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The 1st International Conference of Combinatorics, Graph Theory, and Network Topology

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The First International Conference on Combinatorics, Graph Theory and Network Topology (ICCGANT)

Dafik

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Preface

It is with my great pleasure and honor to organize the First International Conference on Combinatorics, Graph Theory and Network Topology which is held from 25-26 November 2017 in the University of Jember, East Java, Indonesia and present a conference proceeding index by Scopus. It is the first international conference organized by CGANT Research Group University of Jember in cooperation with Indonesian Combinatorics Society (INACOBMS). The conference is held to welcome participants from many countries, with broad and diverse research interests of mathematics especially combinatorical study. The mission is to become an annual international forum in the future, where, civil society organization and representative, research students, academics and researchers, scholars, scientist, teachers and practitioners from all over the world could meet in and exchange an idea to share and to discuss theoretical and practical knowledge about mathematics and its applications. The aim of the first conference is to present and discuss the latest research that contributes to the sharing of new theoretical, methodological and empirical knowledge and a better understanding in the area mathematics, application of mathematics as well as mathematics education.

The themes of this conference are as follows: (1) Connection of distance to other graph properties, (2) Degree/diameter problem, (3) Distance-transitive and distance-regular graphs, (4) Metric dimension and related parameters, (5) Cages and eccentric graphs, (6) Cycles and factors in graphs, (7) Large graphs and digraphs, (8) Spectral Techniques in graph theory, (9) Ramsey numbers, (10) Dimensions of graphs, (11) Communication networks, (12) Coding theory, (13) Cryptography, (14) Rainbow connection, (15) Graph labelings and coloring, (16). Applications of graph theory

The topics are not limited to the above themes but they also include the mathematical application research of interest in general including mathematics education, such as:(1) Applied Mathematics and Modelling, (2) Applied Physics: Mathematical Physics, Biological Physics, Chemistry Physics,(3) Applied Engineering: Mathematical Engineering, Mechanical engineering, Informatics Engineering, Civil Engineering,(4) Statistics and Its Application,(5) Pure Mathematics (Analysis, Algebra and Geometry),(6) Mathematics Education, (7) Literacy of Mathematics,(8) The Use of ICT Based Media In Mathematics Teaching and Learning,(9) Technological, Pedagogical, Content Knowledge for Teaching Mathematics, (10) Students Higher Order Thinking Skill of Mathematics, (11) Contextual Teaching and Realistic Mathematics, (12) Science, Technology, Engineering, and Mathematics Approach, (13) Local Wisdom Based

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Education: Ethnomathematics, (14) Showcase of Teaching and Learning of Mathematics, (16) The 21st Century Skills: The Integration of 4C Skill in Teaching Math.

The participants of this ICCGANT 2017 conference were 200 people consisting research students, academics and researchers, scholars, scientist, teachers and practitioners from many countries. The selected papers to be publish of Journal of Physics: Conference Series are 80 papers. On behalf of the organizing committee, finally we gratefully acknowledge the support from the University of Jember of this conference. We would also like to extend our thanks to all lovely participants who are joining this unforgettable and valuable event.

Prof. Drs. Dafik, M.Sc., Ph.D.

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The Committees of The First International Conference on Combinatorics, Graph Theory and Network Topology (ICCGANT)

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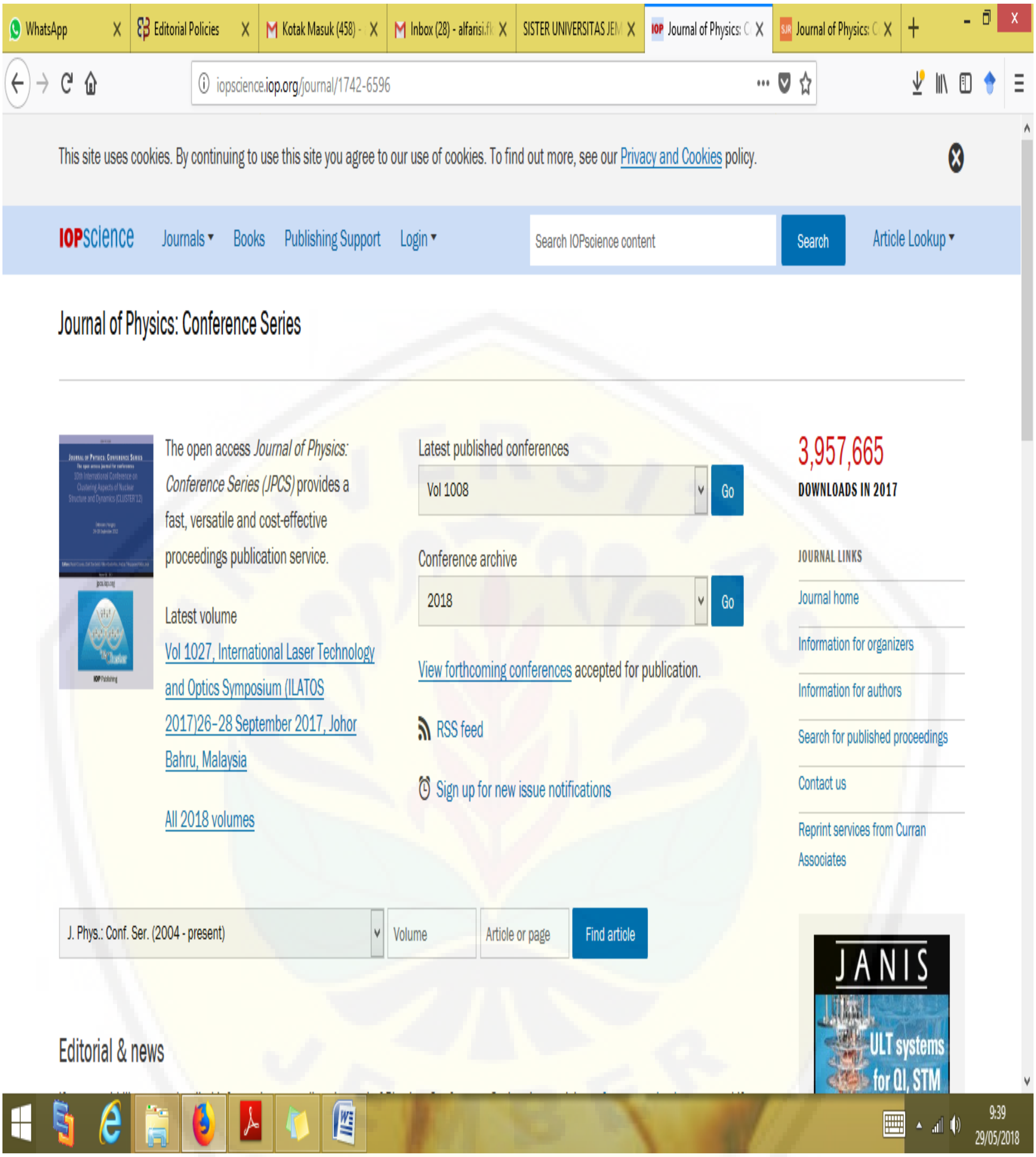

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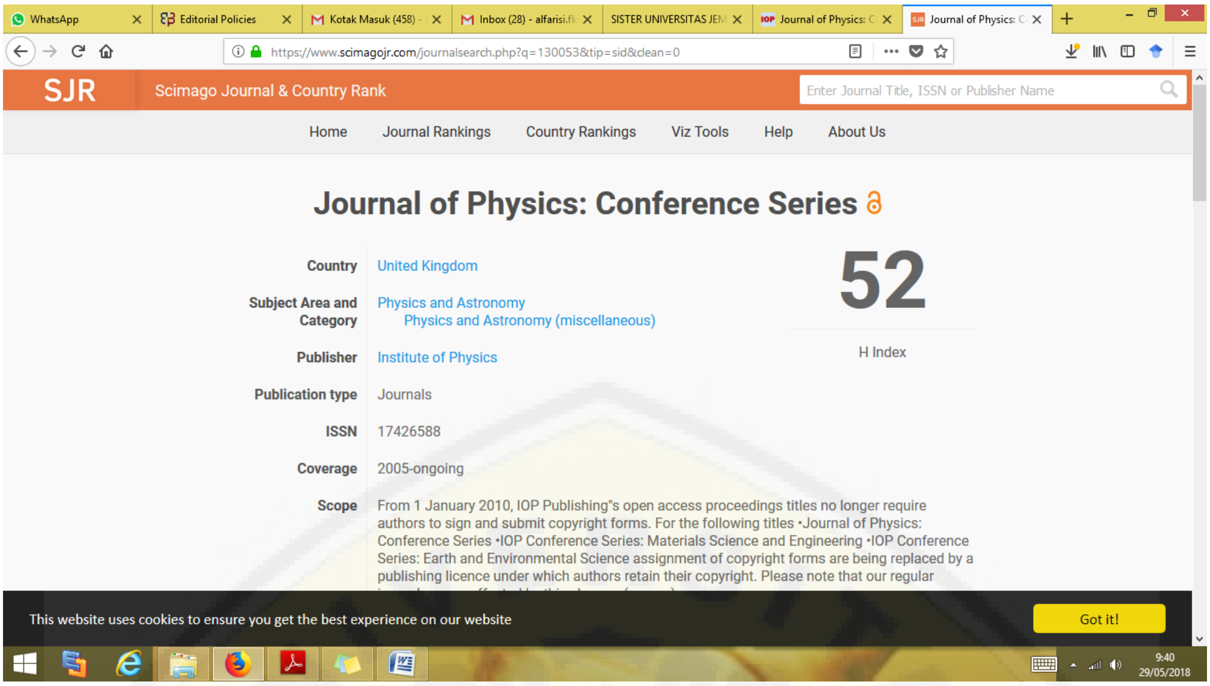

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On the total rainbow connection of the wheel related graphs

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On the total rainbow connection of the wheel related graphs

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Abstract. Let $G = (V(G), E(G))$ be a nontrivial connected graph with an edge coloring $c : E(G) \rightarrow \{1, 2, ..., l\}, l \in N$, with the condition that the adjacent edges may be colored by the same colors. A path P in G is called rainbow path if no two edges of P are colored the same. The smallest number of colors that are needed to make G rainbow edge-connected is called the rainbow edge-connection of G, denoted by $rc(G)$. A vertex-colored graph is rainbow vertex-connected if any two vertices are connected by a path whose internal vertices have distinct colors. The smallest number of colors that are needed to make G rainbow vertex-connected is called the rainbow vertex-connection of G, denoted by $rvc(G)$. A total-colored path is total-rainbow if edges and internal vertices have distinct colours. The minimum number of colour required to color the edges and vertices of G is called the total rainbow connection number of G, denoted by $trc(G)$. In this paper, we determine the total rainbow connection number of some wheel related graphs such as gear graph, antiweb-gear graph, infinite class of convex polytopes, sunflower graph, and closed-sunflower graph.

Keywords: rainbow edge-connected, rainbow vertex-connected, total rainbow connection, wheel related graphs.

1. Introduction

Let G be a nontrivial connected graph on which an edge-coloring $c : E(G) \rightarrow \{1, 2, ..., n\}, n \in N$ is defined, where the adjacent edges may be colored by the same color. A path is called a *rainbow path* if no two edges of the path have the same color. An edge-colored graph G is rainbow connected if any two vertices are connected by a rainbow path [2]. Chartrand et al. [1] defined the rainbow connection number of a connected graph G, denoted by $rc(G)$, as the smallest number of colors that are needed in order to make G rainbow connected.

A vertex-colored graph is rainbow vertex-connected if any two vertices are connected by a path whose internal vertices have distinct colors. The rainbow vertex-connection of a connected graph G, denoted by $rvc(G)$, is the smallest number of colors that are needed in order to make G rainbow vertex-connected. An easy observation is that if G is of order n, we always have $rvc(G) \leq n - 2$, and $rvc(G) = 0$ if and only if G is a complete graph. Notice that $rvc(G) = diam(G) - 1$ with equality if the diameter is 1 or 2 [4]. For rainbow connection and

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rainbow vertex-connection, some examples are given to show that there is no upper bound for one of parameters in terms of the other in [4].

A total-coloured path is total-rainbow if its edges and internal vertices have distinct colours. A total colouring of a k-connected graph G, not necessarily proper, is total-rainbow k-connected if any two vertices of G are connected by k disjoint total-rainbow paths. The total rainbow k-connection number of G, denoted by $trc(G)$, is the minimum integer t such that, there exists a total-rainbow k-connected colouring of G using t colours. We have $trc_k(G)$ is well-defined if and only if G is k-connected. We write $trc(G)$ for $trc_1(G)$, and similarly for $rc(G)$ and $rvc(G)$ [7]. Li *et al.* [6] proved that for any fixed $k \geq 1$ it is NP-complete to decide whether $trc_k(G) = 3$. The following theorem due to Liu *et al.* [7].

Theorem 1.1 *[7] Let G be a nontrivial connected graph, the total rainbow connection number of G is $trc(G) \geq 2diam(G) - 1$.*

In this paper, we investigate the total rainbow connection number $trc_k(G)$ of some wheel related graphs such as gear graph [10], antiweb-gear graph [9], infinite class of convex polytopes [9], sunflower graph [8], and closed-sunflower graph.

2. Main Results

We start this section with the total rainbow connection of a gear graph as follows.

Theorem 2.1 *If J_n is a gear graph with $n \geq 3$, then the total rainbow connection of J_n is*

$$trc(J_n) = \begin{cases} 3 & \text{for } n = 2 \\ 5 & \text{for } n = 3 \\ 7 & \text{for } n \geq 4 \text{ and } n \text{ even} \\ 8 & \text{for } n \geq 5 \text{ and } n \text{ odd.} \end{cases}$$

Proof. Let J_n, for $n \geq 3$, be a gear graph. Then J_n is a connected graph with vertex set $V(J_n) = \{x, y_i, z_i; 1 \leq i \leq n\}$ and $E(J_n) = \{xy_i; 1 \leq i \leq n\} \cup \{y_iz_i; 1 \leq i \leq n\} \cup \{z_iy_{i+1}, z_ny_1; 1 \leq i \leq n-1\}$. The cardinality of vertex set and edge set, respectively are $|V(J_n)| = 2n + 1$ and $|E(J_n)| = 3n$, and $diam(J_n)$ is

$$diam(J_n) = \begin{cases} 2 & \text{for } n = 2 \\ 3 & \text{for } n = 3 \\ 4 & \text{otherwise.} \end{cases}$$

For $n = 2$, based on Theorem 1.1 that $trc(J_2) \geq 2diam(J_2) - 1 = 2(2) - 1 = 3$. However, we can attain the lower bound. Furthermore, we prove that $trc(J_2) \leq 3$, by defining the edge coloring $f_1(e)$ with

$$f_1(e) = \begin{cases} 1 & \text{for } e \in \{y_iz_i | 1 \leq i \leq n\} \cup \{xy_1\} \\ 2 & \text{for } e \in \{z_1y_2, z_2y_1, xy_2\} \end{cases}$$

and by defining the vertex coloring $f_1(v)$ with

$$f_1(v) = 3 \text{ for } v \in \{x\} \cup \{y_i | 1 \leq i \leq n\} \cup \{z_i | 1 \leq i \leq n\}.$$

Hence, it can be seen that it $trc(J_2) \leq 3$. Thus, the total rainbow connection of J_2 is $trc(J_2) = 3$.

For $n = 3$, based on Theorem 1.1 that $trc(J_3) \geq 2diam(J_3) - 1 = 2(3) - 1 = 5$. However, we can attain the lower bound. Furthermore, we prove that $trc(J_3) \leq 5$, by defining the edge coloring $f_2(e)$ with

$$f_2(e) = \begin{cases} i & \text{for } e \in \{xy_i | 1 \le i \le n\} \\ 1 & \text{for } e \in \{y_1z_1, z_2y_3\} \\ 2 & \text{for } e \in \{z_1y_2, y_3z_3\} \\ 3 & \text{for } e \in \{y_2z_2, z_3y_1\} \end{cases}$$

and by defining the vertex coloring $f_2(v)$ with

$$f_2(v) = \begin{cases} 4 & \text{for } v \in \{x\} \\ 5 & \text{for } v \in \{y_i | 1 \le i \le n\} \cup \{z_i | 1 \le i \le n\} \end{cases}$$

Hence, it can be seen that it $trc(J_3) \le 5$. Thus, the total rainbow connection of J_3 is $trc(J_3) = 5$.

For $n \ge 4$ and n even, based on Theorem 1.1 that $trc(J_n) \ge 2diam(J_n) - 1 = 2(4) - 1 = 7$. However, we can attain the lower bound. Furthermore, we prove that $trc(J_n) \le 7$, by defining the edge coloring $f_3(e)$ with

$$f_3(e) = \begin{cases} 1 & \text{for } e \in \{xy_i | 1 \le i \le n \text{ and } i \text{ is odd}\} \\ 2 & \text{for } e \in \{xy_i | 1 \le i \le n \text{ and } i \text{ is even}\} \\ 3 & \text{for } e \in \{y_iz_i | 1 \le i \le n \text{ and } i \text{ is odd}\} \cup \\ & \qquad \{z_iy_{i+1} | 1 \le i \le n-1 \text{ and } i \text{ is even}\} \cup \{z_ny_1\} \\ 4 & \text{for } e \in \{y_iz_i | 1 \le i \le n \text{ and } i \text{ is even}\} \cup \\ & \qquad \{z_iy_{i+1} | 1 \le i \le n-1 \text{ and } i \text{ is odd}\} \end{cases}$$

and by defining the vertex coloring $f_3(v)$ with

$$f_3(v) = \begin{cases} 5 & \text{for } v \in \{y_i | 1 \le i \le n \text{ and } i \text{ is odd}\} \\ 6 & \text{for } v \in \{y_i | 1 \le i \le n \text{ and } i \text{ is even}\} \\ 7 & \text{for } v \in \{x\} \cup \{z_i | 1 \le i \le n\}. \end{cases}$$

Hence, it can be seen that it $trc(J_n) \le 7$. Thus, the total rainbow connection of J_n is $trc(J_n) = 7$.

Now, for $n \ge 5$ and n is odd, we get $f(e)$ and $f(v)$ above such that it show $trc(J_n) \le 7$. But if n is odd, there is a path that has the same color. A path y_i to z_n *(i odd)* has two choices of *path* that can be passed, that is $y_i - x - y_1 - z_n$ *(i odd)* or $y_i - x - y_n - z_n$ *(i odd)*, but of the two options the *path* has the same color, so that when n odd $f(e)$ is not enough if only colored with four colors. Then there is a color change on the edge $x - y_n$, colored with 5. So that $trc(J_n) \ge 8$, when n is odd. □

Figure 1 shows the example of the total rainbow connection of the gear graph J_6 with $trc(J_6) = 7$.

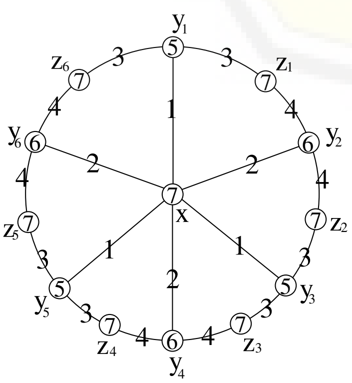

Figure 1. The total rainbow connection of the gear graph J_6 with $trc(J_6) = 7$.

The following theorem presents the total rainbow connection of an antiweb-gear graph.

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Theorem 2.2 *If AWJ_n for $n \geq 3$ is an antiweb-gear graph, then the total rainbow connection of AWJ_n is*

$$trc(AWJ_n) = \begin{cases} 3 & for\ 3 \leq n \leq 4 \\ 5 & for\ 5 \leq n \leq 6 \\ 7 & for\ n \geq 7 \end{cases}$$

Proof. Let AWJ_n for $n \geq 3$ be an antiweb-gear graph. Then AWJ_n is a connected graph with vertex set $V(AWJ_n) = \{x, y_i, z_i; 1 \leq i \leq n\}$ and $E(AWJ_n) = \{xy_i; 1 \leq i \leq n\} \cup \{y_iz_i; 1 \leq i \leq n\} \cup \{z_iy_{i+1}, z_ny_1; 1 \leq i \leq n-1\} \cup \{y_iy_{i+1}, y_ny_1; 1 \leq i \leq n-1\} \cup \{z_iz_{i+1}, z_nz_1; 1 \leq i \leq n-1\}$. The cardinality of vertex set and edge set, respectively are $|V(AWJ_n)| = 2n+1$ and $|E(AWJ_n)| = 5n$, and $diam(AWJ_n)$ is

$$diam(AWJ_n) = \begin{cases} 2 & \text{for } 3 \leq n \leq 4 \\ 3 & \text{for } 5 \leq n \leq 7 \\ 4 & \text{otherwise.} \end{cases}$$

For $3 \leq n \leq 4$, based on Theorem 1.1 that $trc(AWJ_n) \geq 2diam(AWJ_n) - 1 = 2(2) - 1 = 3$. However, we can attain the lower bound. Furthermore, we prove that $trc(AWJ_n) \leq 3$, by defining the edge coloring $f_4(e)$ with

$$f_4(e) = \begin{cases} 1 & \text{for } e \in \{xy_1, xy_2, y_1y_2, y_4y_1, z_1y_2, z_2y_3, y_3z_3, y_4z_4\} \cup \{z_iz_{i+1}|1 \leq i \leq n \text{ and } i \text{ is odd}\} \\ 2 & \text{for } e \in \{xy_3, xy_4, z_3y_4, z_4y_1, y_2y_3, y_3y_4, z_2z_3, z_4z_1, y_1z_1, y_2z_2\} \end{cases}$$

and the vertex coloring $f_4(v)$ with

$$f_4(v) = 3 \text{ for } v \in \{x\} \cup \{y_i|1 \leq i \leq n\} \cup \{z_i|1 \leq i \leq n\}.$$

Hence, it can be seen that it $trc(AWJ_n) \leq 3$. Thus, the total rainbow connection of AWJ_n is $trc(AWJ_n) = 3$.

For $5 \leq n \leq 6$, based on Theorem 1.1 that $trc(AWJ_n) \geq 2diam(AWJ_n) - 1 = 2(3) - 1 = 5$. However, we can attain the lower bound. Furthermore, we prove that $trc(AWJ_n) \leq 5$, by defining the edge coloring $f_5(e)$ with

$$f_5(e) = \begin{cases} 1 & \text{for } e \in \{xy_1, xy_2, xy_3\} \\ 2 & \text{for } e \in \{xy_4, xy_5, xy_6\} \\ 3 & \text{for } e \in \{y_iz_i|1 \leq i \leq n\} \cup \{z_iy_{i+1}|1 \leq i \leq n-1\} \cup \{z_ny_1\} \\ i & \text{for } e \in \{y_1y_2, y_2y_3, y_3y_4, z_1z_2, z_2z_3, z_3z_4\} \\ i-3 & \text{for } e \in \{y_iy_{i+1}|4 \leq i \leq n-1\} \cup \{z_iz_{i+1}|4 \leq i \leq n-1\} \\ n-3 & \text{for } e \in \{y_ny_1, z_nz_1\} \end{cases}$$

and the vertex coloring $f_5(v)$ with

$$f_5(v) = \begin{cases} 4 & \text{for } v \in \{y_i|1 \leq i \leq n\} \cup \{z_i|1 \leq i \leq n \text{ and } i \text{ is odd}\} \\ 5 & \text{for } v \in \{x\} \cup \{z_i|1 \leq i \leq n \text{ and } i \text{ is even}\}. \end{cases}$$

Hence, it can be seen that it $trc(AWJ_n) \leq 5$. Thus, the total rainbow connection of AWJ_n is $trc(AWJ_n) = 5$.

Now, we prove the lower bound for $n = 7$. For $n = 7$ when $diam(AWJ_7) = 3$, based on Theorem 1.1 that $trc(AWJ_7) \geq 5$. But AWJ_7 have a path $z_1 - z_2 - z_3 - \cdots - z_n$, when a path is *cycle graph* with $n = 7$. By [7] $trc(C_7) \geq 6$. So that $trc(AWJ_7) \geq 6$.

For $n \geq 8$, based on Theorem 1.1 that $trc(AWJ_n) \geq 2diam(AWJ_n) - 1 = 2(4) - 1 = 7$. However, we can attain the lower bound. Furthermore, we prove that $trc(AWJ_n) \leq 7$, by defining the edge coloring $f_6(e)$ with

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$$f_6(e) = \begin{cases} 1 & \text{for } e \in \{xy_i | 1 \le i \le n \text{ and } i \text{ is odd}\} \\ 2 & \text{for } e \in \{xy_i | 1 \le i \le n \text{ and } i \text{ is even}\} \\ 3 & \text{for } e \in \{y_iy_{i+1} | 1 \le i \le n-1\} \cup \{y_ny_1\} \cup \{z_iz_{i+1} | 1 \le i \le n-1\} \cup \{z_ny_1\} \cup \\ & \qquad \{y_iz_{i+1} | 1 \le i \le n-1 \text{ and } i \text{ is odd}\} \cup \{z_iy_{i+1} | 1 \le i \le n-1 \text{ and } i \text{ is even}\} \\ 4 & \text{for } e \in \{y_iz_i | 1 \le i \le n \text{ and } i \text{ is even}\} \cup \{z_iy_{i+1} | 1 \le i \le n-1\} \cup \{z_ny_1\} \end{cases}$$

and the vertex coloring $f_6(v)$ with

$$f_6(v) = \begin{cases} 5 & \text{for } v \in \{y_i | 1 \le i \le n \text{ and } i \text{ is odd}\} \\ 6 & \text{for } v \in \{y_i | 1 \le i \le n \text{ and } i \text{ is even}\} \\ 7 & \text{for } v \in \{x\} \cup \{z_i; 1 \le i \le n\}. \end{cases}$$

Hence, it can be seen that it $trc(AWJ_n) \le 7$. Thus, the total rainbow connection of AWJ_n is $trc(AWJ_n) = 7$ for $n \ge 7$. □

Figure 2 shows the example of the total rainbow connection of the antiweb-gear graph AWJ_5 with $trc(AWJ_5) = 5$.

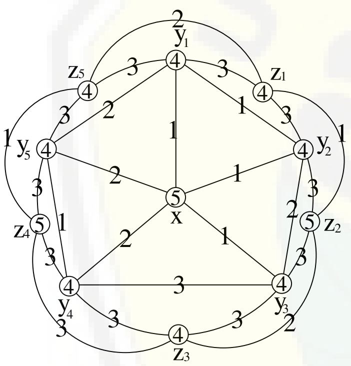

Figure 2. The total rainbow connection of the antiweb-gear graph AWJ_5 with $trc(AWJ_5) = 5$

The total rainbow connection of an infinite class of convex polytopes is presented below.

Theorem 2.3 *If $\mathbf{Q}_n$ for $n \ge 3$ is an infinite class of convex polytopes, then the total rainbow connection of $\mathbf{Q}_n$ is $trc(\mathbf{Q}_n) = 5$.*

Proof. Let $\mathbf{Q}_n$ for $n \ge 3$ be an infinite class of convex polytopes. Then $\mathbf{Q}_n$ is a connected graph with vertex set $V(\mathbf{Q}_n) = \{P, x, y_i, z_i; 1 \le i \le n\}$ and $E(\mathbf{Q}_n) = \{xy_i; 1 \le i \le n\} \cup \{y_iy_{i+1}, y_ny_1; 1 \le i \le n-1\} \cup \{y_iz_i; 1 \le i \le n\} \cup \{z_iy_{i+1}, z_ny_1; 1 \le i \le n-1\} \cup \{z_iz_{i+1}, z_nz_1; 1 \le i \le n-1\} \cup \{Pz_i; 1 \le i \le n\}$. The cardinality of vertex set and edge set, respectively are $|V(\mathbf{Q}_n)| = 2n + 1$ and $|E(\mathbf{Q}_n)| = 5n$, and $diam(\mathbf{Q}_n) = 3$. Based on Theorem 1.1 that $trc(\mathbf{Q}_n) \ge 2diam(\mathbf{Q}_n) - 1 = 2(3) - 1 = 5$. However, we can attain the lower bound. Furthermore, we prove that $trc(\mathbf{Q}_n) \le 5$, by defining the edge coloring $f_7(e)$ with

$$f_7(e) = \begin{cases} 1 & \text{for } e \in \{xy_i | 1 \le i \le n \text{ and } i \text{ is odd}\} \cup \{Pz_i | 1 \le i \le n \text{ and } i \text{ is odd}\} \\ 2 & \text{for } e \in \{xy_i | 1 \le i \le n \text{ and } i \text{ is even}\} \cup \{Pz_i | 1 \le i \le n \text{ and } i \text{ is even}\} \\ 3 & \text{for } e \in \{y_iy_{i+1} | 1 \le i \le n-1\} \cup \{y_ny_1\} \cup \{z_iy_{i+1} | 1 \le i \le n-1\} \cup \\ & \qquad \{z_iz_{i+1} | 1 \le i \le n-1\} \cup \{z_nz_{i+1} | 1 \le i \le n-1\} \cup \{y_iz_i | 1 \le i \le n\} \end{cases}$$

and the vertex coloring $f_7(v)$ with

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$$f_7(v) = \begin{cases} 4 & \text{for } v \in \{y_i | 1 \leq i \leq n\} \cup P \\ 5 & \text{for } v \in \{x\} \cup \{z_i | 1 \leq i \leq n\}. \end{cases}$$

Hence, it can be seen that it $trc(\mathbf{Q}_n) \leq 5$. Thus, the total rainbow connection of $\mathbf{Q}_n$ is $trc(\mathbf{Q}_n) = 5$. □

Figure 3 shows the example of the total rainbow connection of the infinite class of convex polytopes $\mathbf{Q}_5$ with $trc(\mathbf{Q}_5) = 5$.

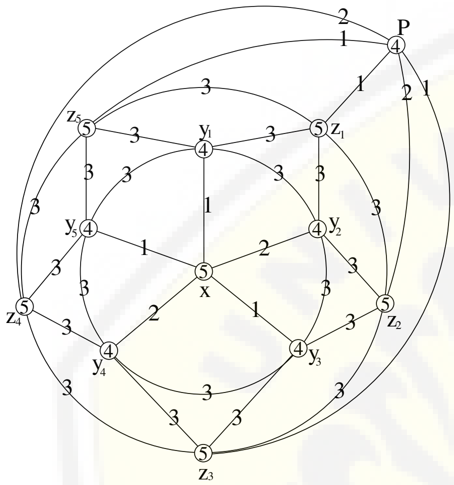

Figure 3. The total rainbow connection of the infinite class of convex polytopes $\mathbf{Q}_5$ with $trc(\mathbf{Q}_5) = 5$

The following theorem provide the total rainbow connection of a sunflower graph.

Theorem 2.4 *If SF_n for $n \geq 3$ is a sunflower graph, then the total rainbow connection of SF_n is*

$$trc(SF_n) = \begin{cases} 3 & \text{for } n = 3 \\ 5 & \text{for } n = 4 \\ 7 & \text{for } n \geq 5. \end{cases}$$

Proof: Let SF_n for $n \geq 3$ be a sunflower graph. Then SF_n is a connected graph with vertex set $V(SF_n) = \{x, y_i, z_i; 1 \leq i \leq n\}$ and $E(SF_n) = \{xy_i; 1 \leq i \leq n\} \cup \{y_iy_{i+1}, y_ny_1; 1 \leq i \leq n-1\} \cup \{y_iz_i; 1 \leq i \leq n\} \cup \{z_iy_{i+1}, z_ny_1; 1 \leq i \leq n-1\}$. The cardinality of vertex set and edge set, respectively are $|V(SF_n)| = 2n + 1$ and $|E(SF_n)| = 4n$, and $diam(SF_n)$ is

$$diam(SF_n) = \begin{cases} 2 & \text{for } n = 3 \\ 3 & \text{for } 4 \leq n \leq 5 \\ 4 & \text{otherwise.} \end{cases}$$

For $n = 3$, based on Theorem 1.1 that $trc(SF_3) \geq 2diam(SF_3) - 1 = 2(2) - 1 = 3$. However, we can attain the lower bound. Furthermore, we prove that $trc(SF_3) \leq 3$, by defining the edge coloring $f_8(e)$ with

$$f_8(e) = \begin{cases} 1 & \text{for } e \in \{xy_i | 1 \leq i \leq n\} \cup \{y_iz_i | 1 \leq i \leq n\} \\ 2 & \text{for } e \in \{y_iy_{i+1} | 1 \leq i \leq n-1\} \cup \{y_ny_1\} \cup \{z_iy_{i+1} | 1 \leq i \leq n-1\} \cup \{z_ny_1\} \end{cases}$$

and the vertex coloring $f_8(v)$ with

$$f_8(v) = 3 \text{ for } v \in \{x\} \cup \{y_i | 1 \leq i \leq n\} \cup \{z_i | 1 \leq i \leq n\}.$$

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Hence, it can be seen that it $trc(SF_3) \leq 3$. Thus, the total rainbow connection of SF_3 is $trc(SF_3) = 3$.

For $n = 4$, based on Theorem 1.1 that $trc(SF_4) \geq 2diam(SF_4) - 1 = 2(3) - 1 = 5$. However, we can attain the lower bound. Furthermore, we prove that $trc(SF_4) \leq 5$, by defining the edge coloring $f_9(e)$ with

$$f_9(e) = \begin{cases} 1 & \text{for } e \in \{xy_i | 1 \leq i \leq 2\} \cup \{y_i z_i | 1 \leq i \leq n\} \\ 2 & \text{for } e \in \{z_i y_{i+1} | 1 \leq i \leq n-1\} \cup \{z_n z_1\} \\ 3 & \text{for } e \in \{xy_i | 3 \leq i \leq 4\} \cup \{y_n y_1\} \cup \{y_i y_{i+1} | 1 \leq i \leq n-1\} \end{cases}$$

and the vertex coloring $f_9(v)$ with

$$f_9(v) = \begin{cases} 4 & \text{for } v \in \{x\} \cup \{y_i | 1 \leq i \leq n \text{ and } i \text{ is odd}\} \\ 5 & \text{for } v \in \{y_i | 1 \leq i \leq n \text{ and } i \text{ is even}\} \cup \{z_i | 1 \leq i \leq n\}. \end{cases}$$

Hence, it can be seen that it $trc(SF_4) \leq 5$. Thus, the total rainbow connection of SF_4 is $trc(SF_4) = 5$.

Now, we prove the lower bound for $n = 5$. For $n = 5$ when $diam(SF_5) = 3$, based on Theorem 1.1 that $trc(SF_5) \geq 5$. But SF_5 have a path $y_1 - y_2 - y_3 - y_4 - y_5 - y_1$, when a path is *cycle graph* with $n = 5$. By [7] $trc(C_5) \geq 3$. Then, by adding colors to the other vertices and other edges, we have $trc(SF_5) \geq 7$.

For $n \geq 6$, based on Theorem 1.1 that $trc(SF_n) \geq 2diam(SF_n) - 1 = 2(4) - 1 = 7$. However, we can attain the lower bound. Furthermore, we prove that $trc(SF_n) \leq 7$, by defining the edge coloring $f_{10}(e)$ with

$$f_{10}(e) = \begin{cases} 1 & \text{for } e \in \{xy_i | 1 \leq i \leq n \text{ and } i \text{ is odd}\} \\ 2 & \text{for } e \in \{xy_i | 1 \leq i \leq n \text{ and } i \text{ is even}\} \\ 3 & \text{for } e \in \{y_i y_{i+1} | 1 \leq i \leq n-1\} \cup \{y_n y_1\} \cup \{y_i z_i | 1 \leq i \leq n \text{ and } i \text{ is odd}\} \cup \\ & \qquad \{z_i y_{i+1} | 1 \leq i \leq n-1 \text{ and } i \text{ is even}\} \cup \{z_n y_1 | n \text{ even}\} \\ 4 & \text{for } e \in \{z_i y_{i+1} | 1 \leq i \leq n-1 \text{ and } i \text{ is odd}\} \cup \{z_n y_1 | n \text{ odd}\} \cup \\ & \qquad \{y_i z_i | 1 \leq i \leq n \text{ and } i \text{ is even}\} \end{cases}$$

and the vertex coloring $f_{10}(v)$ with

$$f_{10}(v) = \begin{cases} 5 & \text{for } v \in \{y_i | 1 \leq i \leq n \text{ and } i \text{ is odd}\} \cup \{z_i | 1 \leq i \leq n\} \\ 6 & \text{for } v \in \{y_i | 1 \leq i \leq n \text{ and } i \text{ is even}\} \\ 7 & \text{for } v \in \{x\}. \end{cases}$$

Hence, it can be seen that it $trc(SF_n) \leq 7$. Thus, the total rainbow connection of SF_n is $trc(SF_n) = 7$ for $n \geq 6$. □

Figure 4 shows the example of the total rainbow connection of the sunflower graph SF_6 with $trc(SF_6) = 7$.

We close this section with the total rainbow connection of a closed-sunflower graph as presented in the following theorem.

Theorem 2.5 *If CSF_n for $n \geq 3$ is a closed-sunflower graph, then the total rainbow connection of CSF_n is*

$$trc(CSF_n) = \begin{cases} 3 & \text{for } 3 \leq n \leq 4 \\ 5 & \text{for } 5 \leq n \leq 6 \\ 7 & \text{for } n \geq 7. \end{cases}$$

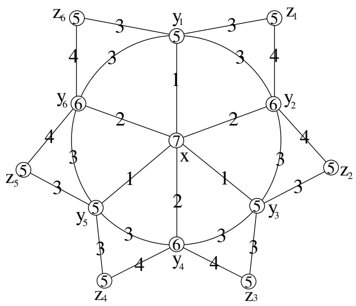

Figure 4. The total rainbow connection of the sunflower graph SF_6 with $trc(SF_6) = 7$

Proof: Let CSF_n for $n \geq 3$ be a closed-sunflower graph. Then CSF_n is a connected graph with vertex set $V(CSF_n) = \{x, y_i, z_i;\ 1 \leq i \leq n\}$ and $E(CSF_n) = \{xy_i; 1 \leq i \leq n\} \cup \{y_iy_{i+1}, y_ny_1; 1 \leq i \leq n-1\} \cup \{y_iz_i; 1 \leq i \leq n\} \cup \{z_iy_{i+1}, z_ny_1; 1 \leq i \leq n-1\} \cup \{z_iz_{i+1}, z_nz_1; 1 \leq i \leq n-1\}$. The cardinality of vertex set and edge set, respectively are $|V(CSF_n)| = 2n+1$ and $|(CSF_n)E| = 5n$, and $diam(CSF_n)$ is

$$diam(CSF_n) = \begin{cases} 2 & \text{for } 3 \leq n \leq 4 \\ 3 & \text{for } 5 \leq n \leq 7 \\ 4 & \text{for } n \geq 8. \end{cases}$$

For $3 \leq n \leq 4$, based on Theorem 1.1 that $trc(CSF_n) \geq 2diam(CSF_n) - 1 = 2(2) - 1 = 3$. However, we can attain the lower bound. Furthermore, we prove that $trc(CSF_n) \leq 3$, by defining the edge coloring $f_{11}(e)$ with

$$f_{11}(e) = \begin{cases} 1 & \text{for } e \in \{xy_i | 1 \leq i \leq n \text{ and } i \text{ is even}\} \cup \{y_iy_{i+1} | 1 \leq i \leq n-1 \text{ and } i \text{ is odd}\} \cup \\ & \qquad \{z_iy_{i+1} | 1 \leq i \leq n-1 \text{ and } i \text{ is odd}\} \cup \{y_iz_i | 1 \leq i \leq n \text{ and } i \text{ is odd}\} \cup \\ & \qquad \{y_ny_1\} \cup \{z_ny_1\} \cup \{z_nz_1 | n \text{ odd}\} \cup \{z_iz_{i+1} | 1 \leq i \leq n-1 \text{ and } i \text{ is odd}\} \\ 2 & \text{for } e \in \{xy_i | 1 \leq i \leq n \text{ and } i \text{ is odd}\} \cup \{y_iy_{i+1} | 1 \leq i \leq n-1 \text{ and } i \text{ is even}\} \cup \\ & \qquad \{z_iy_{i+1} | 1 \leq i \leq n-1 \text{ and } i \text{ is even}\} \cup \{y_iz_i | 1 \leq i \leq n \text{ and } i \text{ is even} \cup \\ & \qquad \{y_ny_1\} \cup \{z_ny_1\} \cup \{z_nz_1 | n \text{ even}\} \cup \{z_iz_{i+1} | 1 \leq i \leq n-1 \text{ and } i \text{ is even}\} \end{cases}$$

and the vertex coloring $f_{11}(v)$ with

$$f_{11}(v) = 3 \text{ for } v \in \{x\} \cup \{y_i | 1 \leq i \leq n\} \cup \{z_i | 1 \leq i \leq n\}.$$

Hence, it can be seen that it $trc(CSF_n) \leq 3$. Thus, the total rainbow connection of CSF_n is $trc(CSF_n) = 3$.

For $5 \leq n \leq 6$, based on Theorem 1.1 that $trc(CSF_n) \geq 2diam(CSF_n) - 1 = 2(3) - 1 = 5$. However, we can attain the lower bound. Furthermore, we prove that $trc(CSF_n) \leq 5$, by defining the edge coloring $f_{12}(e)$ with

$$f_{12}(e) = \begin{cases} 1 & \text{for } e \in \{z_4z_5\} \cup \{xy_i | 1 \leq i \leq n \text{ and } i \text{ is odd}\} \\ 2 & \text{for } e \in \{xy_i | 1 \leq i \leq n \text{ and } i \text{ even}\} \cup \{z_nz_1 | n \text{ odd}\} \cup \{z_{n-1}z_n | n \text{ even}\} \\ 3 & \text{for } e \in \{y_iy_{i+1} | 1 \leq i \leq n-1\} \cup \{z_iy_{i+1} | 1 \leq i \leq n-1\} \cup \{y_ny_1\} \cup \{z_ny_1\} \cup \\ & \qquad \{y_iz_i | 1 \leq i \leq n\} \cup \{z_nz_1 | n \text{ even}\} \\ i & \text{for } e \in \{z_iz_{i+1} | 1 \leq i \leq 3\} \end{cases}$$

and by defining the vertex coloring $f_{12}(v)$ in the following:

$$f_{12}(v) = \begin{cases} 4 & \text{for } v \in \{x\} \cup \{z_i | 1 \leq i \leq n \text{ and } i \text{ is odd}\} \\ 5 & \text{for } v \in \{y_i | 1 \leq i \leq n\} \cup \{z_i | 1 \leq i \leq n \text{ and } i \text{ is even}\} \end{cases}$$

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Hence, it can be seen that it $trc(CSF_n) \leq 5$. Thus, the total rainbow connection of CSF_n is $trc(CSF_n) = 5$.

Now, we prove the lower bound for $n = 7$. For $n = 7$ when $diam(CSF_7) = 3$, based on Theorem 1.1 that $trc(CSF_7) \geq 5$. But CSF_7 have a path $z_1 - z_2 - z_3 - z_4 - z_5 - z_6 - z_7 - z_1$, when a path is *cycle graph* with $n = 7$. By [7] $trc(C_7) \geq 6$. Then, by adding colors to the other vertices and other edges, we have $trc(CSF_7) \geq 7$.

For $n \geq 8$, based on Theorem 1.1 that $trc(CSF_n) \geq 2diam(CSF_n) - 1 = 2(4) - 1 = 7$. However, we can attain the lower bound. Furthermore, we prove that $trc(CSF_n) \leq 7$, by defining the edge coloring $f_{13}(e)$ with

$$f_{13}(e) = \begin{cases} 1 & \text{for } e \in \{xy_i | 1 \leq i \leq n \text{ and } i \text{ is odd}\} \\ 2 & \text{for } e \in \{xy_i | 1 \leq i \leq n \text{ and } i \text{ is even}\} \\ 3 & \text{for } e \in \{y_i y_{i+1} | 1 \leq i \leq n-1\} \cup \{z_i z_{i+1} | 1 \leq i \leq n-1\} \cup \{y_n y_1\} \cup \\ & \qquad \{y_i z_i | 1 \leq i \leq n \text{ and } i \text{ is odd}\} \cup \{e = z_n z_1\} \cup \{z_n z_1 | n \text{ even}\} \cup \\ & \qquad \{z_i y_{i+1} | 1 \leq i \leq n-1 \text{ and } i \text{ is even}\} \\ 4 & \text{for } e \in \{y_i z_i | 1 \leq i \leq n \text{ and } i \text{ is even}\} \cup \{z_i y_{i+1} | 1 \leq i \leq n-1 \text{ and } i \text{ is odd}\} \cup \\ & \qquad \{z_n y_1 | n \text{ odd}\} \end{cases}$$

and the vertex coloring $f_{13}(v)$ with

$$f_{13}(v) = \begin{cases} 5 & \text{for } v \in \{y_i | 1 \leq i \leq n \text{ and } i \text{ is odd}\} \cup \{z_i | 1 \leq i \leq n\} \\ 6 & \text{for } v \in \{y_i | 1 \leq i \leq n \text{ and } i \text{ is even}\} \\ 7 & \text{for } v \in \{x\}. \end{cases}$$

Hence, it can be seen that it $trc(CSF_n) \leq 7$. Thus, the total rainbow connection of CSF_n is $trc(CSF_n) = 7$ for $n \geq 8$. □

Figure 5 shows the example of the total rainbow connection of the closed-sunflower graph CSF_6 with $trc(CSF_6) = 5$.

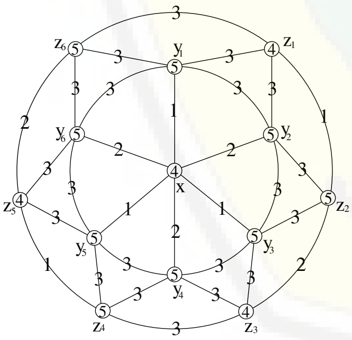

Figure 5. The total rainbow connection of the closed-sunflower graph CSF_6 with $trc(CSF_6) = 5$

3. Conclusion

In this paper we have given an total rainbow connection of some wheel related graphs namely gear graph, antiweb-gear graph, infinite class of convex polytopes, sunflower graph, and closed-sunflower graph. We give the following open problem for future work.

Open Problem 3.1 *Determine the total rainbow connection number of other wheel related graphs that have not been discovered.*

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