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On super $\mathcal{H}-$antimagicness of an edge comb product of graphs with subgraph as a terminal of its amalgamation

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Abstract. All graphs in this paper are simple, finite, and undirected graph. Let r be a edges of H. The edge comb product between L and H, denoted by $L\trianglerighteq H$, is a graph obtained by taking one copy of L and $|E(L)|$ copies of H and grafting the i-th copy of H at the edges r to the i-th edges of L, we call such a graph as an edge comb product of graph with subgraph as a terminal of its amalgamation, denoted by $G = K\trianglerighteq Amal(H, L \subset H, n)$. The graph G is said to admits an (a, d)-H-antimagic total labeling if there exist a bijection $f : V(G) \cup E(G) \rightarrow \{1, 2, \ldots, |V(G)| + |E(G)|\}$ such that for all subgraphs isomorphic to H, the total H-weights $W(H) = \sum_{v\in V(H)} f(v) + \sum_{e\in E(H)} f(e)$ form an arithmetic sequence $\{a, a + d, a + 2d, ..., a + (t - 1)d\}$, where a and d are positive integers and t is the number of all subgraphs isomorphic to H. An (a, d)-H-antimagic total labeling f is called super if the smallest labels appear in the vertices. In this paper, we will study the super $\mathcal{H}-$antimagicness of disjoint union of edge comb product of graphs with subgraph as a terminal of its amalgamation.

Keywords: Graph amalgamation, edge comb product, $\mathcal{H}$-Antimagic total labeling.

1. Introduction

Let $G = (V(G), E(G))$ be a simple, finite, and undirected graph with vertex set $V(G)$ and edge set $E(G)$. The graph G is said to admit an (a, d)-H-antimagic total labeling if there exist a bijection $f : V(G)\cup E(G) \rightarrow \{1, 2, \ldots, |V(G)|+|E(G)|\}$ such that for all subgraphs isomorphic to H, the total H-weights $W(H) = \sum_{v\in V(H)} f(v)+\sum_{e\in E(H)} f(e)$ form an arithmetic sequence $\{a, a + d, a + 2d, ..., a + (t - 1)d\}$, where a and d are positive integers and t is the number of all subgraphs isomorphic to H. An (a, d)-H-antimagic total labeling f is called super if $f : V(G) \rightarrow \{1, 2, \ldots, |V(G)|\}$.

Some results related to the existence of an (a, d)-H-antimagic total labeling can be cited in [2, 3, 6, 7] and [8, 9, 10, 11, 12]. Inayah *et al.* in [6] proved that for any H and any integer $k \geq 2$, $shack(H, v, k)$ which contains exactly k subgraphs isomorphic to H admits H-super antimagic. But they only covered a connected version of shackle of graph when a vertex as a connector, and their paper did not cover all feasible d. Our paper attempt to solve a super (a, d)-H antimagic total labeling of disjoint union of edge comb product of graphs with subgraph as a terminal of its amalgamation, denoted by $G = c(K \trianglerighteq Amal(H, L \subset H, n))$, where c is number of copies of $K \trianglerighteq Amal(H, L \subset H, n)$. In this study, we aim to achieve for all feasible d.

To show those existence, we will use *an integer set partition technique* introduced by [1, 3]. This technique is used in determining the feasible difference d. Let n, m, d and k be positive integers. We consider the partition $\mathcal{P}^n_{m,d}(i, j)$ of the set $\{1, 2, \dots, mn\}$ into n columns, $n \geq 2$, m-rows such that the difference between the sum of the numbers in the $(j + 1)$th m-rows and the sum of the numbers in the jth m-rows is always equal to the constant d, where $j = 1, 2, \dots, n - 1$. Thus, these sums form an arithmetic sequence with the difference d. By the symbol $\mathcal{P}^n_{m,d}(i, j)$, we denote the jth m-rows in the partition with the difference d, where $j = 1, 2, \dots, n$. Let $\sum \mathcal{P}^n_{m,d}(i, j)$ be the sum of the numbers in $\mathcal{P}^n_{m,d}(i, j)$, thus $d = \sum \mathcal{P}^n_{m,d}(j + 1) - \sum \mathcal{P}^n_{m,d}(j)$.

2. Some Useful Lemma and Corollary

Let G be a disjoint union of comb product of graphs with subgraph as a terminal of its amalgamation, denoted by $G = c(K \trianglerighteq Amal(H, L \subset H, n))$. The graph G is a simple and disconnected graph with $|V(G)| = p_G$, $|E(G)| = q_G$, $|V(H)| = p_H$, and $|E(H)| = q_H$. The vertex set and edge set of the graph $G = c(K \trianglerighteq Amal(H, L \subset H, n))$ can be split into following sets: $V(G) = \{x_{i,t}; 1 \leq i \leq p_K; 1 \leq t \leq c\} \cup \{x_{i,k,t}; 1 \leq i \leq p_L - 2, 1 \leq k \leq q_K, 1 \leq t \leq c\} \cup \{x_{i,j,k,t}; 1 \leq i \leq p_H - p_L, 1 \leq k \leq q_K, 1 \leq j \leq n; 1 \leq t \leq c\}$ and $E(G) = \{e_{l,k,t}; 1 \leq l \leq q_L, 1 \leq k \leq q_K, 1 \leq t \leq c\} \cup \{e_{l,j,k,t}; 1 \leq l \leq q_H - q_L; 1 \leq j \leq n; 1 \leq k \leq q_K; 1 \leq t \leq c\}$. Thus, the cardinalities of $|V(G)| = p_G = (p_H - p_L)ncq_K + (p_L - 2)cq_K + cp_K$ and $|E(G)| = q_G = (q_H - q_L)ncq_K + q_Lcq_K$.

The upper bound of feasible d such that $G = c(K \trianglerighteq Amal(H, L \subset H, n))$ admits a super (a, d)-H-antimagic total labeling can be obtained in the following lemma.

Lemma 1. [2] *Let G be a simple graph of order p and size q. If G is super (a, d)-H-antimagic total labeling then $d \leq \frac{(p_G - p_H)p_H + (q_G - q_H)q_H}{n-1}$, for $p_G = |V(G)|$, $q_G = |E(G)|$, $p_H = |V(H)|$, $q_H = |E(H)|$, and $n = |H_k|$, $|H_k|$ is number of subgraph which is isomorphic to the graph H.*

Thus for $p_G = (p_H - p_L)ncq_K + (p_L - 2)cq_K + cp_K$ and $q_G = (q_H - q_L)ncq_K + q_Lcq_K$, we have the following corollary.

Corrollary 1. *For $n \geq 2$, if the graph $G = c(K \trianglerighteq Amal(H, L \subset H, n))$ admits super (a, d)-H-antimagic total labeling then $d \leq {p_H}^2 + {q_H}^2 - \frac{ncq_K(p_Lp_H + q_Lq_H + (p_L-2)cq_Hp_H + cp_Kp_H + cq_Lq_Kq_H}{ncq_K - 1}$*

Theorem 1. [5] *If G is connected graph with p vertices and q edges, then $p \leq q + 1$*

We recall a partition $\mathcal{P}^n_{m,d}(i, k)$ introduced in [4]. We will use this partition for a linear combination to develop a bijection of vertex and edge label of the main theorem.

Lemma 2. *Let n , m and s be positive integers $1 \leq j \leq n; 1 \leq k \leq s$, the $\sum_{i=1}^{m} \mathcal{P}^{n,s}_{m,d_1}(i, j, k) = \{a(ns-1)\lceil \frac{i}{a} \rceil - a(ns-1) + i + (k-1)a + (j-1)as;\ 1 \leq i \leq m\}$ forms an arithmetic sequences of difference $d_1 = am$.*

Proof. For $j = 1, 2, ..., n$; $k = 1, 2, ..., s$, it gives that $\sum_{i=1}^{m} \mathcal{P}^{n,s}_{m,d_1}(i, j, k) = \mathcal{P}^{n,s}_{m,d_1}(j, k) = \{\frac{m^2ns}{2} + \frac{m}{2}(1 - a - asn) - mas + am(js + k)\} \longleftrightarrow \mathcal{P}^{n,s}_{m,d_1}(j, k) = \{\frac{m^2ns}{2} + \frac{m}{2}(1 - a - asn) - mas + am(s+1), \frac{m^2ns}{2} + \frac{m}{2}(1 - a - asn) - mas + am(s+2), \dots, \frac{m^2ns}{2} + \frac{m}{2}(1 - a - asn) - mas + am(2s), \frac{m^2ns}{2} + \frac{m}{2}(1 - a - asn) - mas + am(2s+1), \dots, \frac{m^2ns}{2} + \frac{m}{2}(1 - a - asn) - mas + am(ns+s)\}$ forms an arithmetic sequences of difference $d_1 = am$. It concludes the proof. □

Lemma 3. *Let n , m and s be positive integers $1 \leq j \leq n; 1 \leq k \leq s;$ and $1 \leq t \leq c$, the* $\sum_{i=1}^{m} \mathcal{P}_{m,d_2}^{n,s,c}(i,j,k,t) = \{(t-1)a + (k-1)ac + (j-1)acs + i + (\lceil \frac{i}{a} \rceil - 1)(acsn - 1)\}$ *forms an arithmetic sequences of difference $d_2 = am$.*

Proof. For $j = 1, 2, \ldots, n$; $k = 1, 2, \ldots, s$, and $t = 1, 2, \ldots, c$, it gives that $\sum_{i=1}^{m} \mathcal{P}_{m,d_2}^{n,s,c}(i,j,k,t) = \mathcal{P}_{m,d_2}^{n,s,c}(j,k,t) = \{m[(t-1)a + (k-1)ac + (j-1)acs - acsn + a] + (\frac{m+m^2}{2}) + (acsn - a)(\frac{m^2+am}{2a})\} \longleftrightarrow \mathcal{P}_{m,d_2}^{n,s}(j,k) = \{m[a - acsn] + (\frac{m+m^2}{2}) + (acsn - a)(\frac{m^2+am}{2a}), m[a + (k-1)ac + (j-1)acs - acsn + a] + (\frac{m+m^2}{2}) + (acsn - a)(\frac{m^2+am}{2a}), \ldots, m[(c-1)a + (s-1)ac + (n-1)acs - acsn + a] + (\frac{m+m^2}{2}) + (acsn - a)(\frac{m^2+am}{2a}), \{m[acsn] + (\frac{m+m^2}{2}) + (acsn - a)(\frac{m^2+am}{2a}), m[a + acs - acsn + a] + (\frac{m+m^2}{2}) + (acsn - a)(\frac{m^2+am}{2a}), \ldots, m[(c-1)a + (s-1)ac + (n-1)acs - acsn + a] + (\frac{m+m^2}{2}) + (acsn - a)(\frac{m^2+am}{2a})$ forms an arithmetic sequences of difference $d_2 = am$. It concludes the proof. □

Lemma 4. *Let $m \geq 2$* even *and s be positive integers $1 \leq t \leq c; 1 \leq k \leq s$, the*

$$\sum_{i=1}^{m} \mathcal{P}_{m,d_3}^{c,s}(i,t,k) = \begin{cases} \{csi + t + (k-1)c - cs; i \equiv 1 \pmod{2}\} \\ \{csi - t - (k-1)c + 1; i \equiv 0 \pmod{2}\} \end{cases}$$

form an arithmetic sequences of difference $d_3 = 0$.

Proof. for $j = 1, 2, ..., n$; $k = 1, 2, ..., s$ it gives $\sum_{i=1}^{m} \mathcal{P}_{m,d_3}^{n,s}(i,t,k) = \mathcal{P}_{m,d_3}^{n,s}(t,k) = \{\frac{m^2cs+m}{2}, \frac{m^2cs+m}{2}, \ldots, \frac{m^2cs+m}{2}\}$. It concludes the proof. □

With those lemmas in hand, we ready to show our main result in the following section.

3. Main Results

In this section we will present our main theorem related to the existence of super $(a,d)-\mathcal{H}$ antimagic total labeling of disjoint union of edge comb product of graphs with subgraph as a terminal of its amalgamation. We will describe a construction of how to obtain the $\mathcal{H}$-antimagic total labeling from a smallest weight of edge-antimagic vertex labeling of graph G. We note that if cK is an $(a,d)-EAVL$ graph and H is any graph then $c(K \trianglerighteq Amal(H, L \subset H, n)) \cong G$.

Lemma 5. *Let K be a simple, nontrivial, and connected graph. If K admits an (a,d)-EAVL then $d \leq \frac{2p_K - 4}{q_K - 1}$ or $d \in \{1, 2\}$.*

Proof. Suppose K is a connected graph of order p_K and size q_K. If K admits (a,d)-*edge antimagic vertex labeling* then the bijection $f(V) = \{1, 2, 3, ..., p_K\}$. The set of *edge weights under vertex labeling* f is $w(uv) = f(u) + f(v)$, where $uv \in E(K)$. The weights $w(uv) = \{a, a+d, a+2d, ..., a+(q_K-1)d\}$ where a is the smallest edge weight. The minimum possible edge weight under labeling f is at least: $1+2$, thus $a \geq 3$. The largest label is $p_K + (p_K - 1)$ Hence $a + (q_K - 1)d \leq 2p_K - 1$. Combining the two inequalities, and also based on Theorem 1 we obtain the upper bound of feasible d for the graph K is said to be (a,d)-edge antimagic vertex labeling, namely $d \leq \frac{2(p_K-1)-2}{q_K-1}$. Since the minimum size of graph K is $pK - 1$ then $d \leq \frac{2(p_K-1)-2}{(p_K-1)-1} = \frac{2p_K-4}{p_K-2}$ and thus $d \leq 2$. Furthermore for the upper bound of disjoint union, we suppose $p_K = cp_K$ and $q_K = cq_K$ then $d \leq \frac{2(cp_K-1)-2}{(cp_K-1)-1} = \frac{2cp_K-4}{cp_K-2}$. Now, we will show that $d \neq 0$. By contradiction, suppose If $d = 0 \to 0 \leq \frac{2cp_K-4}{cp_K-2}$ and thus $p_K = 2$. Since we study for a graph of order larger than two then $p_K = 2$ is too trivial, thus $d = 0$ is not our concern. It concludes that the feasible $d \in \{1, 2\}$. □

Now we are ready to show our main theorem.

Theorem 2. *Let K, H be a two simple, nontrivial, and connected graphs. If the graph c_K admits an (a, d^*)-EAVL, then $G = c(K \trianglerighteq Amal(H, L \subset H, n))$ admits a super $(a, d) - \mathcal{H}$-antimagic total labeling with $d = d_v + d_e$, where d_v and d_e are respectively the feasible difference of partitions of integer set of vertex and edge labels.*

Proof. Suppose G has a super $(a, d) - \mathcal{H}$-antimagic total labeling f, we have a map $f : V(G) \cup E(G) \to \{1, 2, \ldots, (p_H - p_L)ncq_K + (p_L - 2)cq_K + cp_K + (q_H - q_L)ncq_K + q_L cq_K\}$, Since the graph cK admits an (a, d^*)-edge antimagic vertex labeling, the edge weight of cK is $\{a, a + d, a + 2d, \ldots, a + (r - 1)d\}$ where $r = 1, 2, \ldots, cqK$. According to lemma 5 the feasible difference of graph cK to be (a, d^*)-antimagic vertex graph is $d^* \in \{1, 2\}$. Thus, to prove the theorem we will prove it into two cases, namely for $d = 1$ and $d = 2$.

Case 1. For $d = 1$, we have $w_1(e_r) = a + (r - 1) \cdot 1$ and the number of $p_L + q_L \in L$ is odd. By using Lemma 2 and Lemma 3, define the vertex and edge labelings f_1 in the following way:

$$\begin{aligned}
f_1(V_{(p_H-p_L)ncq_K}) &= P^{ncq_K}_{p_H-p_L,d_v}(i,j,k,t) \oplus cp_K \\
f_1(V_{(p_L-2)cq_K} \cup E_{(q_L-1)cq_K}) &= P^{cq_K}_{p_L-2+(q_L-1),d_v+d_e}(i,j,k) \oplus cp_K + (p_H - p_L)ncq_K \\
f_1(e_r) &= cq_K + 1 - r + cp_K + (p_H - p_L)ncq_K + (p_L - 2 + q_L - 1)cq_K \\
f_1(E_{q_H-q_L)ncq_K}) &= P^{ncq_K}_{q_H-q_L,d_e}(i,j,k,t) \oplus cp_K + (p_H - p_L)ncq_K + (p_L - 2 + q_L - 1)cq_K \\
&\quad + cq_K
\end{aligned}$$

The labeling f_1 is a bijective function $f_1 : V(G) \cup E(G) \to \{1, 2, \ldots, (p_H - p_L)ncq_K + (p_L - 2)cq_K + cp_K + (q_H - q_L)ncq_K + q_L cq_K\}$. The $\mathcal{H}$ weight under the labeling f_1 constitute

$$\begin{aligned}
W(H_{j,k,t}) &= w_{(e_r)} + \sum_{u \in V(H_{j,k,t})} f(u) + \sum_{e \in V(E_{j,k,t})} f(e) \\
&= (a + (r - 1)) + \sum (P^{ncq_K}_{p_H-p_L,d_v}(i,j,k,t) \oplus cp_K) + \sum (P^{cq_K}_{p_L-2+(q_L-1)}(i,j,k) \oplus \\
&\quad cp_K + (p_H - p_L)ncq_K) + cq_K + 1 - r + cp_K + (p_H - p_L)ncq_K + (p_L - 2)cq_K + \\
&\quad \sum (P^{ncq_K}_{q_H-q_L,d_e}(i,j,k,t) \oplus cp_K + (p_H - p_L)ncq_K + (p_L - 2 + q_L - 1)cq_K + cq_K) \\
&= (a + (r - 1)) + \mathcal{C}^{ncq_K}_{p_H-p_L,d_v} + d_v(j,k,t) + cp_K(p_H - p_L)ncq_K + \mathcal{C}^{cq_K}_{p_L-2+(q_L-1),d_v+d_e} \\
&\quad + (p_L - 2 + q_L - 1)(cp_K + (p_H - p_L)ncq_K) + cq_K + 1 - r + cp_K + (p_H - p_L)ncq_K \\
&\quad + (p_L - 2 + q_L - 1)cq_K + \mathcal{C}^{ncq_K}_{q_H-q_L,d_e} + d_e(j,k,t) + (q_H - q_L)(cp_K + (p_H - p_L)ncq_K \\
&\quad + (p_L - 2 + q_L - 1)cq_K + cq_K) \\
&= a + \mathcal{C}^{ncq_K}_{p_H-p_L,d_v} + cp_K(p_H - p_L)ncq_K + \mathcal{C}^{cq_K}_{p_L-2+(q_L-1),d_v+d_e} \\
&\quad + (p_L - 2 + q_L - 1)(cp_K + (p_H - p_L)ncq_K) + cq_K + cp_K + (p_H - p_L)ncq_K \\
&\quad + (p_L - 2 + q_L - 1)cq_K + \mathcal{C}^{ncq_K}_{q_H-q_L,d_e} + (q_H - q_L)(cp_K + (p_H - p_L)ncq_K \\
&\quad + (p_L - 2 + q_L - 1)cq_K + cq_K) + (d_e + d_v)(j,k,t)
\end{aligned}$$

for subgraph $H_{j,k,t}, 1 \le j \le n; 1 \le k \le cq_K; 1 \le t \le c$.

Case 2. For $d = 2$, we have $w_1(e_r) = a + (r - 1) \cdot 2$ and the number of $p_L + q_L$ is even. By using Lemma 2, Lemma 3 and Lemma 4, define the vertex and edge labelings f_2 in the following way:

$$
\begin{aligned}
f_2(V_{(p_H-p_L)ncq_K}) &= P^{ncq_K}_{p_H-p_L,d_v}(i,j,k,t)\oplus cp_K\\
f_2(V_{(p_L-2)cq_K}\cup E_{(q_L-2)cq_K}) &= P^{cq_K}_{p_L-2+(q_L-2),d_v+d_e}(i,j,k)\oplus cp_K+(p_H-p_L)ncq_K\\
f_2(e^1_r) &= cq_K+1-r+cp_K+(p_H-p_L)ncq_K+(p_L-2+q_L-2)cq_K\\
f_2(E_{e^2\in q_L}) &= cq_K+1-r+cp_K+(p_H-p_L)ncq_K+(p_L-2+q_L-2)cq_K+cq_K\\
f_2(E_{q_H-q_L)ncq_K}) &= P^{ncq_K}_{q_H-q_L,d_e}(i,j,k,t)\oplus cp_K+(p_H-p_L)ncq_K+(p_L-2+q_L-1)cq_K\\
&\quad +2cq_K
\end{aligned}
$$

The labeling f_2 is a bijective function $f_2 : V(G)\cup E(G)\rightarrow\{1,2,\dots,(p_H-p_L)ncq_K+(p_L-2)cq_K+cp_K+(q_H-q_L)ncq_K+q_Lcq_K\}$. The $\mathcal{H}$ weight under the labeling f_2 constitute

$$
\begin{aligned}
W(H_{j,k,t}) &= w_{(e_r)}+\sum_{u\in V(H_{j,k,t})} f(u)+\sum_{e\in V(E_{j,k,t})} f(e)\\
&= (a+2(r-1))+\sum(P^{ncq_K}_{p_H-p_L,d_v}(i,j,k,t)\oplus cp_K)+\sum(P^{cq_K}_{p_L-2+(q_L-2),d_v+d_e}(i,j,k)\oplus\\
&\quad cp_K+(p_H-p_L)ncq_K)+cq_K+1-r+cp_K+(p_H-p_L)ncq_K+(p_L-2)cq_K\\
&\quad +cq_K+1-r+cp_K+(p_H-p_L)ncq_K+(p_L-2+q_L-2)cq_K+cq_K+\\
&\quad \sum(P^{ncq_K}_{q_H-q_L,d_e}(i,j,k,t)\oplus cp_K+(p_H-p_L)ncq_K+(p_L-2+q_L-2)cq_K+2cq_K)\\
&= (a+2(r-1))+\mathcal{C}^{ncq_K}_{p_H-p_L,d_v}+d_v(j,k,t)+cp_K(p_H-p_L)ncq_K+\mathcal{C}^{cq_K}_{p_L-2+(q_L-2),d_v+d_e}\\
&\quad +cq_K+1-r+cp_K+(p_H-p_L)ncq_K+(p_L-2)cq_K+cq_K+1-r+cp_K+\\
&\quad (p_H-p_L)ncq_K+(p_L-2+q_L-2)cq_K+cq_K+\mathcal{C}^{ncq_K}_{q_H-q_L,d_e}+d_e(j,k,t)\\
&\quad +(q_H-q_L)(cp_K+(p_H-p_L)ncq_K+(p_L-2+q_L-2)cq_K+2cq_K)ncq_K\\
&= (a+\mathcal{C}^{ncq_K}_{p_H-p_L,d_v}+cp_K(p_H-p_L)ncq_K+\mathcal{C}^{cq_K}_{p_L-2+(q_L-2),d_v+d_e}+cq_K+cp_K+\\
&\quad (p_H-p_L)ncq_K+(p_L-2)cq_K+cq_K+cp_K+(p_H-p_L)ncq_K+(p_L-2+q_L-2)\\
&\quad cq_K+cq_K+\mathcal{C}^{ncq_K}_{q_H-q_L,d_e}+(q_H-q_L)(cp_K+(p_H-p_L)ncq_K+(p_L-2+q_L-2)cq_K\\
&\quad +2cq_K)ncq_K+(d_e+d_v)(j,k,t)
\end{aligned}
$$

for subgraph $H_{j,k,t}$, $1\le j\le n;\ 1\le k\le cq_K; 1\le t\le c.$

From the two cases above, it is easy to see if the graph cK admits an (a,d^*)-edge antimagic vertex labeling then $G=c(K\trianglerighteq Amal(H,L\subset H,n))$ admits a super $(a,d)-\mathcal{H}$-antimagic total labeling with $d=d_v+d_e$, where d_v and d_e are respectively the feasible difference of partitions of integer set of vertex and edge labels. □

Concluding Remarks

We have shown the existence of super $(a,d)-H$ antimagic total labeling of disjoint union of edge comb product of graphs with subgraph as a terminal of its amalgamation, denoted by $G=c(K\trianglerighteq Amal(H,L\subset H,n))$. We have proved that $G=c(K\trianglerighteq Amal(H,L\subset H,n))$ admits a super(a,d)-H antimagic total labeling for almost feasible difference d, namely $d=d_v+d_e$, where d_v and d_e are respectively the feasible difference of partitions of integer set of vertex and edge labels.

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