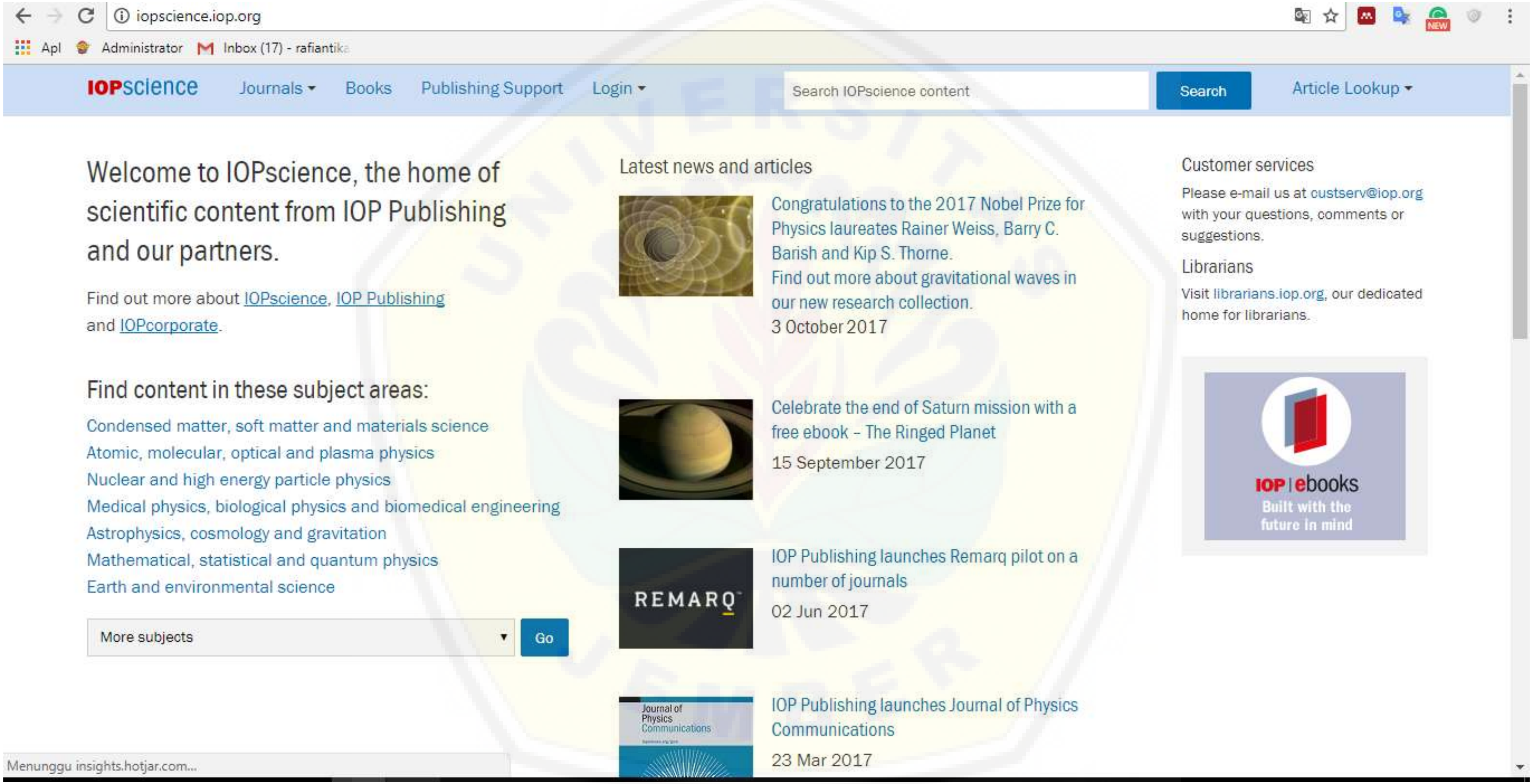
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Journal of Physics: Conference Series

52

H Index

Country United Kingdom

Subject Area and Category Physics and Astronomy
Physics and Astronomy (miscellaneous)

Publisher Institute of Physics

Publication type Journals

ISSN 17426588

Coverage 2005-ongoing

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Super (a^*, d^*)-$\mathcal{H}$-antimagic total covering of second order of shackle graphs

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Abstract. Let H be a simple and connected graph. A *shackle* of graph H, denoted by $G = shack(H, v, n)$, is a graph G constructed by non-trivial graphs $H_1, H_2, \ldots, H_n$ such that, for every $1 \leq s, t \leq n$, H_s and H_t have no a common vertex with $|s - t| \geq 2$ and for every $1 \leq i \leq n-1$, H_i and H_{i+1} share exactly one common vertex v, called *connecting vertex*, and those $k - 1$ connecting vertices are all distinct. The graph G is said to be an (a^*, d^*)-H-antimagic total graph of second order if there exist a bijective function $f : V(G) \cup E(G) \to \{1, 2, \ldots, |V(G)| + |E(G)|\}$ such that for all subgraphs isomorphic to H, the total H-weights $W(H) = \sum_{v \in V(H)} f(v) + \sum_{e \in E(H)} f(e)$ form an arithmetic sequence of second order of $\{a^*, a^* + d^*, a^* + 3d^*, a^* + 6d^*, \ldots, a^* + (\frac{n^2-n}{2})d^*\}$, where a^* and d^* are positive integers and n is the number of all subgraphs isomorphic to H. An (a^*, d^*)-H-antimagic total labeling of second order f is called super if the smallest labels appear in the vertices. In this paper, we study a super (a^*, d^*)-H antimagic total labeling of second order of $G = shack(H, v, n)$ by using a partition technique of second order.

1. Introduction

All graphs in this study are simple, connected an undirected. A graph G is said to be an (a^*, d^*)-H-antimagic total graph of second order if there exist a bijective function $f : V(G) \cup E(G) \to \{1, 2, \ldots, |V(G)| + |E(G)|\}$ such that for all subgraphs of G isomorphic to H, the total H-weights $w(H) = \sum_{v \in V(H)} f(v) + \sum_{e \in E(H)} f(e)$ form an arithmetic sequence of second order $\{a^*, a^* + d^*, a^* + 3d^*, a^* + 6d^*, \ldots, a^* + (\frac{n^2-n}{2})d^*\}$, where a and d are positive integers and n is the number of all subgraphs of G isomorphic to H. If such a function exist then f is called an (a^*, d^*)-H-antimagic total labeling of second order of G. An (a^*, d^*)-H-antimagic total labeling of second order f is called super if $f : V(G) \to \{1, 2, \ldots, |V(G)|\}$. By this notion, the super $(a, d) - H$ antimagic total labeling is classified as the super $(a, d) - H$ antimagic total labeling of first order.

We initiate to study this concept, thus we have not found any relevant results yet. But for the super $(a, d) - H$ antimagic total labeling, we can find many published results, some of them can be cited in [2, 3, 8, 9] and [10, 11, 12, 13, 15]. Inayah *et al.* in [8] proved that, for H is a non-trivial connected graph and $k \geq 2$ is an integer, $shack(H, v, k)$ which contains exactly k subgraphs isomorphic to H is H-super antimagic. All this papers only dealt with d derived from the sequence of order one. Our paper attempt to solve a super (a^*, d^*)-H antimagic total labeling of order two of $G = shack(H, v, n)$.

To show those existence, we will use a special technique, namely *an integer set partition technique*. We consider the partition $\mathcal{P}^n_{m,d}(i,j)$ of the set $\{1, 2, \ldots, mn\}$ into n columns with $n \geq 2$, m-rows such that the difference between the sum of the numbers in the $(j+1)$th m-rows and the sum of the numbers in the jth m-rows is always equal to the constant d, where $j = 1, 2, \ldots, n-1$. We need to establish some lemmas related to the partition $\mathcal{P}^n_{m,d}(i,j)$. Furthermore, the partition will be developed into a second order partition. These lemmas are useful to develop the super (a^*, d^*)-H antimagic total labeling of second order of $G = shack(H, v, n)$.

Let G be a *shackle* of graph denoted by $G = shack(H, v, n)$. Let G and H be a connected graph with $|V(G)| = p_G$, $|E(G)| = q_G$, $|V(H)| = p_H$, and $|E(H)| = q_H$. The vertex set and edge set of the graph $G = shack(H, v, n)$ can be split into following sets: $V = \{x_j; 1 \leq j \leq n+1\} \cup \{x_{ij}; 1 \leq i \leq p_H - 2, 1 \leq j \leq n\}$ and $E = \{e_{ij}; 1 \leq i \leq q_H, 1 \leq j \leq n\}$. Let n, i, j be positive integers with $n \geq 2$. Thus $p_G = |V(G)| = n + 1 + (p_H - 2)n = 1 + np_H + n - 2n = 1 + n(p_H - 1)$ and $q_G = |E(G)| = nq_H$.

We recall a partition $\mathcal{P}^n_{m,d}(i,j)$ introduced in [4]. We will use the partition for a linear combination in developing a bijection of vertex and edge label of the main theorem.

Lemma 1.1. [4] *Let n and m be positive integers. The sum of $\mathcal{P}^n_{m,d_1}(i,j) = \{(i-1)n + j,\ \ 1 \leq i \leq m\}$ and $\mathcal{P}^n_{m,d_2}(i,j) = \{(j-1)m + i;\ \ 1 \leq i \leq m\}$ form an aritmatic sequence of difference $d_1 = m$ and $d_2 = m^2$, respectively.*

2. Main Results

Lemma 2.1. *Let G be a simple graph of order p and size q. If G is super (a^*, d^*)-H-antimagic total labeling of second order then $d \leq \frac{(p_G - p_H)p_H + (q_G - q_H)q_H}{(\frac{n^2-n}{2})}$, for $p_G = |V(G)|$, $q_G = |E(G)|$, $p_H = |V(H)|$, $q_H = |E(H)|$, and $n = |H|$.*

Proof. Given the function $f(V) = 1, 2, 3, \ldots, p_G$ and $f(E) = p_G + 1, p_G + 2, p_G + 3, \ldots, p_G + q_G$. Let (p_G, q_G) admit a super $(a^*, d^*) - \mathcal{H}$ antimagic total labeling with the total second order function, $f(total) = 1, 2, 3, \ldots, p_G + q_G$ then the set of edge weight of a graph is $\{a^*, a^* + d^*, a^* + 3d^*, a^* + 6d^*, \ldots, a + (\frac{n^2-n}{2})d\}$ with a^* is the smallest weight thus:

$$\begin{aligned} 1 + 2 + \cdots + p_H + (p_G + 1) + (p_G + 2) + \cdots + (p_G + q_H) &\leqslant a^* \\ \frac{p_H}{2}(1 + p_H) + q_H p_G + \frac{q_H}{2}(1 + q_H) &\leqslant a^* \\ \frac{p_H}{2} + \frac{p_H^2}{2} + q_H p_G + \frac{q_H}{2} + \frac{q_H^2}{2} &\leqslant a^* \end{aligned}$$

$$\begin{aligned}
a^* + (\frac{n^2-n}{2})d^* &\leqslant p_G + (p_G - 1) + (p_G - 2) + \cdots + (p_G - (p_H - 1)) \\
&\quad +(p_G + q_G) + (p_G + q_G - 1) + (p_G + q_G - 2) + \ldots \\
&\quad +(p_G + q_G - (q_H - 1)) \\
&= p_H p_G - \frac{p_H - 1}{2}(1 + (p_H - 1)) + q_H p_G + q_H q_G \\
&\quad -\frac{q_H - 1}{2}(q + q_H - 1)) \\
&= p_H p_G - \frac{p_H - 1}{2}(p_H) + q_H p_G + q_H q_G - \frac{q_H - 1}{2}(q_H) \\
(\frac{n^2-n}{2})d^* &\leqslant p_H p_G - \frac{p_H - 1}{2}(p_H) + q_H p_G + q_H q_G - \frac{q_H - 1}{2}(q_H) - a \\
&\leqslant p_H p_G - \frac{p_H - 1}{2}(p_H) + q_H p_G + q_H q_G - \frac{q_H - 1}{2}(q_H) - \\
&\quad (\frac{p_H}{2} + \frac{p_H^2}{2} + q_H p_G + \frac{q_H}{2} + \frac{q_H^2}{2}) \\
&= p_H p_G - \frac{p_H^2}{2} + \frac{p_H}{2} + q_H p_G - \frac{q_H^2}{2} + \frac{q_H}{2} - \\
&\quad (\frac{p_H}{2} + \frac{p_H^2}{2} + \frac{q_H}{2} + \frac{q_H^2}{2}) \\
&= p_H p_G + q_H q_G - p_H^2 - q_H^2 \\
&= p_H p_G - p_H^2 + q_H q_G - q_H^2 \\
&= (p_G - p_H)p_H + (q_G - q_H)q_H \\
d^* &\leqslant \frac{(p_G - p_H)p_H + (q_G - q_H)q_H}{(\frac{n^2-n}{2})}
\end{aligned}$$

Based on the inequality above, we get the upper bound of feasible difference $d^* \leq \frac{(p_G-p_H)p_H+(q_G-q_H)q_H}{(\frac{n^2-n}{2})}$ for the graph G to be a super $(a^*, d^*) - \mathcal{H} -$ *antimagic* total *covering* of second order. □

Corollary 2.2. *If the graph $G = shack(H, v, n)$ admits super $(a^*, d^*) - H$-antimagic total labeling of second order for integer $n \geq 2$, then $d \leq \frac{2(p_H^2 - p_H + q_H^2)}{n}$*

The following lemmas are useful for showing the existence of super (a^*, d^*)-H antimagic total labeling $G = shack(H, v, n)$.

Lemma 2.3. *Let n and m be positive integers. For $1 \leq j \leq n$, the sum of $\mathcal{P}^n_{m,d_3}(i,j) = \{1+ni-j; 1 \leq i \leq m\}$ and $\mathcal{P}^n_{m,d_4}(i,j) = \{mn+i-mj; 1 \leq i \leq m\}$ form an arithmetic sequence of differences $d_3 = -m$ and $d_4 = -m^2$.*

Proof. By simple calculation, for $j = 1, 2, ..., n$, it gives $\sum_{i=1}^m \mathcal{P}^n_{m,d_1}(i,j) = \mathcal{P}^n_{m,d_3}(j) \longleftrightarrow \mathcal{P}^n_{m,d_3}(j) = \{\frac{n}{2}(m^2+m) + m - mj\} \longleftrightarrow \mathcal{P}^n_{m,d_1}(j) = \{\frac{n}{2}(m^2+m), \frac{n}{2}(m^2+m) - m, \frac{n}{2}(m^2+m) - 2m, \ldots, \frac{n}{2}(m^2+m)m - mn\}$ and $\sum_{i=1}^m \mathcal{P}^n_{m,d_4}(i,j) = \mathcal{P}^n_{m,d_2}(j) \longleftrightarrow \mathcal{P}^n_{m,d_4}(j) = \{\frac{m}{2}(2mn+m+1) - m^2 j\} \longleftrightarrow \mathcal{P}^n_{m,d_4}(j) = \{\frac{m}{2}(2mn+m+1) - m^2, \frac{m}{2}(2mn+m+1) - 2m^2, \ldots, \frac{m}{2}(2mn+m+1) - m^2 n\}$. It is easy to see that the differences of those sequences are $d_3 = -m, d_4 = -m^2$. It concludes the proof. □

Lemma 2.4. *Let n, m be positive integers and $n = m$. The sum of*

$$\mathcal{P}^{*n}_{m,d_5}(i,j) = \begin{cases} \frac{i(2m+2j+1)-j(2m-1)-i^2-j^2}{2}; \ for\ i-j \geq m-1 \\ \frac{m^2-m+j(2m+1-j)+i(-2m+2j+1-i)}{2}; \ for\ m-1 \geq i-j \end{cases}$$

form an arithmetic sequence of second order with common difference $d_5 = m$.

Proof. By simple calculation, for $j = 1, 2, ..., n$, it gives $\sum_{i=1}^{m} \mathcal{P}^n_{m,d_1}(i,j) = \mathcal{P}^n_{m,d_3}(j) \longleftrightarrow \mathcal{P}^n_{m,d_3}(j) = \{\frac{n}{2}(m^2+m)+m-mj\} \longleftrightarrow \mathcal{P}^n_{m,d_1}(j) = \{\frac{n}{2}(m^2+m), \frac{n}{2}(m^2+m)-m, \frac{n}{2}(m^2+m)-2m, \dots, \frac{n}{2}(m^2+m)m - mn\}$ and $\sum_{i=1}^{m} \mathcal{P}^n_{m,d_4}(i,j) = \mathcal{P}^n_{m,d_2}(j) \longleftrightarrow \mathcal{P}^n_{m,d_4}(j) = \{\frac{m}{2}(2mn+m+1) - m^2 j\} \longleftrightarrow \mathcal{P}^n_{m,d_4}(j) = \{\frac{m}{2}(2mn+m+1) - m^2, \frac{m}{2}(2mn+m+1) - 2m^2, \dots, \frac{m}{2}(2mn+m+1) - m^2 n\}$. It is easy to see that the differences of those sequences are $d_3 = -m, d_4 = -m^2$. It concludes the proof. □

Lemma 2.5. *Let* n, m *be positive integers and* $n = m$. *The sum of*

$$\mathcal{P}^{*n}_{m,d_6}(i,j) = \begin{cases} \frac{2m^2-2mi+i^2-i+2j}{2}; 1 \le i \le m, i+j \le m+1 \\ \frac{4m^2+4m-4mi-4mj-3i-j+2ij+i^2+j^2+2}{2}; 1 \le i \le m, m+1 < i+j \end{cases}$$

form an arithmetic sequence of second order with common difference $d_6 = -m$.

PROOF. By simple calculation, for $j = 1, 2, ..., n$, it gives $\sum_{i=1}^{m} \mathcal{P}^n_{m,d_6^*}(i,j) = \mathcal{P}^n_{m,d_6^*}(j) \longleftrightarrow \mathcal{P}^n_{m,d_6^*}(j) = \{\frac{4m^3-3m^2-(3j^2-9j+1)m}{6}\} \longleftrightarrow \mathcal{P}^n_{m,d_6^*}(j) = \{\frac{2m^3}{3} - \frac{m^2}{2} + \frac{5m}{6}, \frac{2m^3}{3} - \frac{m^2}{2} + \frac{5m}{6}, \frac{2m^3}{3} - \frac{m^2}{2} - \frac{m}{6}, \dots, \frac{2m^3}{3} - \frac{m^2}{2} - \frac{(3n^2-15n+13)m}{6}, \frac{2m^3}{3} - \frac{m^2}{2} - \frac{(3n^2-9n+1)m}{6}\}$. It is easy to see that the differences of those sequences are $d_6^* = -m$. It concludes the proof. □

Lemma 2.6. *Let* d^* *be the common difference of arithmetic sequence of second order and* d *be the common difference of arithmetic sequence of first order, the sum of corresponding terms will form an arithmetic sequence of second order with common difference* d^*.

Proof. An arithmetic sequence of first order is a sequence of the form:

$$a, a+d, a+2d, a+3d, \dots, a+(n-1)d$$

where a is the first term and d is common difference of the sequence. Whilst an arithmetic sequence of second order is of the form $a^*, a^*+d^*, a^*+3d^*, a^*+6d^*, \dots, a^*+(\frac{(n^2-n}{2})d^*$, where a^* and d^* are the first term and common difference of the sequence, respectively. Now, add the corresponding terms of these two expression:

Sequence	a	$a+d$	$a+2d$	$\dots$	$a+(n-1)d$
Sequence	a^*	a^*+d^*	a^*+3d^*	$\dots$	$a^*+(\frac{n^2-n}{2})d^*$ +
Sequence	$a+a^*$ $+d+d^*$	$a+a^*$ $+2d+3d^*$	$a+a^*$ $+3d+6d^*$	$\dots$	$a+a^*+(n-1)d$ $+(\frac{(n^2-n)}{2})d^*$
First order difference	$d+d^*$	$d+2d^*$	$d+3d^*$	$\dots$	$(n-1)d$ $+(\frac{(n^2-n)}{2})d^*$
Second order difference	d^*	d^*	d^*	$\dots$	d^*

It conclude the proof. □

Theorem 2.7. *Let* H *be a connected graph, then the shackle of the connected graph* $G = shack(H, v, n)$ *admits super* $(a^*, d^*) - H$ *antimagic total labeling.*

Proof. Let G be a *shackle* of graph denoted by $G = shack(H, v, n)$. Let G and H be a connected graph with $|V(G)| = p_G$, $|E(G)| = q_G$, $|V(H)| = p_H$, and $|E(H)| = q_H$. The vertex set and edge set of the graph $G = shack(H, v, n)$ can be split into following sets: $V = \{x_j; 1 \leq j \leq n+1\} \cup \{x_{ij}; 1 \leq i \leq p_H - 2, 1 \leq j \leq n\}$ and $E = \{e_{ij}; 1 \leq i \leq q_H, 1 \leq j \leq n\}$. Let n, i, j be positive integers with $n \geq 2$. Thus $p_G = |V(G)| = n + 1 + (p_H - 2)n = 1 + np_H + n - 2n = 1 + n(p_H - 1)$ and $q_G = |E(G)| = nq_H$. Construct a total labeling f,$f : V(shack(H, v, n)) \cup E(shack(H, v, n)) \rightarrow \{1, 2, \ldots, 1 + n(p_H + q_H - 1)\}$ constitute the following set:

$$
\begin{aligned}
f_1(V_{p_H}) &= \{j; 1 \leq j \leq n+1\} \\
f_2(V_{p_H}) &= \{\mathcal{P}^{*n}_{m_1,d_v^*}(i,j) \oplus n+1\} \cup \{\mathcal{P}^n_{m_2,d_v}(i,j) \oplus n(m_1+1)+1\} \\
f(E_{q_H}) &= \{\mathcal{P}^{*n}_{c_1,d_e^*}(i,j) \oplus (p_H-1)n+1\} \cup \{\mathcal{P}^n_{c_2,d_e}(i,j) \oplus (p_H+r_1-1)n+1\}
\end{aligned}
$$

where $m_1 + m_2 = p_H - 2$, $c_1 + c_2 = q_H$, d_v and d_e depends on $p_H - 2$ and q_H, respectively. Furthermore the weight of the subgraphs H_i, $i = 1, 2, \ldots, p_L$ in the following way:

$$
\begin{aligned}
W &= \sum_{v \in V(H_i)} f(v) + \sum_{e \in E(H_i)} f(e) \\
&= (2j+1) + (\sum_{i=1}^{m_1} (\mathcal{P}^{*n}_{m_1,d_v^*}(j) \oplus n+1) + (\sum_{i=1}^{m_2} (\mathcal{P}^n_{m_2,d_v}(j) \oplus n(m_1+1)+1) \\
&\quad +(\sum_{i=1}^{c_1} (\mathcal{P}^{*n}_{c_1,d_e^*}(j) \oplus (p_H-1)n+1)(\sum_{i=1}^{c_2} (\mathcal{P}^n_{c_2,d_e}(j) \oplus (p_H+r_1-1)n+1) \\
&= [2j+1] + [{C^*_{m_1,d_v^*}}^n + d_v^*(\frac{j^2-j+2}{2}) + m_1(n+1)] + [C^n_{m_2,d_v} + d_v j + \\
&\quad m_2(n(m_1+1)+1)] + [{C^*_{c_1,d_e^*}}^n + d_e^*(\frac{j^2-j+2}{2}) + c_1((p_H-1)n+1)] \\
&\quad +[C^n_{c_2,d_e} + d_e j + c_2(n(p_H+r_1-1)+1)]
\end{aligned}
$$

based on Lemma 2.6 we obtained:

$$
\begin{aligned}
&= 1 + {C^*_{m_1,d_v^*}}^n + {C_{m_2,d_v}}^n + {C^*_{c_1,d_e^*}}^n + {C_{c_2,d_e}}^n + m_1(n+1) + m_2(n(m_1+1)+1) \\
&\quad +c_1((p_H-1)n+1 + c_2(n(p_H+r_1-1)+1) + [d_v^* + d_e^* + 2]j
\end{aligned}
$$

3. Special Families

Theorem 3.1. *Suppose $G = shack(C_m, v, n)$, with $s \geq 3$ dan $2s \geq n+1$, graph G admits super (a^*, d^*)-$\mathcal{H}$-antimagic total covering of second order with* $a = 3 + [\frac{2{m_1}^3+4m_1+(3m_1-3m_1)}{6} + m_1(n+1)] + [\frac{{m_2}^2-m_2n}{2} + m_2 + m_2(n(m_1+1)+1)] + [\frac{m_3-{m_3}^2}{2} + {m_3}^2 + m_3(n(m_1+m_2+1)+1)] + [\frac{n}{2}(m_4^2 + m_4) + m_4 - m_4 + m_4(n(\sum_{t=1}^3 m_t + 1) + 1)] + [\frac{m_5}{2}(2m_5n + m_5 + 1) - m_5^2 + m_5(n(\sum_{t=1}^4 m_t + 1) + 1)][\frac{2{c_1}^3+4c_1+(3c_1-3c_1)}{6} + c_1(n(m+1)+1)] + [\frac{{c_2}^2-c_2n}{2} + c_2(n(m+c_1+1)+1)] + [\frac{c_3-{c_3}^2}{2} + c_3(n(m+c_1+c_2+1)+1)] + [\frac{n}{2}(c_4^2+c_4) + c_4 + c_4(n(m+\sum_{t=1}^3 c_t+1)+1)] + [\frac{c_5}{2}(2c_5n+c_5+1) + c_5(n(m+\sum_{t=1}^4 c_t+1)+1)]$ *and* $d = m_1^* + c_1^*$.

Proof. The graph $G = shack(C_m, v, n)$ have vertex set $V = \{x_j; 1 \leq j \leq n+1\} \cup \{x_{ij}; 1 \leq i \leq m-2, 1 \leq j \leq n\}$ and edge set $E = \{e_{ij}; 1 \leq i \leq m, 1 \leq j \leq n\}$. Thus $p_G = |V(G)| = mn - n + 1$ and $q_G = |E(G)| = mn$ where $p_H = m - 2$ and $q_H = m$ respectively are the cardinality of the vertex and edge on one cover H. We can define the vertex labeling

$f_1 : V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p_G + q_G\}$ by using the linear combination of $\mathcal{P}^{*n}_{m,m^*}$, $\mathcal{P}^{*n}_{m,-m^*}$, $\mathcal{P}^n_{m,m}$, $\mathcal{P}^n_{m,-m}$, $\mathcal{P}^n_{m,m^2}$ and $\mathcal{P}^n_{m,-m^2}$. By Lemma 1.1,2.3,2.4 and 2.5 we use m_1 and c_1 for the partition $\mathcal{P}^{*n}_{m,m^*}(i,j)$, m_2 and c_2 for the partition $\mathcal{P}^n_{m,m}(i,j)$, m_3 and c_3 for the partition $\mathcal{P}^n_{m,m^2}(i,j)$, m_4 and c_4 for the partition $\mathcal{P}^{*n}_{m,-m^*}(i,j)$, m_5 and c_5 for the partition $\mathcal{P}^n_{m,-m}(i,j)$ and we use m_6 and c_6 for the partition $\mathcal{P}^n_{m,-m^2}(i,j)$. For $i = 1, 2, \ldots, m$, $l = 1, 2, \ldots, c$ and $j = 1, 2, \ldots, n$, the total labels can be expressed as follows

$$\begin{aligned} f_1(V_{p_H}) &= \{j; 1 \le j \le n+1\} \\ f_2(V_{p_H}) &= \{\mathcal{P}^{*n}_{m_1,d^*_v}(i,j) \oplus n+1\} \cup \{\mathcal{P}^n_{m_2,d_v}(i,j) \oplus n(m_1+1)+1\} \\ f(E_{q_H}) &= \{\mathcal{P}^{*n}_{c_1,d^*_e}(i,j) \oplus (p_H-1)n+1\} \cup \{\mathcal{P}^n_{c_2,d_e}(i,j) \oplus (p_H+r_1-1)n+1\} \end{aligned}$$

The total vertex and edge-weights of $G = shack(C_m, v, n)$ under the labeling f_1, for $1 \le j \le n$, constitute the following sets:

$$\begin{aligned} W &= \sum_{v \in V(H_i)} f(v) + \sum_{e \in E(H_i)} f(e) \\ &= (2j+1) + (\sum_{i=1}^{m_1} (\mathcal{P}^{*n}_{m_1,d^*_v}(j) \oplus n+1) + (\sum_{i=1}^{m_2} (\mathcal{P}^n_{m_2,d_v}(j) \oplus n(m_1+1)+1) \\ &\quad +(\sum_{i=1}^{c_1} (\mathcal{P}^{*n}_{c_1,d^*_e}(j) \oplus (p_H-1)n+1)(\sum_{i=1}^{c_2} (\mathcal{P}^n_{c_2,d_e}(j) \oplus (p_H+r_1-1)n+1) \\ &= [2j+1] + [C^{*}_{m_1,d^*_v}{}^{n} + d^*_v(\frac{j^2-j+2}{2}) + m_1(n+1)] + [C^n_{m_2,d_v} + d_v j + \\ &\quad m_2(n(m_1+1)+1)] + [C^{*}_{c_1,d^*_e}{}^{n} + d^*_e(\frac{j^2-j+2}{2}) + c_1((p_H-1)n+1)] \\ &\quad +[C^n_{c_2,d_e} + d_e j + c_2(n(p_H+r_1-1)+1)] \end{aligned}$$

The total weights of $G = shack(C_m, v, n)$ constitute the following sets:

$$\begin{aligned} W_{f_1} &= w^1_{f_1} + w^2_{f_1} + w^3_{f_1} \\ &= [2j+1] + w^2_{f_1} + w^3_{f_1} \\ &= C^* + C + 1 + [2 + \frac{3m_1j - 3m_1}{2} + m_2 + m_3^2 - m_4 - m_5^2 + \frac{3c_1j - 3c_1}{2} \\ &\quad +c_2 + c_3^2 - c_4 - c_5^2]j; 1 \le j \le n \end{aligned}$$

By simple calculation, for $j = 1, 2, ..., n$, it gives $W_{f_1} = C^* + C + 1 + [2 + \frac{3m_1j-3m_1}{2} + m_2 + m_3^2 - m_4 - m_5^2 + \frac{3c_1j-3c_1}{2} + c_2 + c_3^2 - c_4 - c_5^2]j \longleftrightarrow \{C^*+C+1+m_2+m_3^2-m_4-m_5^2+c_2+c_3^2-c_4-c_5^2, C^*+C+1+m_1+c_1+m_2+m_3^2-m_4-m_5^2+c_2+c_3^2-c_4-c_5^2, C^*+C+1+3(m_1+c_1)+m_2+m_3^2-m_4-m_5^2+c_2+c_3^2-c_4-c_5^2, \ldots, C^*+C+1+3\frac{3(m_1n^2+3c_1n^2)-m_1n-c_1n}{6}+m_2+m_3^2-m_4-m_5^2+c_2+c_3^2-c_4-c_5^2$. with $C^* = [\frac{2{m_1}^3+4m_1}{6} + m_1(n+1)] + [\frac{2{c_1}^3+4c_1}{6} + c_1(n(m+1)+1)]$ $C = [\frac{{m_2}^2-m_2n}{2} + m_2(n(m_1+1)+1)] + [\frac{m_3-{m_3}^2}{2} + m_3(n(m_1+m_2+1)+1)] + [\frac{n}{2}(m_4^2+m_4) + m_4 + m_4(n(\sum_{t=1}^{3} m_t+1)+1)] + [\frac{m_5}{2}(2m_5n + m_5+1) + m_5(n(\sum_{t=1}^{4} m_t+1)+1)] + [\frac{{c_2}^2-c_2n}{2} + c_2(n(m+c_1+1)+1)] + [\frac{c_3-{c_3}^2}{2} + c_3(n(m+c_1+c_2+1)+1)] + [\frac{n}{2}(c_4^2+c_4)+c_4+c_4(n(m+\sum_{t=1}^{3} c_t+1)+1)] + [\frac{c_5}{2}(2c_5n+c_5+1)+c_5(n(m+\sum_{t=1}^{4} c_t+1)+1)]$

It is easy that the set of total edge-weights W_{f_1} consists of an arithmetic sequence of second order with the smallest value $a = 3 + [\frac{2{m_1}^3+4m_1+(3m_1-3m_1)}{6} + m_1(n+1)] + [\frac{{m_2}^2-m_2n}{2} + m_2 +$

$m_2(n(m_1+1)+1)] + [\frac{m_3-{m_3}^2}{2} + {m_3}^2 + m_3(n(m_1+m_2+1)+1)] + [\frac{n}{2}(m_4^2+m_4) + m_4 - m_4 + m_4(n(\sum_{t=1}^{3} m_t+1)+1)] + [\frac{m_5}{2}(2m_5n+m_5+1) - m_5^2 + m_5(n(\sum_{t=1}^{4} m_t+1)+1)][\frac{2{c_1}^3+4c_1+(3c_1-3c_1)}{6} + c_1(n(m+1)+1)] + [\frac{{c_2}^2-c_2n}{2} + c_2(n(m+c_1+1)+1)] + [\frac{c_3-{c_3}^2}{2} + c_3(n(m+c_1+c_2+1)+1)] + [\frac{n}{2}(c_4^2+c_4) + c_4 + c_4(n(m+\sum_{t=1}^{3} c_t+1)+1)] + [\frac{c_5}{2}(2c_5n+c_5+1) + c_5(n(m+\sum_{t=1}^{4} c_t+1)+1)]$ when the total edge weights at $j=1$ and the difference $d=[2+m_1^*+c_1^*]$. It concludes the proof. □

4. Concluding Remarks

We have shown the existence of super antimagic labeling of second order for graph operation $G = shack(H, v, n)$. We have found that $G = shack(H, v, n)$ admits a super(a^*, d^*)-H antimagic labeling of second order for all differences $d = 2+d_v^*+d_e^*$ where d_v^* and d_e^* are respectively feasible difference of second order of integer set partition. We have not found the result for another graph operations. Thus, we propose the following open problems.

Open Problem 4.1. *Analyse the existence of super (a^*, d^*)-H antimagic total labeling of second order of other graph operations.*

4.1. Acknowledgments

We gratefully acknowledge the support from CGANT University of Jember of year 2017.

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