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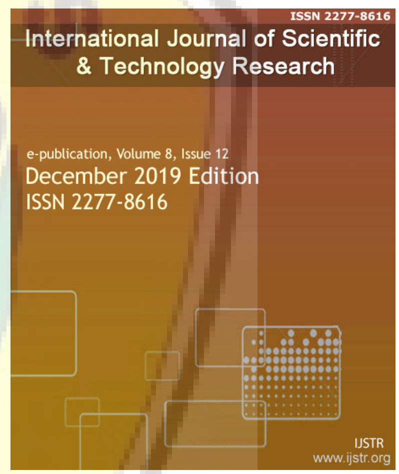

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Metric Chromatic Number Of Unicyclic Graphs

[Full Text]

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KEYWORDS
Metric coloring, metric chromatic number, unicyclic graphs

ABSTRACT
All graphs in this paper are nontrivial and connected graph. Let f: V (G)→{1,2,�,k} be a vertex coloring of a graph Gwhere two adjacent vertices may be colored the same color. Consider the color classes Π= {C_1,C_2,�,C_k}. For a vertex vof G, the representation color of vis the k-vector r(v|Π)= (d(v,,C_1),d(v,C_2),� ,d(v,C_k)), where d(v,C_i)=min{d(v,c);c∈C_i} . If r(u|Π)≠r(v|Π) for every two adjacent vertices uand vof G, then fis a metric coloring of G. The minimum kfor which Ghas a metric k-coloring is called the metric chromatic number of Gand is denoted by μ(G). The metric chromatic numbers of unicyclic graphs namely tadpole graphs, cycle with m-pendants, sun graphs, cycle with two pendants, subdivision of sun graphs.

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Metric Chromatic Number Of Unicyclic Graphs

R. Alfarisi, A.I. Kristiana, E.R.Albirri, R. Adawiyah, Dafik

Abstract: All graphs in this paper are nontrivial and connected graph. Let $f : V(G) \rightarrow \{1,2,\ldots,k\}$ be a vertex coloring of a graph $G$where two adjacent vertices may be colored the same color. Consider the color classes $\Pi = \{C_1, C_2, \ldots, C_k\}$. For a vertex $v$of G, the representation color of $v$is the k-vector $r(v|\Pi) = (d(v, C_1), d(v, C_2), \ldots, d(v, C_k))$, where $d(v, C_i) = \min\{d(v,c); c \in C_i\}$. If $r(u|\Pi) \neq r(v|\Pi)$ for every two adjacent vertices $u$and $v$of G, then $f$is a metric coloring of G. The minimum $k$for which Ghas a metric k-coloring is called the metric chromatic number of $G$and is denoted by $\mu(G)$. The metric chromatic numbers of unicyclic graphs namely tadpole graphs, cycle with m-pendants, sun graphs, cycle with two pendants, subdivision of sun graphs.

Index Terms: Metric coloring, metric chromatic number, unicyclic graphs

1 Introduction

Graphs in this paper are nontrivial and connected graph, for more detail definition of graph see [1]. Let $f : V(G) \rightarrow \{1,2,\ldots,k\}$ be a vertex coloring of a graph G where two adjacent vertices may be colored the same color. Consider the color classes $\pi = \{C_1, C_2, \ldots, C_k\}$.For a vertex $v$in$G$, representation color of v is the k-vector $r(v|\pi) = (d(v, C_1), d(v, C_2), \ldots, d(v, C_k))$,where $d(v, C_i) = \min\{d(v,c); c \in C_i\}$. If $r(u|\pi) \neq r(v|\pi)$ for every two adjacent vertices u and $v$in$G$, then f is a metric coloring of G. The minimum k for which G has a metric k-coloring is called the metric chromatic number of G and is denoted by $\mu(G)$.In recent years, there are some results of the metric chromatic number of some well-known graphs in Chartrand et.al [2] as follows.

Proposition 1.1 *A*nontrivial connected graph G has metric chromatic number 2if and only if G is bipartite.

Proposition 2.2 Let G be a connected graph. If $\chi(G) = 3$, then $\mu(G) = 3$.

Proposition 2.3 Let C_n be a cycle graph, then.

$$\mu(C_n) = \begin{cases} 2, if\ n\ is\ even \\ 3, if\ n\ is\ odd \end{cases}$$

2 Result

In this paper, we investigate the metric chromatic number of unicyclic graphs namely tadpole graphs, cycle with m-pendants, sun graphs, cycle with two pendants, subdivision of sun graphs. Furthermore, we construct the new lemma for any unicyclic graphs as follows.

Observation 2.1. For any two adjacent vertices $u, v \in V(T)$, then we have $d(u,x) \neq d(v,x)$ for $x \in V(T)$.

Observation 2.2. For any two adjacent vertices u', v' in cycle in unicyclic graph, then $\exists\ x \in V(U_n) \ni d(u',x) = d(v',x)$

Lemma 2.1 Consider unicyclic graph U_n for $n \geq 3$. Then

$$\mu(U_n) \geq \begin{cases} 2, if\ n_{c_n} is\ even \\ 3, if\ n_{c_n} is\ odd \end{cases}$$

Proof. Unicyclic graph U_n for $n \geq 3$ has only one cycle subgraph with n vertices. So that, the coloring in the cycle subgraph follows the coloring in the cycle graph. Based on Proposition 2.3. that $\mu(C_n) = 2$ for n is even and $\mu(C_n) = 3$ for n is odd. So, there are several condition in this proof as follows:

i. For any two adjacent vertices u, v not in cycle, based on Observation 2.1. that$d(u,x) \neq d(v,x)$ for $x \in V(T)$ such that $r(u|\Pi) \neq r(v|\Pi)$.

ii. For any two vertices u in cycle and v not in cycle (u, v not adjacent), it may be same representation.

iii. For any two adjacent vertices u in cycle and v not in cycle, based on Observation 2.1. that$d(u,x) \neq d(v,x)$ for $x \in V(T)$ such that $r(u|\Pi) \neq r(v|\Pi)$.

iv. For any two adjacent vertices u, v in cycle, based on Proposition 2.3. that we use the coloring in cycle.

Based on cases i) ,ii) ,iii), iv) that

$$\mu(U_n) \geq \begin{cases} 2, if\ n_{c_n} is\ even \\ 3, if\ n_{c_n} is\ odd \end{cases}$$

Tadpole graph is one of unicyclic graph which obtained by joining a cycle graph C_m to a path graph P_n with a bridge.

Theorem 2.1 Consider tadpole graph $T_{n,m}$ for $n \geq 3$ and $m \geq 1$. Then

$$\mu(T_{n,m}) = \begin{cases} 2, & \text{if n is even} \\ 3, & \text{if n is odd} \end{cases}$$

Proof. Tadpole graph only have one cycle with order n and one pendant path. Thus, This proof divided into two cases as follows.

Case 1. For n is even

Based on Lemma 2.1 that $\mu(T_{n,m}) \geq 2$. Furthermore, we prove that $\mu(T_{n,m}) \leq 2$. Let $f : V(G) \rightarrow \{1,2\}$ be a vertex coloring (two adjacent vertices may be colored same color). The construction of coloring in graph $T_{n,m}$ with the periodic label in cycle $(1,2,1,2,1,2,1,2,\ldots,1,2)$ and the periodic label in path (tail) $(2,1,2,1,2,1,2,1,\ldots)$ with two adjacent vertices in bridge which using color 1 in cycle and color 2 in tail or color 2 in cycle and color 1 in tail. For more detail the label color and the representation of vertices in tadpole graph $T_{n,m}$ respect to with class color $\Pi = \{C_1, C_2\}$ where $C_1 = \{x_i, y_j; i \text{ is odd and } j \text{ is even}\}$ and $C_2 = \{x_i, y_j; i \text{ is even and } j \text{ is odd}\}$ as follows.

$$f(v) = \begin{cases} 1 & , \text{if } v \in \{x_i, y_j; i \text{ is odd and } j \text{ is even} \\ 2 & , \text{if } v \in \{x_i, y_j; i \text{ is evenand } j \text{ is odd} \end{cases}$$

Based on the color label f in tadpole graph $T_{n,m}$. Thus, we have the representation as follows.

$$r(y_j|\Pi) = r(x_i|\Pi) = (0,1); for\ i \text{ is odd and } j \text{ is even}$$
$$r(y_j|\Pi) = r(x_i|\Pi) = (1,0); for\ i \text{ is evenand } j \text{ is odd}$$

It is clear that for every two adjacent vertices has distinct representation, we can see in $r(x_{2k-1}|\Pi) \neq r(x_{2k}|\Pi)$ or $r(y_{2k-1}|\Pi) \neq r(y_{2k}|\Pi)$for $k \in N$. Without the loss generality, we have $\mu(T_{n,m}) \leq 2$. Thus, $\mu(T_{n,m}) = 2$.

Case 2. For n is odd

Based on Lemma 2.1 that $\mu(T_{n,m}) \geq 3$. Furthermore, we prove that $\mu(T_{n,m}) \leq 3$. Let $f : V(G) \to \{1,2,3\}$ be a vertex coloring (two adjacent vertices may be colored same color). The construction of coloring in graph $T_{n,m}$ with the periodic label in cycle $(1,2,1,2,1,2,1,2,\ldots,1,2,3)$ and the periodic label in path (tail) $(2,1,2,1,2,1,2,1,\ldots)$ with two adjacent vertices in bridge which using color 1 in cycle and color 2 in tail or color 2 in cycle and color 1 in tail. For more detail the label color and the representation of vertices in tadpole graph $T_{n,m}$ respect to with class color $\Pi = \{C_1, C_2, C_3\}$ where $C_1 = \{x_i, y_j; i \text{ is odd and } j \text{ is even}\}$, $C_2 = \{x_i, y_j; i \text{ is evenand } j \text{ is odd}\}$ and $C_3 = \{x_n\}$ as follows.

$$f(v) = \begin{cases} 1, & \text{if } v \in \{x_i, y_j; i \text{ is odd and } j \text{ is even} \\ 2, & \text{if } v \in \{x_i, y_j; i \text{ is evenand } j \text{ is odd} \\ 3, & \text{if } v \in \{x_n\} \end{cases}$$

Based on the color label f in tadpole graph $T_{n,m}$. Thus, we have the representation as follows.

$$r(x_i|\Pi) = (0,1,i); \text{for } i \text{ is odd}, 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$$
$$r(x_i|\Pi) = (1,0,i); \text{for } i \text{ is even}, 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$$
$$r(x_i|\Pi) = (0,1,n-i); \text{for } i \text{ is odd}, \left\lfloor \frac{n}{2} \right\rfloor + 1 \leq i \leq n-1$$
$$r(x_i|\Pi) = (1,0,n-i); \text{for } i \text{ is even}, \left\lfloor \frac{n}{2} \right\rfloor + 1 \leq i \leq n-1$$
$$r(y_j|\Pi) = (1,0,j+1); \text{for } j \text{ is odd}$$
$$r(y_j|\Pi) = (0,1,j+1); \text{for } j \text{ is even}$$
$$r(x_n|\Pi) = (1,1,0)$$

It is clear that for every two adjacent vertices has distinct representation, we can see in $r(x_{2k-1}|\Pi) \neq r(x_{2k}|\Pi)$ or $r(y_{2k-1}|\Pi) \neq r(y_{2k}|\Pi)$for $k \in N$. Without the loss generality, we have $\mu(T_{n,m}) \leq 3$. Thus, $\mu(T_{n,m}) = 3$.

//C ycle with m-pendant is one of unicyclic graph which a cycle with one vertex have m-pendant.

Theorem 2.2 Consider cycle with m-pendant C_n^m for $n \geq 3$ and $m \geq 1$. Then

$$\mu(C_n^m) = \begin{cases} 2, \text{if n is even} \\ 3, \text{if n is odd} \end{cases}$$

Proof. Cycle with m-pendant only have one cycle with order n and m-pendant. Thus, This proof divided into two cases as follows.

Case 1. For n is even

Based on Lemma 2.1 that $\mu(C_n^m) \geq 2$. Furthermore, we prove that $\mu(C_n^m) \leq 2$. Let $f : V(G) \to \{1,2\}$ be a vertex coloring (two adjacent vertices may be colored same color). The construction of coloring in graph C_n^m with the periodic label in cycle $(1,2,1,2,1,2,1,2,\ldots,1,2)$ and the periodic label in pendant $(2,2,2,\ldots)$. For more detail the label color and the representation of vertices in cycle with m-pendant graph C_n^m respect to with class color $\Pi = \{C_1, C_2\}$ where $C_1 = \{x_i; i \text{ is odd}\}$ and $C_2 = \{x_i, y_j; i \text{ is even }\}$ as follows.

$$f(v) = \begin{cases} 1, & \text{if } v \in \{x_i; i \text{ is odd} \\ 2, & \text{if } v \in \{x_i, y_j; i \text{ is even} \end{cases}$$

Based on the color label f in cycle with m-pendantgraphC_n^m. Thus, we have the representation as follows.

$$r(x_i|\Pi) = (0,1); \text{for } i \text{ is odd}; \; r(x_i|\Pi) = (1,0); \text{for } i \text{ is even}$$
$$r(y_j|\Pi) = (1,0)$$

It is clear that for every two adjacent vertices has distinct representation, we can see in $r(x_{2k-1}|\Pi) \neq r(x_{2k}|\Pi)$for $k \in N$. Without the loss generality, we have $\mu(C_n^m) \leq 2$. Thus, $\mu(C_n^m) = 2$.

Case 2. For n is odd

Based on Lemma 2.1 that $\mu(C_n^m) \geq 3$. Furthermore, we prove that $\mu(C_n^m) \leq 3$. Let $f : V(G) \to \{1,2,3\}$ be a vertex coloring (two adjacent vertices may be colored same color). The construction of coloring in graph C_n^m with the periodic label in cycle $(1,2,1,2,1,2,1,2,\ldots,1,2,3)$ and the periodic label in pendant $(2,2,2,\ldots)$. For more detail the label color and the representation of vertices in cycle with m-pendant graph C_n^m respect to with class color $\Pi = \{C_1, C_2, C_3\}$where $C_1 = \{x_i; i \text{ is odd }\}$, $C_2 = \{x_i, y_j; i \text{ is even }\}$ and $C_3 = \{x_n\}$ as follows.

$$f(v) = \begin{cases} 1, & \text{if } v \in \{x_i; i \text{ is odd} \\ 2, & \text{if } v \in \{x_i, y_j; i \text{ is even} \\ 3, & \text{if } v \in \{x_n\} \end{cases}$$

Based on the color label f in cycle with m-pendantgraph C_n^m. Thus, we have the representation as follows.

$$r(x_i|\Pi) = (0,1,i); \text{for } i \text{ is odd}, 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$$
$$r(x_i|\Pi) = (1,0,i); \text{for } i \text{ is even}, 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$$
$$r(x_i|\Pi) = (0,1,n-i); \text{for } i \text{ is odd}, \left\lfloor \frac{n}{2} \right\rfloor + 1 \leq i \leq n-1$$
$$r(x_i|\Pi) = (1,0,n-i); \text{for } i \text{ is even}, \left\lfloor \frac{n}{2} \right\rfloor + 1 \leq i \leq n-1$$
$$r(y_j|\Pi) = (1,0,2)$$
$$r(x_n|\Pi) = (1,1,0)$$

It is clear that for every two adjacent vertices has distinct representation, we can see in $r(x_{2k-1}|\Pi) \neq r(x_{2k}|\Pi)$for $k \in N$. Without the loss generality, we have $\mu(C_n^m) \leq 3$. Thus, $\mu(C_n^m) = 3$.

Theorem 2.3 Consider sun graph, $Sun(n)$ for $n \geq 3$. Then

$$\mu(Sun(n)) = \begin{cases} 2, \text{if } n \text{ is even} \\ 3, \text{if } n \text{ is odd} \end{cases}$$

Proof.

Case 1. For n is even

Based on Lemma 2.1 that $\mu(Sun(n)) \geq 2$. Furthermore, we prove that $\mu(Sun(n)) \leq 2$. Let $f : V(G) \to \{1,2\}$be a vertex coloring (two adjacent vertices may be colored same color). The construction of coloring in graph $Sun(n)$ with the periodic label in cycle $(1,2,1,2,1,2,1,2,\ldots,1,2)$ and the pendant $(2,1,2,1,2,1,2,1,\ldots)$ For more detail the label color and the representation of vertices in sun graph $Sun(n)$respect to with class color $\Pi = \{C_1, C_2\}$ where $C_1 = \{x_i, y_j; i \text{ is odd and } j \text{ is even}\}$ and $C_2 = \{x_i, y_j; i \text{ is evenand } j \text{ is odd}\}$ as follows.

$$f(v) = \begin{cases} 1, & \text{if } v \in \{x_i, y_j; i \text{ is odd or } j \text{ is even}\} \\ 2, & \text{if } v \in \{x_i, y_j; i \text{ is evenor } j \text{ is odd}\} \end{cases}$$

Based on the color label f in sun graph $Sun(n)$. Thus, we have the representation as follows.

$$r(y_j|\Pi) = r(x_i|\Pi) = (0,1); for\ i \text{ is odd and } j \text{ is even}$$
$$r(y_j|\Pi) = r(x_i|\Pi) = (1,0); for\ i \text{ is evenand } j \text{ is odd}$$

It is clear that for every two adjacent vertices has distinct representation, we can see in $r(x_{2k-1}|\Pi) \neq r(x_{2k}|\Pi)$ or $r(y_{2k-1}|\Pi) \neq r(y_{2k}|\Pi)$for $k \in N$. Without the loss generality, we have $\mu(Sun(n)) \leq 2$. Thus, $\mu(Sun(n)) = 2$.

Case 2. For n is odd

Based on Lemma 2.1 that $\mu(Sun(n)) \geq 3$. Furthermore, we prove that $\mu(Sun(n)) \leq 3$. Let $f : V(G) \rightarrow \{1,2,3\}$ be a vertex coloring (two adjacent vertices may be colored same color). The construction of coloring in graph $Sun(n)$ with the periodic label in cycle (1,2,1,2,1,2,1,2, ... ,1,2,3) and the pendant in the cycle(2,3,2,3,2,3,2,3, ... ,2,3,2). For more detail the label color and the representation of vertices in sun graph $Sun(n)$ respect to with class color $\Pi = \{C_1, C_2, C_3\}$where $C_1 = \{x_i, y_j; i \text{ is odd}\}$, $C_2 = \{x_i, y_j; i \text{ is evenand } j \text{ is odd}\}$ and $C_3 = \{x_n, y_j; j \text{ is even}\}$ as follows.

$$f(v) = \begin{cases} 1 \text{ , if } v \in \{x_i; i \text{ is odd}\} \\ 2 \text{ , if } v \in \{x_i, y_j; i \text{ is evenand } j \text{ is odd}\} \\ 3 \text{ , if } v \in \{x_n, y_j; j \text{ is even}\} \end{cases}$$

Based on the color label f in sungraph $Sun(n)$. Thus, we have the representation as follows.

$$r(x_1|\Pi) = (0,1,1)$$
$$r(x_n|\Pi) = (1,1,0)$$
$$r(x_i|\Pi) = (0,1,2); \text{for} x_i \text{is odd}, 2 \leq i \leq n-2$$
$$r(x_i|\Pi) = (1,0,1); \text{for} x_i \text{is even}, 2 \leq i \leq n-2$$
$$r(y_1|\Pi) = (1,0,2)$$
$$r(y_n|\Pi) = (2,0,1)$$
$$r(y_j|\Pi) = (2,1,0); \text{for} y_j \text{is even}, 2 \leq j \leq n-2$$
$$r(y_j|\Pi) = (1,0,3); \text{for} y_j \text{is odd}, 2 \leq j \leq n-2$$

It is clear that for every two adjacent vertices has distinct representation, we can see in $r(x_{2k-1}|\Pi) \neq r(x_{2k}|\Pi)$ or $r(y_{2k-1}|\Pi) \neq r(y_{2k}|\Pi)$for $k \in N$. Without the loss generality, we have $\mu(Sun(n)) \leq 3$. Thus, $\mu(Sun(n)) = 3$.

Theorem 2.4 Consider subdivision of sun graph $S(Sun(n))$ for $n \geq 3$. Then$\mu(S(Sun(n))) = 2$

Proof.

Based on Lemma 2.1 that $\mu(S(Sun(n))) \geq 2$. Furthermore, we prove that $\mu(S(n(n))) \leq 2$. Let $f : V(G) \rightarrow \{1,2\}$be a vertex coloring (two adjacent vertices may be colored same color). The construction of coloring in graph $S(Sun(n))$ with the periodic label in cycle (1,2,1,2,1,2,1,2, ... ,1,2) and the pendant in cycle (2,2,2,2,2,2, ... ,2) and (1,1,1,1, ... ,1) For more detail the label color and the representation of vertices in subdivision of sun graph $S(Sun(n))$respect to with class color $\Pi = \{C_1, C_2\}$ where $C_1 = \{x_i, z_j; i \text{ is odd}, 1 \leq i \leq 2n \text{ and } 1 \leq j \leq n\}$ and $C_2 = \{x_i, y_j; i \text{ is even}, 1 \leq i \leq 2n \text{ and } 1 \leq j \leq n\}$as follows.

$$f(v) = \begin{cases} 1 \text{ , if } v \in \{x_i, z_j; i \text{ is odd}, 1 \leq i \leq 2n \text{ and } 1 \leq j \leq n\} \\ 2 \text{ , if } v \in \{x_i, y_j; i \text{ is even}, 1 \leq i \leq 2n \text{ and } 1 \leq j \leq n\} \end{cases}$$

Based on the color label f in subdivision of sun graph $S(Sun(n))$. Thus, we have the representation as follows.

$$r(x_i|\Pi) = r(z_j|\Pi) = (0,1); \text{for } i \text{ is odd}, 1 \leq i \leq 2n \text{ and } 1 \leq j \leq n$$
$$r(x_i|\Pi) = r(y_j|\Pi) = (1,0); \text{for } i \text{ is even}, 1 \leq i \leq 2n \text{ and } 1 \leq j \leq n$$

It is clear that for every two adjacent vertices has distinct representation, we can see in $r(x_{2k-1}|\Pi) \neq r(x_{2k}|\Pi)$ or $r(y_{2k-1}|\Pi) \neq r(y_{2k}|\Pi)$for $k \in N$. Without the loss generality, we have $\mu(S(Sun(n))) \leq 2$. Thus, $\mu(S(Sun(n))) = 2$.

Theorem 2.5 Consider cycle with two pendants C_n^2 for $n \geq 3$. Then

$$\mu(C_n^2) = \begin{cases} 2, if\ n\ is\ even \\ 3, if\ n\ is\ odd \end{cases}$$

Proof. Cycle with 2-pendant only have one cycle with order n and 2-pendant. Thus, This proof divided into two cases as follows.

Case 1. For n is even

Based on Lemma 2.1 that $\mu(C_n^2) \geq 2$. Furthermore, we prove that $\mu(C_n^2) \leq 2$. Let $f : V(G) \rightarrow \{1,2\}$ be a vertex coloring (two adjacent vertices may be colored same color). The construction of coloring in graph C_n^2 with the periodic label in cycle (1,2,1,2,1,2,1,2, ... ,1,2) and the in pendant (1,1). For more detail the label color and the representation of vertices in cycle with 2-pendant graph C_n^2 respect to with class color $\Pi = \{C_1, C_2\}$ where $C_1 = \{x_i, y_1, y_n; i \text{ is odd}\}$ and $C_2 = \{x_i; i \text{ is even}\}$ as follows.

$$f(v) = \begin{cases} 1 \text{ , if } v \in \{x_i, y_1, y_n; i \text{ is odd}\} \\ 2 \text{ , if } v \in \{x_i; i \text{ is even}\} \end{cases}$$

Based on the color label f in cycle with 2-pendantgraphC_n^2. Thus, we have the representation as follows.

$$r(x_i|\Pi) = r(y_n|\Pi) = (0,1); \text{for } i \text{ is odd};$$
$$r(x_i|\Pi) = (1,0); \text{for } i \text{ is even}$$
$$r(y_1|\Pi) = (0,2)$$

It is clear that for every two adjacent vertices has distinct representation, we can see in $r(x_{2k-1}|\Pi) \neq r(x_{2k}|\Pi)$for $k \in N$. Without the loss generality, we have $\mu(C_n^2) \leq 2$. Thus, $\mu(C_n^2) = 2$.

Case 2. For n is odd

Based on Lemma 2.1 that $\mu(C_n^2) \geq 3$. Furthermore, we prove that $\mu(C_n^2) \leq 3$. Let $f : V(G) \rightarrow \{1,2,3\}$ be a vertex coloring (two adjacent vertices may be colored same color). The construction of coloring in graph C_n^2 with the periodic label in cycle (1,2,1,2,1,2,1,2, ... ,1,2,3) and the in pendant (1,1) For more detail the label color and the representation of vertices in cycle with 2-pendant graph C_n^2 respect to with class color $\Pi = \{C_1, C_2, C_3\}$ where $C_1 = \{x_i, y_1, y_n; i \text{ is odd}\}$, $C_2 = \{x_i; i \text{ is even}\}$ and $C_3 = \{x_n\}$ as follows.

$$f(v) = \begin{cases} 1 \text{ , if } v \in \{x_i, y_1, y_n; i \text{ is odd}\} \\ 2 \text{ , if } v \in \{x_i; i \text{ is even}\} \\ 3 \text{ , if } v \in \{x_n\} \end{cases}$$

Based on the color label f in cycle with 2-pendantgraph C_n^2. Thus, we have the representation as follows.

$$r(x_i|\Pi) = (0,1,i); \text{for } i \text{ is odd}, 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$$
$$r(x_i|\Pi) = (1,0,i); \text{for } i \text{ is even}, 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$$
$$r(x_i|\Pi) = (0,1,n-i); \text{for } i \text{ is odd}, \left\lfloor \frac{n}{2} \right\rfloor + 1 \leq i \leq n-1$$
$$r(x_i|\Pi) = (1,0,n-i); \text{for } i \text{ is even}, \left\lfloor \frac{n}{2} \right\rfloor + 1 \leq i \leq n-1$$
$$r(x_n|\Pi) = (1,1,0)$$
$$r(y_1|\Pi) = (0,2,2)$$

$$r(y_1|\Pi) = (0,2,1)$$

It is clear that for every two adjacent vertices has distinct representation, we can see in $r(x_{2k-1}|\Pi) \neq r(x_{2k}|\Pi)$for $k \in N$. Without the loss generality, we have $\mu(C_n^2) \leq 3$. Thus, $\mu(C_n^2) = 3$.

3 Conclusion

In this paper we have shown some results the lower bound of metric chromatic number of unicyclic graphs. However, to obtain the exact values of some special graphs is not easy job. Hence we propose to study other families apart from the families that we have studied in this paper

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Metric Chromatic Number Of Unicyclic Graphs

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Abstract: All graphs in this paper are nontrivial and connected graph. Let $f : V(G) \rightarrow \{1,2,\dots,k\}$ be a vertex coloring of a graph $G$where two adjacent vertices may be colored the same color. Consider the color classes $\Pi = \{C_1, C_2, \dots, C_k\}$. For a vertex $v$of G, the representation color of $v$is the k-vector $r(v|\Pi) = (d(v,, C_1), d(v, C_2), \dots, d(v, C_k))$, where $d(v, C_i) = \min\{d(v,c); c \in C_i\}$. If $r(u|\Pi) \neq r(v|\Pi)$ for every two adjacent vertices $u$and $v$of G, then $f$is a metric coloring of G. The minimum $k$for which Ghas a metric k-coloring is called the metric chromatic number of $G$and is denoted by $\mu(G)$. The metric chromatic numbers of unicyclic graphs namely tadpole graphs, cycle with m-pendants, sun graphs, cycle with two pendants, subdivision of sun graphs.

Index Terms: Metric coloring, metric chromatic number, unicyclic graphs

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1 Introduction

Graphs in this paper are nontrivial and connected graph, for more detail definition of graph see [1]. Let $f : V(G) \rightarrow \{1,2,\dots,k\}$ be a vertex coloring of a graph G where two adjacent vertices may be colored the same color. Consider the color classes $\pi = \{C_1, C_2, \dots, C_k\}$.For a vertex $v$in$G$, representation color of v is the k-vector $r(v|\pi) = (d(v,, C_1), d(v, C_2), \dots, d(v, C_k))$.where $d(v, C_i) = \min\{d(v,c); c \in C_i\}$. If $r(u|\pi) \neq r(v|\pi)$ for every two adjacent vertices u and $v$in$G$, then f is a metric coloring of G. The minimum k for which G has a metric k-coloring is called the metric chromatic number of G and is denoted by $\mu(G)$.In recent years, there are some results of the metric chromatic number of some well-known graphs in Chartrand et.al [2] as follows.

Proposition 1.1 Anontrivial connected graph G has metric chromatic number 2if and only if G is bipartite.

Proposition 2.2 Let G be a connected graph. If $\chi(G) = 3$, then $\mu(G) = 3$.

Proposition 2.3 Let C_n be a cycle graph, then.

$$\mu(C_n) = \begin{cases} 2, if\ n\ is\ even \\ 3, if\ n\ is\ odd \end{cases}$$

2 Result

In this paper, we investigate the metric chromatic number of unicyclic graphs namely tadpole graphs, cycle with m-pendants, sun graphs, cycle with two pendants, subdivision of sun graphs. Furthermore, we construct the new lemma for any unicyclic graphs as follows.

Observation 2.1. For any two adjacent vertices $u, v \in V(T)$, then we have $d(u,x) \neq d(v,x)$ for $x \in V(T)$.

Observation 2.2. For any two adjacent vertices u', v' in cycle in unicyclic graph, then $\exists\ x \in V(U_n) \ni d(u',x) = d(v',x)$

Lemma 2.1 Consider unicyclic graph U_n for $n \geq 3$. Then

$$\mu(U_n) \geq \begin{cases} 2, if\ n_{c_n} is\ even \\ 3, if\ n_{c_n} is\ odd \end{cases}$$

Proof. Unicyclic graph U_n for $n \geq 3$ has only one cycle subgraph with n vertices. So that, the coloring in the cycle subgraph follows the coloring in the cycle graph. Based on Proposition 2.3. that $\mu(C_n) = 2$ for n is even and $\mu(C_n) = 3$ for n is odd. So, there are several condition in this proof as follows:

i. For any two adjacent vertices u, v not in cycle, based on Observation 2.1. that$d(u,x) \neq d(v,x)$ for $x \in V(T)$ such that $r(u|\Pi) \neq r(v|\Pi)$.

ii. For any two vertices u in cycle and v not in cycle (u, v not adjacent), it may be same representation.

iii. For any two adjacent vertices u in cycle and v not in cycle, based on Observation 2.1. that$d(u,x) \neq d(v,x)$ for $x \in V(T)$ such that $r(u|\Pi) \neq r(v|\Pi)$.

iv. For any two adjacent vertices u, v in cycle, based on Proposition 2.3. that we use the coloring in cycle.

Based on cases i) ,ii) ,iii), iv) that

$$\mu(U_n) \geq \begin{cases} 2, if\ n_{c_n} is\ even \\ 3, if\ n_{c_n} is\ odd \end{cases}$$

Tadpole graph is one of unicyclic graph which obtained by joining a cycle graph C_m to a path graph P_n with a bridge.

Theorem 2.1 Consider tadpole graph $T_{n,m}$ for $n \geq 3$ and $m \geq 1$. Then

$$\mu(T_{n,m}) = \begin{cases} 2, & \text{if n is even} \\ 3, & \text{if n is odd} \end{cases}$$

Proof. Tadpole graph only have one cycle with order n and one pendant path. Thus, This proof divided into two cases as follows.

Case 1. For n is even

Based on Lemma 2.1 that $\mu(T_{n,m}) \geq 2$. Furthermore, we prove that $\mu(T_{n,m}) \leq 2$. Let $f : V(G) \rightarrow \{1,2\}$ be a vertex coloring (two adjacent vertices may be colored same color). The construction of coloring in graph $T_{n,m}$ with the periodic label in cycle $(1,2,1,2,1,2,1,2,\dots,1,2)$ and the periodic label in path (tail) $(2,1,2,1,2,1,2,1,\dots)$ with two adjacent vertices in bridge which using color 1 in cycle and color 2 in tail or color 2 in cycle and color 1 in tail. For more detail the label color and the representation of vertices in tadpole graph $T_{n,m}$ respect to with class color $\Pi = \{C_1, C_2\}$ where $C_1 = \{x_i, y_j; i \text{ is odd and } j \text{ is even}\}$ and $C_2 = \{x_i, y_j; i \text{ is even and } j \text{ is odd}\}$ as follows.

$$f(v) = \begin{cases} 1\ , if\ v \in \{x_i, y_j; i \text{ is odd and } j \text{ is even} \\ 2\ , if\ v \in \{x_i, y_j; i \text{ is evenand } j \text{ is odd} \end{cases}$$

Based on the color label f in tadpole graph $T_{n,m}$. Thus, we have the representation as follows.

$$r(y_j|\Pi) = r(x_i|\Pi) = (0,1); for\ i \text{ is odd and } j \text{ is even}$$

$$r(y_j|\Pi) = r(x_i|\Pi) = (1,0); for\ i \text{ is evenand } j \text{ is odd}$$

It is clear that for every two adjacent vertices has distinct representation, we can see in $r(x_{2k-1}|\Pi) \neq r(x_{2k}|\Pi)$ or $r(y_{2k-1}|\Pi) \neq r(y_{2k}|\Pi)$for $k \in N$. Without the loss generality, we have $\mu(T_{n,m}) \leq 2$. Thus, $\mu(T_{n,m}) = 2$.

Case 2. For n is odd

Based on Lemma 2.1 that $\mu(T_{n,m}) \geq 3$. Furthermore, we prove that $\mu(T_{n,m}) \leq 3$. Let $f : V(G) \rightarrow \{1,2,3\}$ be a vertex coloring (two adjacent vertices may be colored same color). The construction of coloring in graph $T_{n,m}$ with the periodic label in cycle (1,2,1,2,1,2,1,2, ...,1,2,3) and the periodic label in path (tail) (2,1,2,1,2,1,2,1, ...) with two adjacent vertices in bridge which using color 1 in cycle and color 2 in tail or color 2 in cycle and color 1 in tail. For more detail the label color and the representation of vertices in tadpole graph $T_{n,m}$ respect to with class color $\Pi = \{C_1, C_2, C_3\}$ where $C_1 = \{x_i, y_j; i \text{ is odd and } j \text{ is even}\}$, $C_2 = \{x_i, y_j; i \text{ is evenand } j \text{ is odd}\}$ and $C_3 = \{x_n\}$ as follows.

$$f(v) = \begin{cases} 1 \text{ , if } v \in \{x_i, y_j; i \text{ is odd and } j \text{ is even} \\ 2 \text{ , if } v \in \{x_i, y_j; i \text{ is evenand } j \text{ is odd} \\ 3 \text{ , if } v \in \{x_n\} \end{cases}$$

Based on the color label f in tadpole graph $T_{n,m}$. Thus, we have the representation as follows.

$$r(x_i|\Pi) = (0,1,i); \text{for } i \text{ is odd}, 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$$
$$r(x_i|\Pi) = (1,0,i); \text{for } i \text{ is even}, 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$$
$$r(x_i|\Pi) = (0,1,n-i); \text{for } i \text{ is odd}, \left\lfloor \frac{n}{2} \right\rfloor + 1 \leq i \leq n-1$$
$$r(x_i|\Pi) = (1,0,n-i); \text{for } i \text{ is even}, \left\lfloor \frac{n}{2} \right\rfloor + 1 \leq i \leq n-1$$
$$r(y_j|\Pi) = (1,0,j+1); \text{for } j \text{ is odd}$$
$$r(y_j|\Pi) = (0,1,j+1); \text{for } j \text{ is even}$$
$$r(x_n|\Pi) = (1,1,0)$$

It is clear that for every two adjacent vertices has distinct representation, we can see in $r(x_{2k-1}|\Pi) \neq r(x_{2k}|\Pi)$ or $r(y_{2k-1}|\Pi) \neq r(y_{2k}|\Pi)$for $k \in N$. Without the loss generality, we have $\mu(T_{n,m}) \leq 3$. Thus, $\mu(T_{n,m}) = 3$.

//Cycle with m-pendant is one of unicyclic graph which a cycle with one vertex have m-pendant.

Theorem 2.2 Consider cycle with m-pendant C_n^m for $n \geq 3$ and $m \geq 1$. Then

$$\mu(C_n^m) = \begin{cases} 2, \text{if n is even} \\ 3, \text{if n is odd} \end{cases}$$

Proof. Cycle with m-pendant only have one cycle with order n and m-pendant. Thus, This proof divided into two cases as follows.

Case 1. For n is even

Based on Lemma 2.1 that $\mu(C_n^m) \geq 2$. Furthermore, we prove that $\mu(C_n^m) \leq 2$. Let $f : V(G) \rightarrow \{1,2\}$ be a vertex coloring (two adjacent vertices may be colored same color). The construction of coloring in graph C_n^m with the periodic label in cycle (1,2,1,2,1,2,1,2, ...,1,2) and the periodic label in pendant (2,2,2, ...) . For more detail the label color and the representation of vertices in cycle with m-pendant graph C_n^m respect to with class color $\Pi = \{C_1, C_2\}$ where $C_1 = \{x_i; i \text{ is odd}\}$ and $C_2 = \{x_i, y_j; i \text{ is even}\}$ as follows.

$$f(v) = \begin{cases} 1 \text{ , if } v \in \{x_i; i \text{ is odd} \\ 2 \text{ , if } v \in \{x_i, y_j; i \text{ is even} \end{cases}$$

Based on the color label f in cycle with m-pendantgraphC_n^m. Thus, we have the representation as follows.

$$r(x_i|\Pi) = (0,1); \text{for } i \text{ is odd}; \ r(x_i|\Pi) = (1,0); \text{for } i \text{ is even}$$
$$r(y_j|\Pi) = (1,0)$$

It is clear that for every two adjacent vertices has distinct representation, we can see in $r(x_{2k-1}|\Pi) \neq r(x_{2k}|\Pi)$for $k \in N$. Without the loss generality, we have $\mu(C_n^m) \leq 2$. Thus, $\mu(C_n^m) = 2$.

Case 2. For n is odd

Based on Lemma 2.1 that $\mu(C_n^m) \geq 3$. Furthermore, we prove that $\mu(C_n^m) \leq 3$. Let $f : V(G) \rightarrow \{1,2,3\}$ be a vertex coloring (two adjacent vertices may be colored same color). The construction of coloring in graph C_n^m with the periodic label in cycle (1,2,1,2,1,2,1,2, ... ,1,2,3) and the periodic label in pendant (2,2,2, ...). For more detail the label color and the representation of vertices in cycle with m-pendant graph C_n^m respect to with class color $\Pi = \{C_1, C_2, C_3\}$where $C_1 = \{x_i; i \text{ is odd}\}$, $C_2 = \{x_i, y_j; i \text{ is even}\}$ and $C_3 = \{x_n\}$ as follows.

$$f(v) = \begin{cases} 1 \text{ , if } v \in \{x_i; i \text{ is odd} \\ 2 \text{ , if } v \in \{x_i, y_j; i \text{ is even} \\ 3 \text{ , if } v \in \{x_n\} \end{cases}$$

Based on the color label f in cycle with m-pendantgraph C_n^m. Thus, we have the representation as follows.

$$r(x_i|\Pi) = (0,1,i); \text{for } i \text{ is odd}, 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$$
$$r(x_i|\Pi) = (1,0,i); \text{for } i \text{ is even}, 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$$
$$r(x_i|\Pi) = (0,1,n-i); \text{for } i \text{ is odd}, \left\lfloor \frac{n}{2} \right\rfloor + 1 \leq i \leq n-1$$
$$r(x_i|\Pi) = (1,0,n-i); \text{for } i \text{ is even}, \left\lfloor \frac{n}{2} \right\rfloor + 1 \leq i \leq n-1$$
$$r(y_j|\Pi) = (1,0,2)$$
$$r(x_n|\Pi) = (1,1,0)$$

It is clear that for every two adjacent vertices has distinct representation, we can see in $r(x_{2k-1}|\Pi) \neq r(x_{2k}|\Pi)$for $k \in N$. Without the loss generality, we have $\mu(C_n^m) \leq 3$. Thus, $\mu(C_n^m) = 3$.

Theorem 2.3 Consider sun graph, $Sun(n)$ for $n \geq 3$. Then

$$\mu(Sun(n)) = \begin{cases} 2, if \ n \ is \ even \\ 3, if \ n \ is \ odd \end{cases}$$

Proof.

Case 1. For n is even

Based on Lemma 2.1 that $\mu(Sun(n)) \geq 2$. Furthermore, we prove that $\mu(Sun(n)) \leq 2$. Let $f : V(G) \rightarrow \{1,2\}$be a vertex coloring (two adjacent vertices may be colored same color). The construction of coloring in graph $Sun(n)$ with the periodic label in cycle (1,2,1,2,1,2,1,2, ... ,1,2) and the pendant (2,1,2,1,2,1,2,1, ...) For more detail the label color and the representation of vertices in sun graph $Sun(n)$respect to with class color $\Pi = \{C_1, C_2\}$ where $C_1 = \{x_i, y_j; i \text{ is odd and } j \text{ is even}\}$ and $C_2 = \{x_i, y_j; i \text{ is evenand } j \text{ is odd}\}$ as follows.

$$f(v) = \begin{cases} 1 \text{ , if } v \in \{x_i, y_j; i \text{ is odd or } j \text{ is even}\} \\ 2 \text{ , if } v \in \{x_i, y_j; i \text{ is evenor } j \text{ is odd}\} \end{cases}$$

Based on the color label f in sun graph $Sun(n)$. Thus, we have the representation as follows.

$$r(y_j|\Pi) = r(x_i|\Pi) = (0,1); for\ i \text{ is odd and } j \text{ is even}$$
$$r(y_j|\Pi) = r(x_i|\Pi) = (1,0); for\ i \text{ is evenand } j \text{ is odd}$$

It is clear that for every two adjacent vertices has distinct representation, we can see in $r(x_{2k-1}|\Pi) \neq r(x_{2k}|\Pi)$ or $r(y_{2k-1}|\Pi) \neq r(y_{2k}|\Pi)$for $k \in N$. Without the loss generality, we have $\mu(Sun(n)) \leq 2$. Thus, $\mu(Sun(n)) = 2$.

Case 2. For n is odd

Based on Lemma 2.1 that $\mu(Sun(n)) \geq 3$. Furthermore, we prove that $\mu(Sun(n)) \leq 3$. Let $f : V(G) \to \{1,2,3\}$ be a vertex coloring (two adjacent vertices may be colored same color). The construction of coloring in graph $Sun(n)$ with the periodic label in cycle (1,2,1,2,1,2,1,2, ...,1,2,3) and the pendant in the cycle(2,3,2,3,2,3,2,3, ... ,2,3,2). For more detail the label color and the representation of vertices in sun graph $Sun(n)$ respect to with class color $\Pi = \{C_1, C_2, C_3\}$where $C_1 = \{x_i, y_j; i \text{ is odd}\}$, $C_2 = \{x_i, y_j; i \text{ is evenand } j \text{ is odd}\}$ and $C_3 = \{x_n, y_j; j \text{ is even}\}$ as follows.

$$f(v) = \begin{cases} 1\ \text{, if } v \in \{x_i; i \text{ is odd}\} \\ 2\ \text{, if } v \in \{x_i, y_j; i \text{ is evenand } j \text{ is odd}\} \\ 3\ \text{, if } v \in \{x_n, y_j; j \text{ is even}\} \end{cases}$$

Based on the color label f in sungraph $Sun(n)$. Thus, we have the representation as follows.

$$r(x_1|\Pi) = (0,1,1)$$
$$r(x_n|\Pi) = (1,1,0)$$
$$r(x_i|\Pi) = (0,1,2); \text{for } x_i \text{ is odd}, 2 \leq i \leq n-2$$
$$r(x_i|\Pi) = (1,0,1); \text{for } x_i \text{ is even}, 2 \leq i \leq n-2$$
$$r(y_1|\Pi) = (1,0,2)$$
$$r(y_n|\Pi) = (2,0,1)$$
$$r(y_j|\Pi) = (2,1,0); \text{for } y_j \text{ is even}, 2 \leq j \leq n-2$$
$$r(y_j|\Pi) = (1,0,3); \text{for } y_j \text{ is odd}, 2 \leq j \leq n-2$$

It is clear that for every two adjacent vertices has distinct representation, we can see in $r(x_{2k-1}|\Pi) \neq r(x_{2k}|\Pi)$ or $r(y_{2k-1}|\Pi) \neq r(y_{2k}|\Pi)$for $k \in N$. Without the loss generality, we have $\mu(Sun(n)) \leq 3$. Thus, $\mu(Sun(n)) = 3$.

Theorem 2.4 Consider subdivision of sun graph $S(Sun(n))$ for $n \geq 3$. Then$\mu(S(Sun(n))) = 2$

Proof.

Based on Lemma 2.1 that $\mu(S(Sun(n))) \geq 2$. Furthermore, we prove that $\mu(S(Sun(n))) \leq 2$. Let $f : V(G) \to \{1,2\}$be a vertex coloring (two adjacent vertices may be colored same color). The construction of coloring in graph $S(Sun(n))$ with the periodic label in cycle (1,2,1,2,1,2,1,2, ... ,1,2) and the pendant in cycle (2,2,2,2,2,2, ... ,2) and (1,1,1,1, ... ,1) For more detail the label color and the representation of vertices in subdivision of sun graph $S(Sun(n))$respect to with class color $\Pi = \{C_1, C_2\}$ where $C_1 = \{x_i, z_j; i \text{ is odd}, 1 \leq i \leq 2n \text{ and } 1 \leq j \leq n\}$ and $C_2 = \{x_i, y_j; i \text{ is even}, 1 \leq i \leq 2n \text{ and } 1 \leq j \leq n\}$as follows.

$$f(v) = \begin{cases} 1\ \text{, if } v \in \{x_i, z_j; i \text{ is odd}, 1 \leq i \leq 2n \text{ and } 1 \leq j \leq n\} \\ 2\ \text{, if } v \in \{x_i, y_j; i \text{ is even}, 1 \leq i \leq 2n \text{ and } 1 \leq j \leq n\} \end{cases}$$

Based on the color label f in subdivision of sun graph $S(Sun(n))$. Thus, we have the representation as follows.

$$r(x_i|\Pi) = r(z_j|\Pi) = (0,1); \text{for } i \text{ is odd}, 1 \leq i \leq 2n \text{ and } 1 \leq j \leq n$$
$$r(x_i|\Pi) = r(y_j|\Pi) = (1,0); \text{for } i \text{ is even}, 1 \leq i \leq 2n \text{ and } 1 \leq j \leq n$$

It is clear that for every two adjacent vertices has distinct representation, we can see in $r(x_{2k-1}|\Pi) \neq r(x_{2k}|\Pi)$ or $r(y_{2k-1}|\Pi) \neq r(y_{2k}|\Pi)$for $k \in N$. Without the loss generality, we have $\mu(S(Sun(n))) \leq 2$. Thus, $\mu(S(Sun(n))) = 2$.

Theorem 2.5 Consider cycle with two pendants C_n^2 for $n \geq 3$. Then

$$\mu(C_n^2) = \begin{cases} 2, if\ n\ is\ even \\ 3, if\ n\ is\ odd \end{cases}$$

Proof. Cycle with 2-pendant only have one cycle with order n and 2-pendant. Thus, This proof divided into two cases as follows.

Case 1. For n is even

Based on Lemma 2.1 that $\mu(C_n^2) \geq 2$. Furthermore, we prove that $\mu(C_n^2) \leq 2$. Let $f : V(G) \to \{1,2\}$ be a vertex coloring (two adjacent vertices may be colored same color). The construction of coloring in graph C_n^2 with the periodic label in cycle (1,2,1,2,1,2,1,2, ... ,1,2) and the in pendant (1,1). For more detail the label color and the representation of vertices in cycle with 2-pendant graph C_n^2 respect to with class color $\Pi = \{C_1, C_2\}$ where $C_1 = \{x_i, y_1, y_n; i \text{ is odd}\}$ and $C_2 = \{x_i; i \text{ is even}\}$ as follows.

$$f(v) = \begin{cases} 1\ \text{, if } v \in \{x_i, y_1, y_n; i \text{ is odd}\} \\ 2\ \text{, if } v \in \{x_i; i \text{ is even}\} \end{cases}$$

Based on the color label f in cycle with 2-pendantgraphC_n^2. Thus, we have the representation as follows.

$$r(x_i|\Pi) = r(y_n|\Pi) = (0,1); \text{for } i \text{ is odd};$$
$$r(x_i|\Pi) = (1,0); \text{for } i \text{ is even}$$
$$r(y_1|\Pi) = (0,2)$$

It is clear that for every two adjacent vertices has distinct representation, we can see in $r(x_{2k-1}|\Pi) \neq r(x_{2k}|\Pi)$for $k \in N$. Without the loss generality, we have $\mu(C_n^2) \leq 2$. Thus, $\mu(C_n^2) = 2$.

Case 2. For n is odd

Based on Lemma 2.1 that $\mu(C_n^2) \geq 3$. Furthermore, we prove that $\mu(C_n^2) \leq 3$. Let $f : V(G) \to \{1,2,3\}$ be a vertex coloring (two adjacent vertices may be colored same color). The construction of coloring in graph C_n^2 with the periodic label in cycle (1,2,1,2,1,2,1,2, ... ,1,2,3) and the in pendant (1,1) For more detail the label color and the representation of vertices in cycle with 2-pendant graph C_n^2 respect to with class color $\Pi = \{C_1, C_2, C_3\}$ where $C_1 = \{x_i, y_1, y_n; i \text{ is odd}\}$, $C_2 = \{x_i; i \text{ is even}\}$ and $C_3 = \{x_n\}$ as follows.

$$f(v) = \begin{cases} 1\ \text{, if } v \in \{x_i, y_1, y_n; i \text{ is odd}\} \\ 2\ \text{, if } v \in \{x_i; i \text{ is even}\} \\ 3\ \text{, if } v \in \{x_n\} \end{cases}$$

Based on the color label f in cycle with 2-pendantgraph C_n^2. Thus, we have the representation as follows.

$$r(x_i|\Pi) = (0,1,i); \text{for } i \text{ is odd}, 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$$
$$r(x_i|\Pi) = (1,0,i); \text{for } i \text{ is even}, 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$$
$$r(x_i|\Pi) = (0,1,n-i); \text{for } i \text{ is odd}, \left\lfloor \frac{n}{2} \right\rfloor + 1 \leq i \leq n-1$$
$$r(x_i|\Pi) = (1,0,n-i); \text{for } i \text{ is even}, \left\lfloor \frac{n}{2} \right\rfloor + 1 \leq i \leq n-1$$
$$r(x_n|\Pi) = (1,1,0)$$
$$r(y_1|\Pi) = (0,2,2)$$

$$r(y_1|\Pi) = (0,2,1)$$

It is clear that for every two adjacent vertices has distinct representation, we can see in $r(x_{2k-1}|\Pi) \neq r(x_{2k}|\Pi)$for $k \in N$. Without the loss generality, we have $\mu(C_n^2) \leq 3$. Thus, $\mu(C_n^2) = 3$.

3 Conclusion

In this paper we have shown some results the lower bound of metric chromatic number of unicyclic graphs. However, to obtain the exact values of some special graphs is not easy job. Hence we propose to study other families apart from the families that we have studied in this paper

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