \leq i \leq n\} \cup \{z_{ij}; 1 \leq i \leq n; 1 \leq j \leq m\}$. The order of this graph is $p = |V(K_n \odot S_m)| = mn + 2n$. Thus the maximum degree of graph $G = K_n \odot S_m$ is $\Delta(K_n \odot S_m) = m+n$ and the minimum degree of graph $G = K_n \odot S_m$ is $\delta(K_n \odot S_m) = 2$. To find the exact value of r-dynamic chromatic number of $G = K_n \odot S_m$, we define two cases, namely for $\chi_{1 \leq r \leq n-1}(K_n \odot S_m)$ and $\chi_{r \geq m+n+1}(K_n \odot S_m)$.

Case 1. For $1 \leq r \leq n-1$, based on Lemma 1.1 and Observation 1.2, $\chi_r(K_n) = n$ then we have the lower bound of the $r-$dynamic chromatic number of $G = K_n \odot S_m$ is $\chi_{1 \leq r \leq n-1}(K_n \odot S_m) \geq \chi_r(K_n) = n$. We will show that the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot S_m$ is $\chi_{1 \leq r \leq n-1}(K_n \odot S_m) \leq \chi_r(K_n)$, define a map of the r-dynamic coloring of $G = K_n \odot S_m$ is $c_1 : V(K_n \odot S_m) \rightarrow \{1, 2, \ldots, k\}$ where $n \geq 4, m \geq 3$, by the following:

$$c_1(x_i) = i, \ 1 \leq i \leq n$$

$$c_1(y_i) = \begin{cases} 1, & \text{for } i = n \\ i+1, & \text{for } 1 \leq i \leq n-1 \end{cases}$$

$$c_1(y_{ij}) = \begin{cases} i+2, & \text{for } j \equiv 1(mod\ 3), 1 \le i \le n-2 \\ 1, & \text{for } j \equiv 1(mod\ 3),\ i = n-1 \\ 2, & \text{for } j \equiv 1(mod\ 3),\ i = n \end{cases}$$

$$c_1(y_{ij}) = \begin{cases} i+3, & \text{for } j \equiv 2(mod\ 3), 1 \le i \le n-3 \\ 1, & \text{for } j \equiv 2(mod\ 3), i = n-2 \\ 2, & \text{for } j \equiv 2(mod\ 3), i = n-1 \\ 3, & \text{for } j \equiv 2(mod\ 3), i = n \end{cases}$$

$$c_1(y_{ij}) = \begin{cases} i-1, & \text{for } j \equiv 0(mod\ 3), 2 \le i \le n \\ n, & \text{for } j \equiv 0(mod\ 3), i = 1 \end{cases}$$

It easy to see that c_1 is a map $c_1 : V(K_n \odot S_m) \to \{1, 2, \cdots n\}$. It is clearly that c_1 gives the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot S_m$ is $\chi_{1 \le r \le n-1}(K_n \odot S_m) \le n$. It concludes that $r-$dynamic chromatic number of $G = K_n \odot S_m$ is $\chi_{1 \le r \le n-1}(K_n \odot S_m) = n$.
Case 2. For $r \ge m+n+1$, based on Lemma 1.1 with $|V(S_m)| = m+1$ then we have the lower bound of the $r-$dynamic chromatic number of $G = K_n \odot S_m$ is $\chi_{r \ge m+n+1}(K_n \odot S_m) \ge n + |V(S_m) = n + (m+1) = m+n+1$. We will show that the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot S_m$ is $\chi_{r \ge m+n+1}(K_n \odot S_m) \le m+n+1$, define a map of the r-dynamic coloring of $G = K_n \odot S_m$ is $c_2 : V(K_n \odot S_m) \to \{1, 2, \ldots, k\}$ where $n \ge 4, m \ge 3$, by the following:

$$c_2(x_i) = i, 1 \le i \le n$$
$$c_2(y_i) = n + i, 1 \le i \le n$$
$$c_2(x_{ij}) = n + j + 1, 1 \le i \le n, 1 \le j \le m$$

It easy to see that c_2 is a map $c_2 : V(K_n \odot S_m) \to \{1, 2, \cdots n+m+1\}$. It is clearly that c_2 gives the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot S_m$ is $\chi_{r \ge m+n+1}(K_n \odot S_m) \le m+n+1$. It concludes that $r-$dynamic chromatic number of $G = K_n \odot S_m$ is $\chi_{r \ge m+n+1}(K_n \odot S_m) = m+n+1$. It concludes the proof. □

Theorem 1.2 *Let $G = K_n \odot F_m$ be a corona graph of K_n and F_m, for $n \ge 4, m \ge 3$, the r-dynamic chromatic number is:*

$$\chi_r(G) = \begin{cases} n, & \text{for } 1 \le r \le n-1 \\ r+1, & \text{for } n \le r \le m+n \\ m+n+1, & \text{for } r \ge\ m+n+1 \end{cases}$$

Proof. The graph $G = K_n \odot F_m$ is a connected graph with vertex set $V(K_n \odot F_m) = \{x_i; 1 \le i \le n\} \cup \{y_i; 1 \le i \le n\} \cup \{x_{ij}; 1 \le i \le n; 1 \le j \le m\}$. The order of this graph is $p = |V(K_n \odot F_m)| = mn + 2n$. Thus the maximum degree of graph $G = K_n \odot F_m$ is $\Delta(K_n \odot F_m) = m + n$ and the minimum degree of graph $G = K_n \odot F_m$ is $\delta(K_n \odot F_m) = 3$. To find the exact value of r-dynamic chromatic number of $G = K_n \odot F_m$, we define two cases, namely for $\chi_{1 \le r \le n-1}(K_n \odot F_m)$ and $\chi_{r \ge m+n+1}(K_n \odot F_m)$.
Case 1. For $1 \le r \le n-1$, based on Lemma 1.1 and Observation 1.2, $\chi_r(K_n) = n$ then we have the lower bound of the $r-$dynamic chromatic number of $G = K_n \odot F_m$ is $\chi_{1 \le r \le n-1}(K_n \odot F_m) \ge \chi_r(K_n) = n$. We will show that the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot F_m$ is $\chi_{1 \le r \le n-1}(K_n \odot F_m) \le \chi_r(K_n)$, define a map of the r-dynamic coloring of $G = K_n \odot F_m$ is $c_3 : V(K_n \odot F_m) \to \{1, 2, \ldots, k\}$ where $n \ge 4, m \ge 3$, by the following:

$$c_3(x_i) = i, 1 \le i \le n$$

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$$c_3(y_i) = \begin{cases} 1, & \text{for } i = n \\ i+1, & \text{for } 1 \le i \le n-1 \end{cases}$$

$$c_3(y_{ij}) = \begin{cases} i+2, & \text{for } j \text{ odd}, 1 \le j \le m, 1 \le i \le n-2 \\ 1, & \text{for } j \text{ odd}, 1 \le j \le m, i = n-1 \\ 2, & \text{for } j \text{ odd}, 1 \le j \le m, i = n \\ i-1, & \text{for } j \text{ even}, 1 \le j \le m, 2 \le i \le n \\ n, & \text{for } j \text{ even}, 1 \le j \le m, i = 1 \end{cases}$$

It easy to see that c_3 is a map $c_3 : V(K_n \odot F_m) \to \{1, 2, \cdots n\}$. It is clearly that c_3 gives the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot F_m$ is $\chi_{1 \le r \le n-1}(K_n \odot F_m) \le n$. It concludes that $r-$dynamic chromatic number of $G = K_n \odot F_m$ is $\chi_{1 \le r \le n-1}(K_n \odot F_m) = n$.

Case 2. For $r \ge m + n + 1$, based on Lemma 1.1 with $|V(F_m)| = m + 1$ then we have the lower bound of the $r-$dynamic chromatic number of $G = K_n \odot F_m$ is $\chi_{r \ge m+n+1}(K_n \odot F_m) \ge n + |V(F_m)| = n + (m+1)$. We will show that the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot F_m$ is $\chi_{r \ge m+n+1}(K_n \odot F_m) \le m + n + 1$, define a map of the r-dynamic coloring of $G = K_n \odot F_m$ is $c_4 : V(K_n \odot F_m) \to \{1, 2, \ldots, k\}$ where $n \ge 4, m \ge 3$, by the following:

$$c_4(x) = i, 1 \le i \le n$$
$$c_4(y_i) = n + m + 1, 1 \le i \le n$$
$$c_4(x_{ij}) = j + i, 1 \le i \le n, 1 \le j \le m$$

It easy to see that c_4 is a map $c_4 : V(K_n \odot F_m) \to \{1, 2, \cdots m + n + 1\}$. It is clearly that c_4 gives the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot F_m$ is $\chi_{r \ge m+n+1}(K_n \odot F_m) \le n+m+1$. It concludes that $r-$dynamic chromatic number of $G = K_n \odot F_m$ is $\chi_{r \ge m+n+1}(K_n \odot F_m) = m + n + 1$. It concludes the proof. □

Lemma 1.2 *Let $G = H \odot K_m$ be a corona graph of $H \ne C_n, W_n$ and graph K_m, for $m \ge 4$, the lower bound of the r-dynamic chromatic number is:*

$$\chi_r(G) \ge \begin{cases} \chi_r(K_m) + 1, & 1 \le r \le m \\ r + 1, & m + 1 \le r \le m + \Delta(H) \\ \Delta(H) + m + 1, & r \ge m + \Delta(H) + 1 \end{cases}$$

Proof. Based defined of corona graph, $V(H \odot K_m) = V(H) \cup \bigcup_{i=1}^{n} V((K_m)_i)$ and the maximum degree of $G = H \odot K_m$ is $\Delta(H \odot K_m) = \Delta(H) + |V(K_m)| = \Delta(H) + m$. For $1 \le r \le m$, based Observation 1.1 and choosen $r = m$, we get $\chi_r(G = H \odot K_m) \ge \min\{ r, \Delta(G)\} + 1 = \min\{ m, \Delta(H) + m\} + 1 = m + 1$. From Observation 1.2 $\chi(K_m) = m$ so proven $\chi_r(G = H \odot K_m) \ge \chi_r(K_m) + 1$ for $1 \le r \le m$. For $r \ge m + \Delta(H) + 1$, based Observation 1.1 and choosen $r = m + \Delta(H) + 1$, we get $\chi_r(G = H \odot K_m) \ge \min\{ r, \Delta(G)\} + 1 = \min\{ m + \Delta(H) + 1, \Delta(H) + m\} + 1 = m + \Delta(H) + 1$. It concludes the proof. □

Theorem 1.3 *Let $G = S_n \odot K_m$ be a corona graph of S_n and K_m, for $n \ge 3, m \ge 4$, the r-dynamic chromatic number is:*

$$\chi_r(G) = \begin{cases} r + 1, & n \le r \le m + n \\ m + n + 1, & r \ge m + n + 1 \end{cases}$$

Proof. The graph $g = S_n \odot K_m$ is a connected graph with vertex set $V(S_n \odot K_m) = \{x\} \cup \{x_i; 1 \le i \le n\} \cup \{y_j; 1 \le i \le m\} \cup \{x_{ij}; 1 \le i \le n; 1 \le j \le m\}$. The order of this graph is $p = |V(S_n \odot K_m)| = mn + m + n + 1$. Thus the maximum degree of graph $G = S_n \odot K_m$ is $\Delta(S_n \odot K_m) = m + n$ and the minimum degree of graph $G = S_n \odot K_m$ is $\delta(S_n \odot K_m) = m$.

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To find the exact value of r-dynamic chromatic number of $G = S_n \odot K_m$, we define two cases, namely for $\chi_{1\le r\le m}(S_n \odot K_m)$ and $\chi_{r\ge m+n+1}(S_n \odot K_m)$.

Case 1. For $1 \le r \le n-1$, based on Lemma 1.2 and Observation 1.2, $\chi_r(K_m) = m$ then we have the lower bound of the $r-$dynamic chromatic number of $G = S_n \odot K_m$ is $\chi_{1\le r\le m}(S_n \odot K_m) \ge \chi_r(K_m)+1 = m+1$. We will show that the upper bound of the $r-$dynamic chromatic number of $G = S_n \odot K_m$ is $\chi_{1\le r\le m}(S_n \odot K_m) \le m+1$, define a map of the r-dynamic coloring of $G = S_n \odot K_m$ is $c_5 : V(S_n \odot K_m) \to \{1, 2, \ldots, k\}$ where $n \ge 3, m \ge 4$, by the following:

$$c_5(x) = 1$$
$$c_5(x_i) = 2, 1 \le i \le n$$
$$c_5(x_{ij}) = \begin{cases} 1, & \text{for } j = 1, 1 \le i \le n \\ 1+j, & \text{for } 2 \le j \le m, 1 \le i \le n \end{cases}$$
$$c_5(y_j) = \begin{cases} 2, & \text{for } j = 1 \\ 1+j, & \text{for } 2 \le j \le m \end{cases}$$

It easy to see that c_5 is a map $c_5 : V(S_n \odot K_m) \to \{1, 2, \cdots m+1\}$. It is clearly that c_5 gives the upper bound of the $r-$dynamic chromatic number of $G = S_n \odot K_m$ is $\chi_{1\le r\le m}(S_n \odot K_m) \le m+1$. It concludes that $r-$dynamic chromatic number of $G = S_n \odot K_m$ is $\chi_{1\le r\le m}(S_n \odot K_m) = m+1$.

Case 2. For $r \ge r \ge n+m+1$, based on Lemma 1.2 with $\Delta(S_n) = n$ then we have the lower bound of the $r-$dynamic chromatic number of $G = S_n \odot K_m$ is $\chi_{r\ge n+m+1}(S_n \odot K_m) \ge \Delta(S_n) + m + 1 = n + m + 1$. We will show that the upper bound of the $r-$dynamic chromatic number of $G = S_n \odot K_m$ is $\chi_{r\ge r\ge n+m+1}(S_n \odot K_m) \le m + n + 1$, define a map of the r-dynamic coloring of $G = S_n \odot K_m$ is $c_6 : V(S_n \odot K_m) \to \{1, 2, \ldots, k\}$ where $n \ge 3, m \ge 4$, by the following:

$$c_6(x) = 1$$
$$c_6(x_i) = 1 + i, 1 \le i \le n$$
$$c_6(x_{ij}) = 1 + n + j, 1 \le i \le n, 1 \le j \le m$$
$$c_6(y_j) = 1 + n + j, 1 \le j \le m$$

It easy to see that c_6 is a map $c_6 : V(S_n \odot K_m) \to \{1, 2, \cdots m + n + 1\}$. It is clearly that c_6 gives the upper bound of the $r-$dynamic chromatic number of $G = S_n \odot K_m$ is $\chi_{r\ge m+n+1}(S_n \odot K_m) \le m+n+1$. It concludes that $r-$dynamic chromatic number of $G = S_n \odot K_m$ is $\chi_{r\ge m+n+1}(S_n \odot K_m) = m + n + 1$. It concludes the proof. □

Theorem 1.4 *Let $G = F_n \odot K_m$ be a corona graph of F_n and K_m, for $n \ge 3, m \ge 4$, the r-dynamic chromatic number is:*

$$\chi_r(G) = \begin{cases} m+1, & 1 \le r \le m \\ r+1, & n \le r \le m+n \\ m+n+1, & r \ge m+n+1 \end{cases}$$

Proof. The graph $G = F_n \odot K_m$ is a connected graph with vertex set $V(S_n \odot K_m) = \{x\} \cup \{x_i; 1 \le i \le n\} \cup \{y_j; 1 \le i \le m\} \cup \{x_{ij}; 1 \le i \le n; 1 \le j \le m\}$. The order of this graph is $p = |V(F_n \odot K_m)| = mn + m + n + 1$. Thus the maximum degree of graph $G = F_n \odot K_m$ is $\Delta(F_n \odot K_m) = m + n$ and the minimum degree of graph $G = F_n \odot K_m$ is $\delta(F_n \odot K_m) = m$. To find the exact value of r-dynamic chromatic number of $G = F_n \odot K_m$, we define two cases, namely for $\chi_{1\le r\le m}(F_n \odot K_m)$ and $\chi_{r\ge m+n+1}(F_n \odot K_m)$.

Case 1. For $1 \le r \le m$, based on Lemma 1.2 with $\chi_r(K_m) = m$ then we have the lower bound of the $r-$dynamic chromatic number of $F_n \odot K_m$ is $\chi_{1\le r\le m}(F_n \odot K_m) \ge \chi_r(K_m) + 1 = m + 1$.

We will show that the upper bound of the $r-$dynamic chromatic number of $G = F_n \odot K_m$ is $\chi_{1 \le r \le m}(F_n \odot K_m) \le m+1$, define a map of the r-dynamic coloring of $G = F_n \odot K_m$ is $c_7 : V(F_n \odot K_m) \to \{1, 2, \dots, k\}$ where $n \ge 3, m \ge 4$, by the following:

$$c_7(x) = 1$$

$$c_7(x_i) = \begin{cases} 2, & \text{for } i \text{ odd}, 1 \le i \le n \\ 3, & \text{for } i \text{ even}, 1 \le i \le n \end{cases}$$

$$c_7(x_i) = \begin{cases} 1, & \text{for } j = 1, 1 \le i \le n \\ 2, & \text{for } j = 2, i \text{ even}, 1 \le i \le n \\ 3, & \text{for } j = 2, i \text{ odd}, 1 \le i \le n \\ 1+j, & \text{for } 3 \le j \le m, 1 \le i \le n \end{cases}$$

$$c_7(y_j) = j + 1, 2 \le i \le m$$

It easy to see that c_7 is a map $c_7 : V(F_n \odot K_m) \to \{1, 2, \cdots m+1\}$. It is clearly that c_7 gives the upper bound of the $r-$dynamic chromatic number of $G = F_n \odot K_m$ is $\chi_{1 \le r \le m}(F_n \odot K_m) \le m+1$. It concludes that $r-$dynamic chromatic number of $G = F_n \odot K_m$ is $\chi_{1 \le r \le m}(F_n \odot K_m) = m+1$.

Case 2. For $r \ge m+n+1$, based on Lemma 1.2 with $\Delta(F_n) = n+1$ then we have the lower bound of the $r-$dynamic chromatic number of $G = F_n \odot K_m$ is $\chi_{r \ge m+n+1}(F_n \odot K_m) \ge \Delta(F_n) + m + 1 = n + m + 2$. We will show that the upper bound of the $r-$dynamic chromatic number of $G = F_n \odot K_m$ is $\chi_{r \ge m+n+1}(F_n \odot K_m) \le m + n + 1$, define a map of the r-dynamic coloring of $G = F_n \odot K_m$ is $c_8 : V(F_n \odot K_m) \to \{1, 2, \dots, k\}$ where $n \ge 3, m \ge 4$, by the following:

$$c_8(x) = 1$$
$$c_8(x_i) = 1 + i, 1 \le i \le n$$
$$c_8(x_{ij}) = 1 + n + j, 1 \le i \le n, 1 \le j \le m$$
$$c_8(y_j) = 1 + n + j, 1 \le j \le m$$

It easy to see that c_8 is a map $c_8 : V(F_n \odot K_m) \to \{1, 2, \cdots m+n+1\}$. It is clearly that c_6 gives the upper bound of the $r-$dynamic chromatic number of $G = F_n \odot K_m$ is $\chi_{r \ge m+n+1}(F_n \odot K_m) \le m+n+1$. It concludes that $r-$dynamic chromatic number of $G = F_n \odot K_m$ is $\chi_{r \ge m+n+1}(F_n \odot K_m) = m+n+1$. It concludes the proof. □

Theorem 1.5 *Let $G = K_n \odot K_m$ be a corona graph of K_n and K_m, for $m, n \ge 4$, the r-dynamic chromatic number is:*

i. For $n \le m$

$$\chi_r(G) = \begin{cases} m+1, & 1 \le r \le m \\ r+1, & m+1 \le r \le m+n-1 \\ m+n, & r \ge m+n \end{cases}$$

ii. For $n > m$

$$\chi_r(G) = \begin{cases} n, & 1 \le r \le n-1 \\ r+1, & n \le r \le m+n-1 \\ m+n, & r \ge m+n \end{cases}$$

Proof. The graph $G = K_n \odot K_m$ is a connected graph with vertex set $V(K_n \odot K_m) = \{x_i; 1 \le i \le n\} \cup \{x_{ij}; 1 \le i \le n \; 1 \le j \le m\}$. The order of this graph is $p = |V(K_n \odot K_m)| = n(m+1)$. Thus the maximum degree of graph $G = K_n \odot K_m$ is $\Delta(K_n \odot K_m) = m + n - 1$ and the minimum degree of graph $G = K_n \odot K_m$ is $\delta(K_n \odot K_m) = m$. To find the exact value of r-dynamic chromatic number of $K_n \odot K_m$, we define two cases, namely for $\chi_{1 \le r \le m}(K_n \odot K_m)$ and $\chi_{r \ge m+n}(K_n \odot K_m)$.

Case 1. We take for $n \leq m$. For $1 \leq r \leq m$, based on Lemma 1.2 and Observation 1.2, $\chi_r(K_m) = m$ then we have the lower bound of the $r-$dynamic chromatic number of $G = K_n \odot K_m$ is $\chi_{1 \leq r \leq m}(K_n \odot K_m) \geq \chi_r(K_m) + 1 = m + 1$. We will show that the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot K_m$ is $\chi_{1 \leq r \leq m}(K_n \odot K_m) \leq m + 1$, define a map of the r-dynamic coloring of $G = K_n \odot K_m$ is $c_9 : V(K_n \odot K_m) \to \{1, 2, \ldots, k\}$ where $m, n \geq 4$, by the following:

$$c_9(x_i) = i, 1 \leq i \leq n$$

$$c_1(x_{ij}) = \begin{cases} i+1, & \text{for } j = 1, 1 \leq i \leq n-1 \\ 1, & \text{for } j = 1, i = n \\ j+1, & \text{for } 1 \leq i \leq n, 2 \leq j \leq m \end{cases}$$

It easy to see that c_9 is a map $c_9 : V(K_n \odot K_m) \to \{1, 2, \cdots m+1\}$. It is clearly that c_9 gives the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot K_m$ is $\chi_{1 \leq r \leq m}(K_n \odot K_m) \leq m+1$. It concludes that $r-$dynamic chromatic number of $G = K_n \odot K_m$ is $\chi_{1 \leq r \leq m}(K_n \odot K_m) = m+1$.

Case 2. For $r \geq m+n$, based on Lemma 1.1 with $\Delta(K_n) = n-1$ then we have the lower bound of the $r-$dynamic chromatic number of $G = K_n \odot K_m$ is $\chi_{r \geq m+n}(K_n \odot K_m) \geq \Delta(K_n) + m + 1 = (n-1) + m + 1 = m + n$. We will show that the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot K_m$ is $\chi_{r \geq m+n}(K_n \odot K_m) \leq m + n$, define a map of the r-dynamic coloring of $G = K_n \odot K_m$ is $c_{10} : V(K_n \odot K_m) \to \{1, 2, \ldots, k\}$ where $m, n \geq 4$, by the following:

$$c_{10}(x_i) = i, 1 \leq i \leq n$$
$$c_{10}(x_{ij}) = i + j, 1 \leq i \leq n, 1 \leq j \leq m$$

It easy to see that c_{10} is a map $c_{10} : V(K_n \odot K_m) \to \{1, 2, \cdots n + m\}$. It is clearly that c_{10} gives the upper bound of the $r-$dynamic chromatic number of $G = K_n \odot K_m$ is $\chi_{r \geq m+n}(K_n \odot K_m) \leq m + n$. It concludes that $r-$dynamic chromatic number of $G = K_n \odot K_m$ is $\chi_{r \geq m+n}(K_n \odot K_m) = m + n$. It concludes the proof. □

Conclusion

We have found the lower bound of $\chi_r(K_n \odot H)$ and $\chi_r(H \odot K_m)$ where $H \neq C_m, W_m$. We also find the exact value some r-dynamic chromatic number of corronation by complete graphs, namely $\chi_r(K_n \odot S_m), \chi_r(K_n \odot F_m), \chi_r(S_n \odot K_m), \chi_r(F_n \odot K_m)$ and $\chi_r(K_n \odot K_m)$. We got $\chi_r(K_n \odot S_m) = \chi_r(K_n \odot F_m)$ and $\chi_r(S_n \odot K_m) = \chi_r(F_n \odot K_m)$. For the characterization of the lower bound of $\chi_r(G \odot H)$ for any connected graphs G and H, we have not found any result yet, thus we propose the following open problem.

Open Problem 1.1 *Given the any connected graphs G and H. Determine the sharp lower bound of $\chi_r(G \odot H)$.*

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