Journal of Physics: Conference Series

PAPER • **OPEN ACCESS**

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To cite this article: Dafik *et al* 2017 *J. Phys.: Conf. Ser.* **943** 012003

IOP Conf. Series: Journal of Physics: Conf. Series **943** (2017) 012003 doi:10.1088/1742-6596/943/1/012003

A study of local domination number of $Sn \trianglerighteq H$ graph

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Abstract. All graphs in this paper are undirected, connected and simple graph. Let $G = (V, E)$ be a graph of order $|V|$ and size $|E|$. We define a set D as a dominating set if for every vertex $u \in V - D$ is adjacent to some vertex $v \in D$. The domination number $\gamma(G)$ is the minimum cardinality of dominating set. By a locating dominating set of graph $G = (V, E)$, we define for every two vertices $u, v \in V(G) - D$, $N(v) \cap D \neq \emptyset$. Locating dominating set is a special case of dominating set with an extra constrain above. The minimum cardinality of a locating dominating set is locating dominating number $\gamma_L(G)$. The value of locating dominating number is $\gamma_L(G) \subseteq V(G)$. This paper studies locating dominating set of edge comb product of graphs, denoted by GH. The graph $G \trianglerighteq H$ is a graph obtained by taking one copy of G and $|E(G)|$ copies of H and grafting the i-th copy of H at the edge e to the i-th edge of G, where G is star graph S_n and H is any special graph.

1. Introduction

Let $G = (V(G), E(G))$ be a simple graph with vertex set $V(G)$ and edge set $E(G)$. A subset D of $V(G)$ is called a vertex dominating set of G if every vertex not in D is adjacent to some vertices in D. A graph $G = (V, E)$ is called a locating dominating set if for every two vertices $u, v \in V(G) - D$, $N(v) \cap D \neq \emptyset$. Slater [5], [6] defined the locating-dominating number $\gamma_L(G)$ of a graph G is the minimum cardinality of a locating-dominating set of G. The value of locating dominating number is $\gamma_L(G) \subseteq V(G)$. In this paper, we will initiate to analyze locating dominating set of edge comb product of two graphs, denoted by $G \trianglerighteq H$, where G is path graph and H is any special graph.

Saputro *et. al* [7] firstly introduction a comb product of graph. Let G and H be two connected graphs. Let o be a vertex of H. The comb product between G and H, denoted by $G \triangleright H$, is a graph obtained by taking one copy of G and $|V(G)|$ copies of H and grafting the i-th copy of H at the vertex o to the i-th vertex of G. By the definition of comb product, we can say that $V(G \triangleright H) = \{(a, v)|a \in V(G), v \in V(H)\}$ and $(a, v)(b, w) \in E(G \triangleright H)$ whenever $a = b$ and $vw \in E(G)$ and $v = w = o$.

A natural extension of comb product of graph is an edge comb product of graph. Let G and H be two connected graphs. Let e be an edge of H. The edge comb product between G and H, denoted by $G \trianglerighteq H$, is a graph obtained by taking one copy of G and $|E(G)|$ copies of H and grafting the i-th copy of H at the edge e to the i-th edge of G. By the definition of comb product. We can say that $p = |V(G \trianglerighteq H)| = q_1(p_2 - 2) + p_1$ and $q = |E(G \trianglerighteq H)| = q_1q_2$, see [8] for detail.

Let G be path P_n with vertex set $V(P_n) = \{x_i; 1 \le i \le n\}$, and edge set $E(G) = x_ix_{i+1}; 1 \le i \le n-1$ so $|V(P_n)| = n$, $|E(P_n)| = n - 1$ and H is helm graph H_m with vertex set and edge set are $V(H_m) = \{A\} \cup \{x_i; 1 \le i \le m\} \cup \{y_i; 1 \le i \le m\}$, $E(H_m) = \{Ax_i; 1 \le i \le m\} \cup \{x_ix_{i+1}; 1 \le im - 1\} \bigcup \{x_mx_1\} \cup \{x_iy_i; 1 \le i \le m\}$. Thus $|V(H_m)| = 2m + 1$, $|E(H_m)| = 3m$. Furthermore, the graph $P_n \trianglerighteq H_m$ has a vertex set $V(P_n \trianglerighteq H_m) = \{A_i; 1 \le i \le n-1\} \cup \{x_{i,j}; 1 \le i \le n; 1 \le j \le m-1\} \cup \{y_{i,j}; 1 \le i \le n-1; 1 \le j \le m\}$ and edge set $E(P_n \trianglerighteq H_m) = \{x_{i,j}x_{i+1,j+1}; 1 \le i \le n-1; 1 \le j \le m-2\} \cup \{A_ix_{i,j}; 1 \le i \le n-1; 1 \le j \le m-1\} \cup \{x_{i,j}y_{i,j}; 1 \le i \le n; 1 \le j \le m-1\}$. The order and size of $(P_n \trianglerighteq H_m), n \ge 3, m \ge 3$ are $|V(P_n \trianglerighteq H_m)| = 2nm - m - 2$ and $|E(P_n \trianglerighteq K_m)| = 3mn - 3m$. We can say that $p = V|G \trianglerighteq H| = q_1(p_2 - 2) + p_1$ and $q = |E(G \trianglerighteq H)| = q_1q_2$. See Figure 1 as an example of edge comb product of graphs.

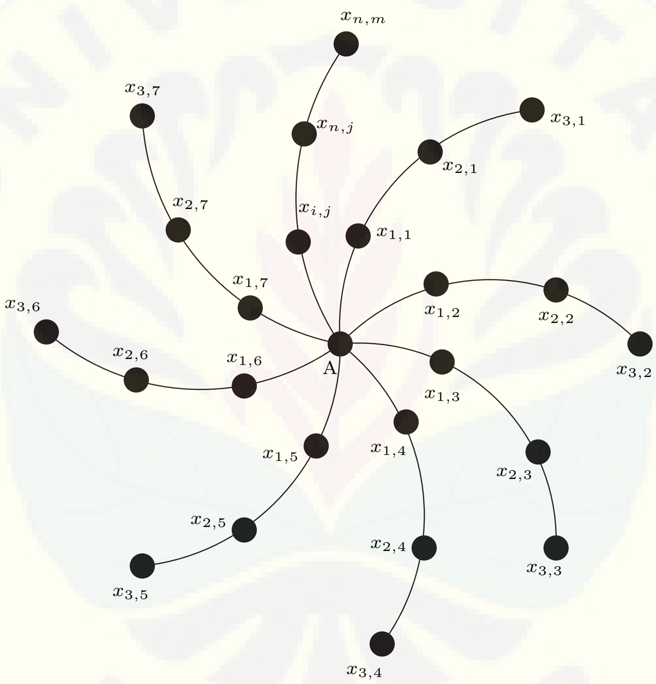

Figure 1. Locating dominating set of edge comb product $S_8 \trianglerighteq P_4$

2. Main Results

In this section, we determine the exact values of locating dominating number of some edge comb product of graphs, namely $S_n \trianglerighteq H$. In this paper, H are complete graph K_m, star graph S_m, triangular book Bt_m and path graph P_n.

Theorem 2.1. *Let x_ix_{i+1} be an edge of K_m, as well as be a grafting edge of K_m. The locating domination number of comb product graph $S_n \trianglerighteq K_m$ is $\gamma_L\ (S_n \trianglerighteq K_m) = nm - 2n$.*

Proof. An edge comb product graph $S_n \trianglerighteq K_m$, $n \ge 3$ and $m \ge 4$, is a connected graph with vertex set $V(S_n \trianglerighteq K_m) = \{A\} \bigcup \{x_{i,j}; 1 \le i \le n; 1 \le j \le m-1\}$ and edge set $E(S_n \trianglerighteq K_m) = \{Ax_{i,j}; 1 \le i \le n; 1 \le j \le m-1\} \bigcup \{x_{i,j}x_{i,j}; 1 \le i \le n; 1 \le j \le m-1\}$. The order and size of $(S_n \trianglerighteq K_m), n \ge 3, m \ge 4$ are $|V(S_n \trianglerighteq K_m)| = nm - n + 1$ and $|E(S_n \trianglerighteq K_m)| = (n)(\dfrac{m(m-1)}{2})$.

First we analysis the lower bound. We claim the lower bound is $\gamma_L(S_n \trianglerighteq K_m) \geq nm - 2n$. To show that is the lower bound, we prove by contradiction. Assume $\gamma_L(P_n \trianglerighteq K_m) < nm - 2n$, we choose locating dominating set of $(S_n \trianglerighteq K_m)$ namely $D = \{x_{i,j}; 2 \leq i \leq n-1; 1 \leq j \leq m-1\} \cup \{x_{1,j}; 1 \leq j \leq m-2\}$ with cardinality of D is $|D| = nm - 2n - 1$. The vertex set without locating dominating set of $(S_n \trianglerighteq K_m)$ is $V - D = \{A\} \cup \{x_{i_j}; 2 \leq i \leq n; j = m-1\} \cup \{x_{1,j}; 1 \leq j \leq m-2 \leq j \leq m-1\}$. The intersection between vertex set $\forall v \in (V - D)$ and D are as follows.

$$\begin{aligned} N(A) \cap D &= \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m-2\} \\ N(x_{i,j}) \cap D &= \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m-2\} \\ N(x_{1,j}) \cap D &= \{x_{1,j}; 1 \leq j \leq m-2\} \end{aligned}$$

For the vertices $x_{1,m-1}, x_{1,m-2} \in (V - D)$, it can be seen that $N(x_{1,m-1}) \cap D = N(x_{1,m-2}) \cap D$, and D does not comply the properties of locating dominating set. So, we can conclude that $\gamma_L(S_n \trianglerighteq K_m) < nm-2n$ is a contradiction. Hence, the location domination number of $(S_n \trianglerighteq K_m)$ is $\gamma_L(S_n \trianglerighteq K_m) \geq nm - 2n$.

Furthermore, we show that $\gamma_L(S_n \trianglerighteq K_m) \leq nm - 2n$, by choosing locating dominating set of $(S_n \trianglerighteq K_m)$ is $D = \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m-2\}$ with $|D| = nm - 2n$. The vertex set without locating dominating set of $(S_n \trianglerighteq K_m)$ is $V - D = \{A\} \cup \{x_{i,j}; 1 \leq i \leq n; j = m-2\}$. Intersection between vertex set $\forall v \in (V - D)$ and D are as follows.

$$\begin{aligned} N(A) \cap D &= \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m-3\} \\ N(x_{i,j}) \cap D &= x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m-3\} \end{aligned}$$

For the vertices $x_A, x_{i,j} \in (V - D)$ can be seen that $N(x_A) \cap D \neq \emptyset$ and $N(x_{i,j}) \cap D \neq \emptyset$, D is locating dominating set of $(S_n \trianglerighteq K_m)$ because D satisfied the definition 1. In the otherhand $N(x_A) \cap D \neq N(x_{i,j}) \cap D$, it conclude that D satisfied to be locating dominating set, so D satisfied the definition 2. Hence the location domination number of $(S_n \trianglerighteq K_m)$ is $\gamma_L(S_n \trianglerighteq K_m) \leq nm-2n$. Because $\gamma_L(S_n \trianglerighteq K_m) \geq nm - 2n$ and $\gamma_L(S_n \trianglerighteq K_m) \leq nm - 2n$, then we can say that $\gamma_L(S_n \trianglerighteq K_m) = nm - 2n$. □

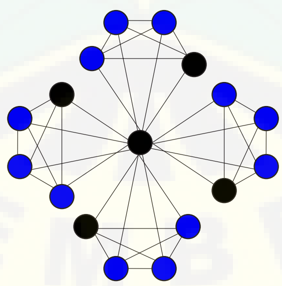

Figure 2. Edge Comb Product $S_4 \trianglerighteq K_5$

Theorem 2.2. *Given that a comb product graph $S_n \trianglerighteq S_m$. Suppose Ax_i is a grafting edge of S_m. Then the locating domination number of γ_L $(S_n \trianglerighteq S_m) = nm - n$.*

Proof. An edge comb product graph $S_n \trianglerighteq S_m$ $n \geq 3$ and $m \geq 3$, is a connected graph with vertex set $V(S_n \trianglerighteq S_m) = \{A\} \cup \{x_i; 1 \leq i \leq n\} \cup \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m-1\}$ and edge set $E(S_n \trianglerighteq S_m) = \{Ax_i; 1 \leq i \leq n\} \cup \{x_i x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m-1\}$. The order and size of $(S_n \trianglerighteq S_m), n \geq 3, m \geq 3$ are $|V(S_n \trianglerighteq S_m)| = nm + 1$ and $|E(S_n \trianglerighteq S_m)| = nm$.

First we analysis the lower bound. We claim the lower bound is γ_L $(S_n \trianglerighteq S_m) \leq nm - n$. To show that is the lower bound, we prove by contradiction. Assume $\gamma_L(S_n \trianglerighteq S_m) < nm - n$, we choose locating dominating set of $(S_n \trianglerighteq K_m)$ namely $D = \{x_{i,j}; 2 \leq i \leq n-1; 1 \leq j \leq m-1\} \cup \{x_{1,j}; 1 \leq j \leq m-2\}$ with cardinality of D is $|D| = nm-2n-1$. The vertex set without locating dominating set of $S_n \trianglerighteq S_m$ namely $D = \{x_i; 1 \leq i \leq n-1\} \cup \{x_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m-2\}$ with cardinality of D is $|D| = nm-n-1$. The vertex set without locating dominating set of $S_n \trianglerighteq S_m$ is $V - D = \{A\} \cup \{x_n\} \cup \{x_{i,j}; 1 \leq i \leq n; j = m-2\}$ Intersection between vertex set $\forall v \in (V-D)$ and D are as follows.

$$\begin{aligned} N(A) \cap D &= \{x_i; 1 \leq i \leq n-1\} \\ N(x_{i,j}) \cap D &= \{x_i; 1 \leq i \leq n-1\} \\ N(x_n) \cap D &= \emptyset \end{aligned}$$

For the vertices $x_n \in (V-D)$, it can be seen that $N(x_n) \cap D = \emptyset$, and D not satisfied properties of locating dominating set. So, we can conclude that $\gamma_L(S_n \trianglerighteq S_m) < nm-n$ is a contradiction. Hence, the location domination number of $(S_m \trianglerighteq S_m)$ is $\gamma_L(S_n \trianglerighteq S_m) \geq nm-n$.

Furthermore, we show that $\gamma_L(S_n \trianglerighteq S_m) \leq nm-n$, by choosing locating dominating set of $(S_m \trianglerighteq S_m)$ is $D = \{x_i; 1 \leq i \leq n\} \cup \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m-2\}$ with $|D| = nm-n$. The vertex set without locating dominating set of $(S_m \trianglerighteq S_m)$ is $V - D = \{A\} \cup \{x_{i,m-1}; 1 \leq i \leq n\}$. Intersection between vertex set $\forall v \in (V-D)$ and D are as follows.

$$\begin{aligned} N(A) \cap D &= \{x_i; 1 \leq i \leq n\} \\ N(x_{i,m-1}) \cap D &= \{x_i; 1 \leq i \leq n\} \end{aligned}$$

For the vertices $A, x_{i,m-1} \in (V-D)$ can be seen that $N(A) \cap D \neq \emptyset$ and $N(x_{i,m-1}) \cap D \neq \emptyset$, D is locating dominating set of $S_n \trianglerighteq S_m$ because D satisfied the definition 1. In the otherhand $N(A) \cap D \neq N(x_{i,m-1}) \cap D$ it conclude that D satisfied to be locating dominating set, so D satisfied the definition 2. Hence the location domination number of $S_n \trianglerighteq S_m$ is $\gamma_L(S_n \trianglerighteq S_m) \leq nm-n$. Because $\gamma_L(S_n \trianglerighteq S_m) \geq nm-n$ and $\gamma_L(S_n \trianglerighteq S_m) \leq nm-n$, then we can say that $\gamma_L(S_n \trianglerighteq S_m) = nm-n$. □

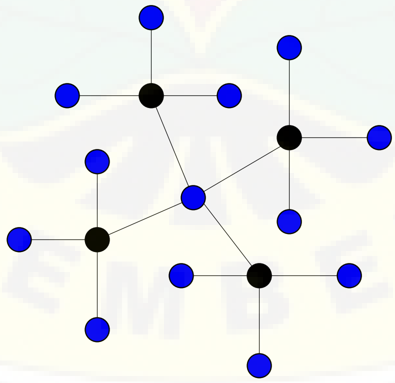

Figure 3. Edge Comb Product $S_4 \trianglerighteq S_4$

Theorem 2.3. *Given that a comb product graph $S_n \trianglerighteq Bt_m$ and AB is a graft edge of Bt_m. Then locating domination number of γ_L $(S_n \trianglerighteq Bt_m) = nm$.*

Proof. An edge comb product graph $S_n \trianglerighteq Bt_m$, $n \geq 3$ and $m \geq 3$, is a connected graph with vertex set $V(S_n \trianglerighteq Bt_m) = \{A\} \cup \{x_i; 1 \leq i \leq n\} \cup \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m\}$, and edge set $E(S_n \trianglerighteq Bt_m) = \{Ax_i; 1 \leq i \leq n\} \cup \{Ax_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m\} \cup \{x_i x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m\}$. The order and size of $(S_n \trianglerighteq Bt_m), n \geq 3, m \geq 3$ are $|V(S_n \trianglerighteq Bt_m)| = n + nm + 1$ and $|E(S_n \trianglerighteq P_m)| = n + 2nm$.

First we analysis the lower bound. We claim the lower bound is $\gamma_L(S_n \trianglerighteq Bt_m) \geq nm$. To show that is the lower bound, we prove by contradiction. Assume $\gamma_L(S_n \trianglerighteq Bt_m) < nm$, we choose locating dominating set of $S_n \trianglerighteq Bt_m$ namely $D = \{x_{i,j}; 2 \leq i \leq n-1; 1 \leq j \leq m\} \cup \{x_{1,j}; 2 \leq m\}$ with cardinality of D is $|D| = nm - 1$. The vertex set without locating dominating set of $S_n \trianglerighteq Bt_m$ is $V - D = \{A\} \cup \{x_{1,1}\}$ Intersection between vertex set $\forall v \in (V - D)$ and D are as follows.

$$\begin{aligned} N(A) \cap D &= \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m\} \\ N(x_{1,1}) \cap D &= \emptyset \end{aligned}$$

For the vertices $x_{1,1} \in (V - D)$, it can be seen that $N(x_{1,1}) \cap D = \emptyset$, and D does not comply the properties of locating dominating set. So, we can conclude that $\gamma_L(S_n \trianglerighteq Bt_m) < nm$ is a contradiction. Hence, the location domination number of $(S_n \trianglerighteq Bt_m)$ is $\gamma_L(S_n \trianglerighteq Bt_m) \geq nm$.

Furthermore, we show that $\gamma_L(S_n \trianglerighteq Bt_m) \leq nm$, by choosing locating dominating set of $(S_n \trianglerighteq Bt_m)$ is $D = \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m\}$ with $|D| = nm$. The vertex set without locating dominating set of $(S_n \trianglerighteq Bt_m)$ is $V - D = \{A\} \cup \{x_i; 1 \leq i \leq n\}$. Intersection between vertex set $\forall v \in (V - D)$ and D are as follows.

$$\begin{aligned} N(A) \cap D &= \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m\} \\ N(x_i) \cap D &= \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m\} \end{aligned}$$

For the vertices $A, x_{(i)} \in (V - D)$ can be seen that $N(A) \cap D \neq \emptyset$ and $N(x_i) \cap D \neq \emptyset$, D is locating dominating set of $S_n \trianglerighteq Bt_m$ because D satisfied the definition 1. In the otherhand $N(A) \cap D \neq N(x_i) \cap D$ it conclude that D satisfied to be locating dominating set, so D satisfied the definition 2. Hence the location domination number of $S_n \trianglerighteq Bt_m$ is $\gamma_L(S_n \trianglerighteq Bt_m) \leq nm$. Because $\gamma_L(S_n \trianglerighteq Bt_m) \geq nm$ and $\gamma_L(S_n \trianglerighteq Bt_m) \leq nm$, then we can say that $\gamma_L(S_n \trianglerighteq Bt_m) = nm$.
□

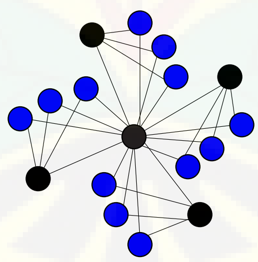

Figure 4. Edge Comb Product $S_4 \trianglerighteq Bt_3$

Theorem 2.4. *Given that a comb product graph $S_n \trianglerighteq P_m$ and x_1x_2 is a grafting edge of P_m. Then locating domination number of γ_L $(S_n \trianglerighteq P_m) = n(\lfloor \frac{2m}{5} \rfloor)$.*

Proof. An edge comb product graph $S_n \trianglerighteq P_m$, $n \geq 3$ and $m \geq 5$, is a connected graph with vertex set $V(S_n \trianglerighteq P_m) = \{A\} \cup \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m-1\}$, and edge set $E(S_n \trianglerighteq P_m) = \{Ax_{i,j}; 1 \leq i \leq n; j = 1\} \cup \{x_{i,j}x_{i,j+1}; 1 \leq i \leq n; 1 \leq j \leq m-2\}$. The order and size of $(S_n \trianglerighteq P_m), n \geq 3, m \geq 5$ are $|V(S_n \trianglerighteq P_m)| = nm - n + 1$ and $|E(S_n \trianglerighteq P_m)| = nm - n$.

First we analysis the lower bound. We claim the lower bound is $\gamma_L(S_n \trianglerighteq P_m) \geq n(\lfloor \frac{2m}{5} \rfloor)$. To show that is the lower bound, we prove by contradiction. Assume $\gamma_L(S_n \trianglerighteq P_m) < n(\lfloor \frac{2m}{5} \rfloor)$, we choose locating dominating set of $S_n \trianglerighteq P_m$ namely $D = \{x_{i,j}; 1 \leq i \leq n-1; j \equiv 2 + 0 \text{ mod}$

$2\} \cup \{x_n, j; j \geq 3; j \equiv 2+0 \bmod 2\}$ with cardinality of D is $|D| = n(\lfloor\frac{2m}{5}\rfloor) - 1$. The vertex set without locating dominating set of $S_n \trianglerighteq P_m$ is $V - D = \{A\} \cup \{x_{i,j}; 1 \leq i \leq n; j \equiv 1 \bmod 2\} \cup \{x_{n,j}; 1 \leq j \leq 2\}$ Intersection between vertex set $\forall v \in (V-D)$ and D are as follows.

$$\begin{aligned} N(A) \cap D &= \{x_{i,1}; 1 \leq i \leq n-1\} \\ N(x_{i,j} \cap D &= x_{i,j}; 1 \leq i \leq n; j \equiv 2+0 \bmod 2 \\ N(x_{n,j}) \cap D &= \emptyset \end{aligned}$$

For the vertices $x_{n,j} \in (V-D)$, it can be seen that $N(x_{n,j}) \cap D = \emptyset$, and D does not comply the properties of locating dominating set. So, we can conclude that $\gamma_L(S_n \trianglerighteq P_m) < n(\lfloor\frac{2m}{5}\rfloor)$ is a contradiction. Hence, the locating dominating number of $(S_n \trianglerighteq P_m)$ is $\gamma_L(S_n \trianglerighteq P_m) \geq n(\lfloor\frac{2m}{5}\rfloor)$.

Furthermore, we show that $\gamma_L(S_n \trianglerighteq P_m) \leq n(\lfloor\frac{2m}{5}\rfloor)$, by choosing locating dominating set of $(S_n \trianglerighteq P_m)$ is $D = \{x_{i,j}; 1 \leq i \leq n; j \equiv 2+0 \bmod 2\}$ with $|D| = n(\lfloor\frac{2m}{5}\rfloor)$. The vertex set without locating dominating set of $(S_n \trianglerighteq P_m)$ is $V - D = \{A\} \cup \{x_{i,j}; 1 \leq i \leq n; j \equiv 1 \bmod 2\}$. Intersection between vertex set $\forall v \in (V-D)$ and D are as follows.

$$\begin{aligned} N(A) \cap D &= \{x_{i,1}; 1 \leq i \leq n\} \\ N(x_{i,j} \cap D &= \{x_{i,j}; 1 \leq i \leq n; j \equiv 2+0 \bmod 2\} \end{aligned}$$

For the vertices $x_A, x_{i,j} \in (V-D)$ can be seen that $N(x_A) \cap D \neq \emptyset$ and $N(x_{i,j}) \cap D \neq \emptyset$, D is locating dominating set of $S_n \trianglerighteq P_m$ because D satisfied the definition 1. In the otherhand $N(x_A) \cap D \neq N(x_{i,j}) \cap D$ satisfied to be locating dominating set, so D satisfied the definition 2. Hence the location domination number of $S_n \trianglerighteq P_m$ is $\gamma_L(S_n \trianglerighteq P_m) \leq n(\lfloor\frac{2m}{5}\rfloor)$. Because $\gamma_L(S_n \trianglerighteq P_m) \geq n(\lfloor\frac{2m}{5}\rfloor)$ and $\gamma_L(S_n \trianglerighteq P_m) \leq n(\lfloor\frac{2m}{5}\rfloor)$, then we can say that $\gamma_L(S_n \trianglerighteq P_m) = n(\lfloor\frac{2m}{5}\rfloor)$. □

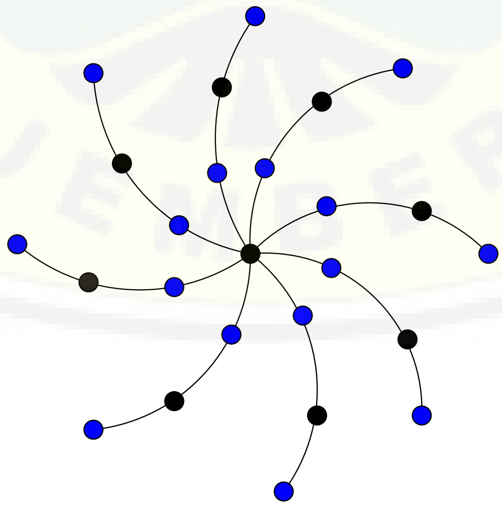

Figure 5. Edge Comb Product $S_8 \trianglerighteq P_4$

3. Concluding Remarks

In this paper, we have obtained the exact values of locating dominating number of some edge comb product graph, namely $S_n \trianglerighteq H$. In this paper, we studied H as complete graph K_m, star graph S_m, triangular book Bt_m and path graph P_n. We have found he exact values of their locating dominating number. However for H is any graph we have found any result yet. Therefore we proposed the following open problem.

Open Problem 3.1. *Determine the sharp lower bound or upper bound of locating dominating number of* $S_n \trianglerighteq H$ *for* H *is any graph.*

3.1. Acknowledgments

We gratefully acknowledge the support from CGANT University of Jember of year 2017.

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