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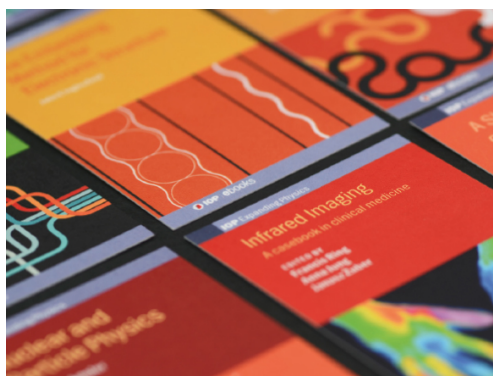
The anxiety of students on deduction level in proving the geometry theorem

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Abstract. This study aims to describe the anxiety of university students who were on deduction level in proving the geometry theorem. This was conducted to find out student anxiety, so that it can be handled and anticipated. Anxiety is the state of feeling students' uncomfortable, worried, and afraid. The type of this research is qualitative-descriptive. The subjects were taken as many as 3 students who were at the deduction level based on the Van Hiele geometry ability test. Data was collected through theorem proofing tests and interviews conducted after the test. The results of this study indicate that the symptoms of cognitive anxiety (uneasy, nervousness and confuse) occur when the subject begins to understand the problem in the theorem. Symptoms of physiological anxiety (sweating, body feels hot) and behavioral symptoms (physical tension, impatience) appeared when the subjects connected the elements of proof on theorem in geometry. The results of this study are expected to be used by students or lecturers, especially in learning to teach how to proof theorem in geometry.

1. Introduction

Proving theorem in geometry is very important to ensure what has been assumed to be true is definitely true. It is undeniable that there are many truths on the mathematics facts as it has only been trusted without having any suspicion of the truth. The students or the students at university level are reluctant to prove the truth on mathematics themselves due to the difficulties in proving it. The difficulties of learning and teaching about the evidence that had been recognized internationally [1]. Some research which show the difficulties experienced by the students or the students at university level in proving the theorem had been carried out by several researchers, including [2], [1]. The researchers also carried out a survey of several prospective students who would be the future teachers at the Faculty of Teacher Training and Education, Mathematics and Natural Sciences Department, Mathematics Education Study Program, University of Jember, which once took Geometry course, and the results obtained from the survei (through distributing the questionnaires and conducting the interview with some students) found difficulties in proving the theorems or other truth statements, so that in the end, the learning objectives could not be maximally achieved. Several other research had shown that the results of mathematics learning about proof were very low also conducted by Hoyles, et al (2007). Hoyles said that many students did not know where to start proving that made it even more difficult to prove and impacted learning outcomes that were considered very low.

The researchers also conducted the Van Hiele test to several students in Jember majoring Mathematics Education to find out the stages or levels of thinking possessed by students in learning

geometry. The results obtained from the Van Hiele test were that most students were still in the first three levels of Van Hiele. In the process of studying geometry, Van Hiele argues that one will go through sequential levels of thinking. Hoffer (in Burger & Shaughnessy, 1986) explains these stages as follows:

- Level of Visualization/Introduction (Level 0)
  At this level, the students are only new to geometric shapes and look at the geometry as a whole. For instance, the new students recognize rectangles as rectangular objects such as blackboard, book, door, etc.
- Level of Analysis (level 1)
  At this level of analysis, students are familiar with the properties of geometric shapes. For example, students already know that a rectangle has two pairs of faces that are equal in length, the diagonal length is the same.
- Level of Sorting (level 2)
  At this level, students already understand the order of geometric shapes, for example students already know that a square is a rectangle, a rectangle is a square.
- Level of deduction (level 3)
  At this level, students have been able to draw conclusions from specific things deductively. Students at this stage have understood the importance of the role of elements that are not defined, besides the defined elements, axioms, and theorems. At this level students do not understand the usefulness of a deductive system yet.
- Level of Rigor/accuracy (level 4)
  At this level, students has begun to understand the importance of the accuracy of the basic principles in a proof. This level of thinking has been categorized to a high level of thinking, complicated, and complex. So, not all students can be at this level, and not surprisingly, even though they are already in senior high school, students is still not at this level.

One of the factors that can cause the learning objectives not achieved is due to anxiety [3]. [4] provides an understanding of anxiety as an emotional state that has physiological arousal characteristics, unpleasant tense feelings, and concerns that something bad will happen. Feeling concern and fear when have to do work related to mathematics is called math anxiety, Hambree in [5]. Some studies show that anxiety has an influence on the achievement of learning including the research conducted by Whinnie et al (2017) and Anita (2014). This means that when a person is filled with a sense of anxiety he will has an impact on the learning outcomes (knowledge aspects and skills aspects), one of which is the skill to prove theorems on geometry.

Anxiety is a response - a normal biological reaction when we feel threatened [6]. Taylor (1953) in the Tailor Manifest Anxiety Scale (TMAS) suggests that anxiety is a subjective feeling about nervous tension as a general reaction to the inability to overcome a problem or lack of security. According to the American Psychiatric Association [APA] (2000) there are six main categories of anxiety disorders including phobias, panic disorder (with or without agoraphobia) general anxiety disorders, obsessive-compulsive disorder, acute stress disorder and posttraumatic stress disorder [7].

Among the existing anxieties, mathematical anxiety differs from specific generalized anxiety [8]. According to Hambree in [5] Mathematical anxiety is interpreted as feeling of fear when have to do work related to mathematics.

The indicators used in this research were indicators that have been put forward by Stuart (2006), but not all indicators were used. Only indicators that showed rapid responses were used. Insomnia, loss of appetite, diarrhea were examples of indicators that were not used in this research. The indicators of this research are explained in the following table:

Table 1. Indicator of anxiety

Response	Indicator	Sub-indicator
Physiological symptoms	Cardiovascular	Beating heart
		Unconsciousness
	Breathing	Chest pressure
	Neuromuscular	Back and forth
	Gastrointestinal	Loss of appetite
		Nausea
	Urinary tract	Cannot hold urine
	Skin	Sweating
		Blushing face
Behavior, Cognitive, and affective	Behavior	Restless
		Physical tension
		Tremor
		Surprised reaction
		Talk fast
		Escape from problems
	Cognitive	Attention disturbed
		Poor concentration
		Forgetful
		Barriers to think
		Confused
		Fear
	Affective	Easily disturbed
		Impatient
		Restless
		Tense
		Nervous
		Worry
		Guilty feeling
		Shy

In this research, the definition of anxiety used was the mathematical anxiety, especially geometry anxiety that was a response to describe the feeling of inconvenience, worry and fear which associated both emotional and physical sensation that commonly happened when someone worried or nervous in facing geometry problems. Among of them were spatial problems and proof problems of geometry theorem. The anxiety indicators used in this research were the anxiety indicators proposed by Stuart (2006).

According to [9] several methods of proof in Mathematics are as follows:

- Direct proof

  Direct proof is usually applied to prove theorems in the form of implications $p \Rightarrow q$. Logically this direct proof is equivalent to proving that the statement $p \Rightarrow q$ is true where it is known that p is true.

- Indirect proof

  We know that the truth value of an implication $p \Rightarrow q$ is equivalent to its contraposcopic truth value $-q \Rightarrow -p$. So the work of proving the truth of the statement of implication is proven by its counterposis.

- Blank proof

If the p hypothesis of the implication p ⇒ q is false, the implication of p ⇒ q is always true of any truth value of q. So if we can show that p is wrong then we have succeeded in proving the truth p ⇒ q.

- Trivial proof
  If at the implication p ⇒ q, it can be shown that q is true, then this implication always has any true value from p. So if we can show that q is true, we have succeeded in proving the truth p ⇒ q.

- Evidence with contradictions
  In proving the truth of the implications of p ⇒ we depart from known p and - q. Departing from these two assumptions we will arrive at a contradiction.

- Existential proof
  There are two types of existential evidence, namely constructive and not constructive. In constructive methods, their existence is resolved explicitly. While in the unconstructive method, its existence is not explicitly shown.

- Proof of unity
  In proving singularity, first it must be shown the existence of an object, say the object is x. There are two approaches that can be taken to prove that x is the one and only object that satisfies, that is
  - Any object taken, say y then meeting y = x, or
  - Let y be any other object with $y \neq y$, finding a contradiction. this method does not use the contradiction method as previously provided.

- Proof with counter example.
  To prove a conjecture sometimes we need a fairly long and difficult translation. But if we can find only one case that does not meet, the conjecture is not validated.

- Prooving with mathematical induction
  The principle of mathematical induction is for inference to statements about n where n runs on a set of integers, usually the set of natural numbers N or in the set of natural numbers, $N1 \subset N$. Usually statements about natural numbers n are expressed as P (n).

- Bidirectional evidence,
  Proving the validity of the p ⇔ q implications means proving the truth of the two implications p ⇒ q and q ⇒ p. Furthermore, it can use direct, indirect or perhaps contradictory evidence.

Based on the background explained, therefore the research problem of this research was ―what was the anxiety of the deduction level students in proving the Geometry theorem?

2. Research Method

This research is a descriptive research with qualitative approach. This research aimed at describing the anxiety of the deduction level students in proving the geometry theorem. In this research, the researchers acted as the key instruments, it means that the existence of the researchers cannot be replaced by other people or something else.

The subjects of this research was the students of mathematics education at a university in Jember which had studied geometry. The samples were selected by using Van Hiele test adopted from Usiskin (1982). The sample was taken as many as 3 students who were at the deduction level. The data collection techniques used were through proof theorem test, observation and interview. The proof theorem test, observation and interview were given to the sample of this research. The theorem given to the students was ―If two lines are cut by a transverse line so that the interior angles on the same side of the transversal are supplementary (180^0), then these lines are parallel.

Data analysis carried out include data reduction, data presentation, and conclusions. Data reduction in this research is defined as activities focusing on important things and discarding unnecessary ones. All data are selected according to the need to answer the research question. The presentation of data in

this research includes the classification and identification of data, namely writing organized and categorized data sets. Based on the presentation, conclusions are made.

3. Results and discussion

3.1 Results

In this research, the researchers divided the stages or steps in proving the geometry theorem into three, as follows: understanding the theorem problems, connecting the elements of geometry system and concluding the proof of theorem.

3.1.1   Understanding the theorem problems

When starting the process of geometry proof theorem, first the students read the theorem several times then sketched and visualized the theorem with an image. The same thing also done by the second and third students, the second and third students also read the theorem several times then sketched it into an image.
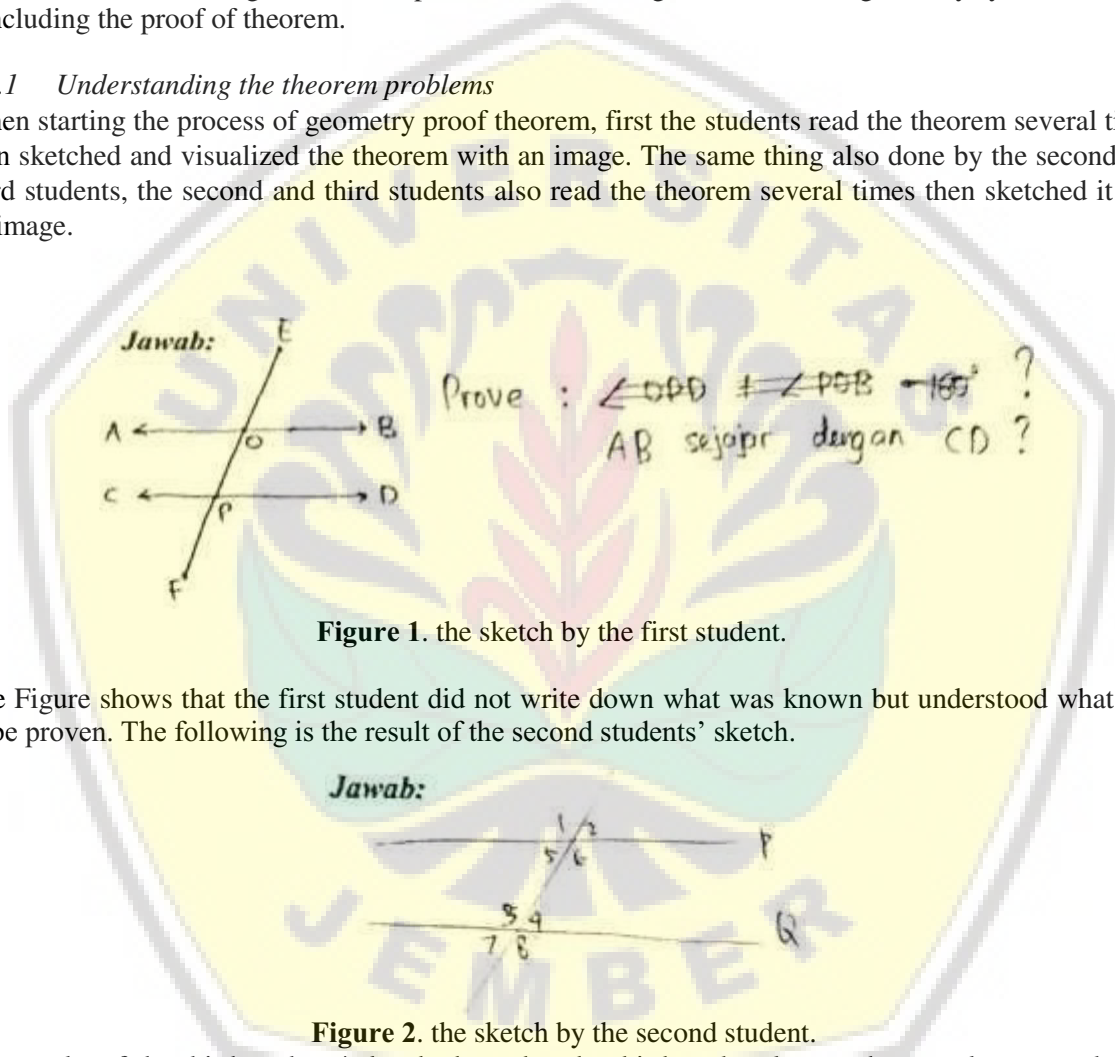

Figure 1. the sketch by the first student.

The Figure shows that the first student did not write down what was known but understood what had to be proven. The following is the result of the second students' sketch.

Figure 2. the sketch by the second student.

The results of the third students' sketch show that the third student knew what was known and what had to be proven. Based on this, it can be concluded that the three students had been able to show sketches correctly and were able to determine what need to be proven.

The anxiety symptoms happened in three students were relatively similar when starting the proof theorem. It can be seen from the face expression that showed the expression of confused, nervous and anxious. This was confirmed through the interview that they experienced confusion so that it emerged the nervous reaction and anxiety. But from the image that had been made and the confirmation results of the interview with subjects can be known that they are actually understood the problems.

The following was the result of the interview with the third student.

Researcher   : *How did you feel when you prove the theorem?*

Student : *It is quite dizzy, Mom.*

Researcher   : *Why was that?*

Student : *I learned this material in the last semester, so I was forgot.*
Researcher   : *Did you feel any heartbeat or pressure in your chest?*
Student : *Not really, Mom.. I just felt sweat.*
Researcher   : *Why were you sweaty?*
Student : *Maybe because I felt tensioned, Mom.. (smiling)*
Researcher   : *Why did you feel so?*
Student : *Probably because I learned this material last semester, so I was forgot.*
Researcher   : *Why did you look like rushed and impatient?*
Student : *Probably because I felt tensioned and panic so I looked like rushed and impatient.*

3.1.2   Connecting the elements of Geometry System
The following was the result of theorem proving test by the students.

Figure 3. The result of theorem proving test by the first student.

Figure 4. The result of theorem proving test by the second student.

Figure 5. The result of theorem proving test by the third student.

The symptoms of anxiety showed by the first student, second student, and third student when connecting the elements of geometry system was physiological and behavioral anxiety symptoms. The symptom of physiological anxiety has been shown by the first student in the form of sweaty face. Whereas, the behavioral anxiety showed by him was shaking legs for some times. After the confirmation through interview, it was known that the first student felt anxious and tense.

The following was the excerpt of the interview with the first student.

Researcher   : *How did you feel when you prove the theorem?*
Student   : *I was a bit tense, Mom.. It was because I was forgot about the process of the proof*
Researcher   : *Was that one of the causes that made you sweat?*
Student   : *Yes, probably, Mom.*
Researcher   : *Do you understand the elements of geometry? Especially the element used to prove the theorem?*
Student   : *a little, Mom.*
Researcher   : *Why did you shake your legs?*
Student   : *Yeah, Mom.. It was because I was forgot and felt anxious and tense as well.*

The symptoms of physiologically anxiety showed by the second student was sweaty and shivers. Whereas, the symptoms of behavioral anxiety that was seen from the second student were anxious and tense. This was known from the behavior that appear by playing the pen with tense expression. The following was the excerpt of the interview with the second student.

Researcher   : *How did you feel when you prove the theorem?*
Student   : *I felt tense, Mom..*
Researcher   : *Why?*
Student   : *Because I was forgot the proof.. (smiling)*
Researcher   : *Why were you sweaty?*
Student   : *Probably, because I felt tense, Mom.. So I was sweaty, I also felt hot and cold at the same time.*
Researcher   : *Why did you shake your legs or playing your pen?*
Student   : *I was thinking, Mom..*

The symptoms of physiological anxiety and the symptom of behavioral anxiety showed by the third student were sweaty, tense, and rushed or impatient. This was known from the result of the observation and interview with the student. The following was the result of the interview with the third student.

Researcher	: *How did you feel when you prove the theorem?*
Student	: *It was quite dizzy, Mom..*
Researcher	: *Why was that?*
Student	: *I learned this material in the last semester, so I was forgot.*
Researcher	: *Did you felt your heart beat fast or pressure in your chest?*
Student	: *Not really, Mom.. I just feel sweaty.*
Researcher	: *Why were you sweaty?*
Student	: *Maybe because I felt tense, Mom.. (smiling)*
Researcher	: *Why did you feel so?*
Student	: *Probably because I learned this material last semester, so I was forgot.*
Researcher	: *Why did you look rushed and impatient?*
Student	: *Probably because I felt tense and panic.*

3.1.3   Concluding the theorem proof
During concluding the proof, there was no sign of anxiety from those three students. After being confirmed by the interview, it turned out that the three students did not feel anxious when concluding the theorem proof.

3.2 Discussion
Based on the research results above it can be seen that it turns out that the level of deduction have the symptoms of cognitive, physiological and behavioral anxiety responses. Based on the test results it can be seen that the first, second and third students have been able to prove the theorem even though it is not perfect, this is in accordance with the theory expressed by Van Hiele [10] that is, at this level a person has begun to develop and deduction reasoning as a way to build geometrical structures in axiomatic systems that have been understood. This has been demonstrated by students at this level by proving a statement about geometry using logical and deductive reasons.

Based on the description of the results of the study, researchers can find out that the level of education also experiences danger that threatens him, anxiety in the form of disease and seen in several forms, anxiety due to feeling guilty or guilty because doing things that are contrary to the convictions of conscience as well as the level of analysis. So this is what makes the level of deduction more visible anxiety than the level of visualization and analysis.

4. Conclusion
The results of this research showed that the symptom of cognitive anxiety (anxious, nervous, and confused) occurred when the subjects started to understand the problem within the theorem. The symptoms of physiological anxiety (sweaty, shivers) and behavioral symptom (physical tense, impatient) appeared when the subjects connected the element of prove in the theorem in geometry.

Acknowledgent

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