# On local adjacency metric dimension of some wheel related graphs with pendant points

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# On Local Adjacency Metric Dimension of Some Wheel Related Graphs with Pendant Points

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**Abstract.** Let $G = (V(G), E(G))$ be any connected graph of order $n = |V(G)|$ and measure $m = |E(G)|$. For an order set of vertices $S = \{ s_1, s_2, ..., s_k\}$ and a vertex $v$ in $G$, the adjacency representation of $v$ with respect to $S$ is the ordered $k$-tuple $r_A(v|S) = (d_A(v, s_1), d_A(v, s_2), ..., d_A(v, s_k))$, where $d_A(u,v)$ represents the adjacency distance between the vertices $u$ and $v$. The set $S$ is called a local adjacency resolving set of $G$ if for every two distinct vertices $u$ and $v$ in $G$, $u$ adjacent $v$ then $r_A(u|S) \neq r_A(v|S)$ . A minimum local adjacency resolving set for $G$ is a local adjacency metric basis of $G$. Local adjacency metric dimension for $G$, $\dim_{A,l}(G)$, is the cardinality of vertices in a local adjacency metric basis for $G$. In this paper, we study and determine the local adjacency metric dimension of some wheel related graphs $G$ (namely gear graph, helm, sunflower and friendship graph) with pendant points, that is edge corona product of $G$ and a trivial graph $K_1$, $G \Diamond K_1$. Moreover, we compare among the local adjacency metric dimension of $G \Diamond K_1$ graph,of $W_n \Diamond K_1$ graph and metric dimension of $W_n$ .

## INTRODUCTION

This section presents about some definitions and notions that are using in this research. These concepts are taken from [4]. We begin with, $G = (V(G), E(G))$ is a simple, finite and connected graph with a set of vertices $V(G)$ and a set of edges $E(G)$, of cardinality $n$ and $m$, respectively. Two adjacent vertices $u$ and $v$ will be write $u \sim v$ and two vertex $u$ and $v$ that is not adjacent with $u \nsim v$. The distance between two vertices $u$ and $v$ in $G$, $d(u,v)$ is the lenght of shortest path joining $u$ and $v$. The adjacency distance between $u$ and $v$ denoted by $d_A(u,v)$, and defines by [9],

$$d_A(u,v_i) = \begin{cases} 0 \;\; if \; u = v_i, \\ 1 \;\; if \; u \sim v_i, \\ 2 \;\; if \; u \nsim v_i. \end{cases}$$

Let $S = \{s_1, s_2, ..., s_k\} \subseteq V(G)$ be an order set of vertices and $v$ is a vertex in $G$. The adjacency representation of $v$ with respect to $S$ is the ordered $k$-tuple $r_A(v|S) = (d_A(v, s_1), d_A(v, s_2), ..., d_A(v, s_k))$. $S$ is called a local adjacency resolving set of $G$, if a pair of adjacent distinct vertex in $G$ have different adjacency representations. A minimum local adjacency resolving set for $G$ is a local adjacency metric basis of $G$. Adjacency metric dimension for $G$, $\dim_{A.l}(G)$, is the cardinality of vertices in a local adjacency metric basis for $G$.

A Concept about local metric dimension of a graph has introduced by Okamoto et al. [3]. Research about local metric dimension of corona graphs have done by Rodriguez et al. [8] and local metric dimension of edge-corona graph by Rinurwati et al. [12]. Then, Rodriguez and Fernau [7], continuoued their reseach that is about local adjacency metric dimension of corona graphs. Their research is developing of the concept about adjacency metric dimension of graphs that has introduced by Jannesari and Omoomi [9]. Farthes before, Harary and Melter [2] have been introduced about resolving set in 1976 and independently, Slater [10] introduce this concept in 1975. This

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concept is a basic concept that must be known when a research results metric dimension of graphs. To prove that set $S$ is resolving set of a graph $G$, we only present that every vertex in $V(G)$-$S$ has distinct representation, because In vertex $v$ in $S$ is unix vertex with $d(v, v) = 0$.

Motivated by results in [1], [5], [6], and [7], we study and determine the local adjacency metric dimension of *some wheel related graphs G with pendant points* (edge-corona of graphs $G\Diamond H$ when $H\cong K_1$ or $H\cong mK_1$ for $m\geq 2$ ). *Edge-corona of graphs G and H*, denoted by $G\Diamond H$, is defined as a graph formed by taking $G$ and $m = |E(G)|$ copies of $H$ then joining two end-vertices $s_i$, $s_h$ of edge $e_j = s_i s_h$ of $G$ to every vertex in the $j^{th}$-copy of $H$ [13]. In this paper, as $G$, we use gear graph ($G_{2n}$), helm ($H_n$), sunflower ($SF_n$) and friendship ($f_n$) graphs. All of these graphs are obtained from wheel graph, that is graph trivial $K_1$ that joining with an edge to all vertices of cycle graph, $C_n$. Moreover, we compare the local adjacency metric dimension of these graphs, respectively with a wheel graph with pendant points.

## RESULTS

In the following, we present some useful results on the local adjacency metric dimension of some wheel related graphs with pendant points.

### Local Adjacency Metric Dimension of Gear Graphs with Pendant Points.

*A* gear graph $G_{2n}$ is a graph obtained from a wheel graph $W_n \cong K_1 + C_n$ by adding a vertex between every pair of adjacent vertices of the cycle $C_n$ [6]. A gear graph with pendant points denoted by $G_{2n}\Diamond K_1$, that is a graph obtained from edge-corona of a gear graph $G_{2n}$ and a trivial graph $K_1$. Let $G \cong G_{2n}\Diamond K_1$ with a set of vertices $V(G)=\{c\}\cup\{v_1,v_2,\ldots,v_n\}\cup\{w_1,w_2,\ldots,w_n\}\cup\{a_1,a_2,\ldots,a_n\}\cup\{b_{11},b_{12},b_{21},b_{22},\ldots,b_{n1},b_{n2}\}$, where

$c$ : the vertex of $K_1$ of $W_n$
$v_j$ : $j$-th vertex of a cycle $C_n$ of $W_n$
$w_j$ : an adding vertex between every pair of $j$-th adjacent vertices of the cycle $C_n$ of $W_n$
$a_j$ : a pendant point (vertex) joining two end-vertices $c, v_j$ of rim edge $e_j = cv_j$ of $W_n$
$b_{j1}$ : a pendant point (vertex) joining two end-vertices $v_j, w_j$ of edge $e_j = v_j w_j$
$b_{j2}$ : a pendant point (vertex) joining two end-vertices $w_j, v_{j+1}$ of edge $e_j = w_j v_{j+1}$
$j \in \{1,2,\ldots,n\}$.

As illustration, we can see FIGURE 1.

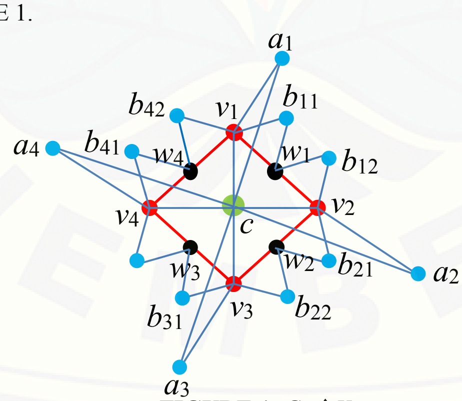

**FIGURE 1.** $G_{24}\Diamond K_1$

The local adjacency metric dimension of gear graphs with pendant points is mentioned in the following theorem.

**Theorem 1.** *Let* $G \cong G_{2n}\Diamond K_1$ *with* $|V(G)| = 5n+1$, *then* $dim_{A,l}(G) = n$ *for* $n\geq 3$.

**Proof.** Choose $S=\{v_1,v_2,\ldots,v_n; v_{n+1}=v_1\}\subseteq V(G)$. We will show that $S$ is a local adjacency resolving set of $G$. The local adjacency representations of vertices from $V(G)-S$ are as follows:

$$r_A(c \mid S) = (1,1,\ldots,1)$$
$$r_A(a_j \mid S) = (2,\ldots,2,\underbrace{1}_{j-term},2,\ldots,2) = r(b_{j1} \mid S);\quad j \in \{1,2,\ldots,n\},\text{ but } a_j \nsim b_{j1}$$
$$r_A(b_{(j-1)2} \mid S) = r(b_{j1} \mid S);\quad j \in \{2,\ldots,n\},\text{ but } b_{j1} \nsim b_{(j-1)2} \nsim a_j.$$
$$r_A(b_{n2} \mid S) = r(b_{11} \mid S),\text{ and } b_{n2} \nsim b_{11}.$$
$$r_A(w_n \mid S) = (1,2,\ldots,2,1)$$
$$r_A(w_j \mid S) = (2,\ldots,2,\underbrace{1}_{j-term},\underbrace{1}_{(j+1)-term},2,\ldots,2);\quad j \in \{1,2,\ldots,n-1\}.$$

As we see that all of the adjacency representation of adjacent vertices are distinct. So, $S = \{v_1, v_2, \ldots, v_n; v_{n+1} = v_1\}$ is a local adjacency resolving set for $G$. The cardinality of $S$, $|S| = n$ is minimum, because if $|S| < n$ certainly there are $x \neq y \in V(G) - S$ such that $r(x \mid S) = r(y \mid S)$. Let $S_1 = \{v_1, v_2, \ldots, v_{n-1}\}, |S_1| = n-1 < n$, then $r(v_n \mid S_1) = (2,2,\ldots,2) = r(b_{n2} \mid S_1)$ and $b_{n2} \sim v_n$, also $r(w_n \mid S_1) = (2,2,\ldots,2,1) = r(b_{n1} \mid S_1)$ and $b_{n1} \sim w_n$. Thus, $\dim_{A,l}(G) = n$.

**Local Adjacency Metric Dimension of Helm Graphs with Pendant Points.**

A helm graph $H_n$ is a graph obtained from a wheel graph $W_n \cong K_1 + C_n$ with cycle $C_n$ having a pendant edge attached to each vertex of the cycle [6]. A helm graph with pendant points, $H_n \Diamond K_1$ is a graph obtained from edge-corona of a helm graph $H_n$ and a trivial graph $K_1$. Let $G \cong H_n \Diamond K_1$ with a set of vertices $V(G) = \{c\} \cup \{v_1, v_2, \ldots, v_n\} \cup \{w_1, w_2, \ldots, w_n\} \cup \{u_1, u_2, \ldots, u_n\} \cup \{x_1, x_2, \ldots, x_n\} \cup \{a_1, a_2, \ldots, a_n\}$, where

$c$ : the vertex of $K_1$ of $W_n$
$v_j$ : $j$-th vertex of a cycle $C_n$ of $W_n$
$u_j$ : a pendant point (vertex) joining two end-vertices $c, v_j$ of rim edge $e_j = cv_j$ of $W_n$
$a_j$ : a pendant point (vertex) joining two end-vertices $c, v_j$ of pendant edge $e_j = v_jx_j$ of $C_n$ of $W_n$
$w_j$ : a pendant point (vertex) joining two end-vertices $v_j$, $v_{j+1}$ of edge $e_j = v_jv_{j+1}$ of $C_n$ of $W_n$
$j \in \{1,2,\ldots,n\}$.

The following figure is an example of $H_n \Diamond K_1$ graphs.

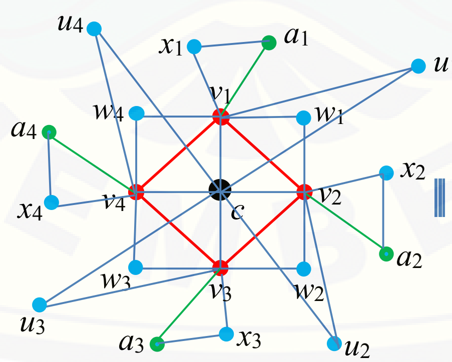


FIGURE 2. $H_4 \Diamond K_1$

The following theorem is the local adjacency metric dimension of helm graphs with pendant points.

**Theorem 2.** *Let* $G \cong H_n \Diamond K_1$ *with* $|V(G)| = 5n+1$, *then* $\dim_{A,l}(G) = n + 1$ *for* $n \geq 3$.

**Proof.** Choose $S=\{a_1,a_2,\ldots,a_n,c\}\subseteq V(G)$. Adjacency representation of vertices in $V(G)-S$ as follows:

$$r_A(c\mid S)=(2,2,\ldots,2,0).$$

$$r_A(a_j\mid S)=(2,2,\ldots,2,\underbrace{1}_{j-term},2,\ldots,2,2).$$

$r_A(u_j\mid S)=(2,2,\ldots,2,1)$, for every $j\in\{1,2,\ldots,n\}$, $u_i\nsim u_j$, and $i\neq j$.

$r_A(w_j\mid S)=(2,2,\ldots,2,2)$, for every $j\in\{1,2,\ldots,n\}$, $w_i\nsim w_j$, and $i\neq j$.

$r_A(x_j\mid S)=(2,2,\ldots,2,\underbrace{1}_{j-term},2,\ldots,2,2)$, $j\in\{1,2,\ldots,n\}$.

$r_A(v_j\mid S)=(2,2,\ldots,2,\underbrace{1}_{j-term},2,\ldots,2,1)$, $j\in\{1,2,\ldots,n\}$.

So, $S=\{a_1,a_2,\ldots,a_n,c\}$ is a local adjacency resolving set for $G$.
$|S|=n+1$ is minimum, because if $|S|<n+1$ certainly there are $x\neq y\in V(G)-S$ such that $r(x\mid S)=r(y\mid S)$. Let $S_1=\{a_1,a_2,\ldots,a_n\}, |S_1|=n<n+1$, then $r(u_j\mid S_1)=(2,2,\ldots,2)=r(c\mid S_1)$ and $c\sim u_j$ also for every $i\in\{1,2,\ldots,n\}$. Thus, $\dim_{A,l}(G)=n+1$. □

**Local Adjacency Metric Dimension of Sunflower Graphs with Pendant Points.**

**A** Sunflower graph $SF_n$ is a graph obtained from a wheel graph $W_n\cong K_1+C_n$ with $K_1$ as a central vertex $c$ and $C_n$ as an $n$-cycle $w_0, w_1, w_2, \ldots, w_{n-1}$, and additional $n$ vertices $v_0, v_1, v_2, \ldots, v_{n-1}$ where $v_j$ is joined by edges to $w_j$, $w_{j+1}$ for $j\in\{1,2,\ldots,n-1\}$ with $j+1$ is taken modulo $n$.Order of a sunflower graph $SF_n$ is $2n+1$ and its measure is $4n$ [6].

A Sunflower graph with pendant points, $SF_n\lozenge K_1$ is a graph obtained from edge-corona of a sunflower graph $SF_n$ and a trivial graph $K_1$. Let $G\cong SF_n\lozenge K_1$ with a set of vertices
$V(G)=\{c\}\cup\{w_0,w_1,\ldots,w_{n-1}\}\cup\{v_0,v_1,\ldots,v_{n-1}\}\cup\{a_0,a_1,\ldots,a_{n-1}\}\cup\{b_0,b_1,\ldots,b_{n-1}\}\cup\{u_0,u_1,\ldots,u_{n-1}\}\cup\{x_0,x_1,\ldots,x_{n-1}\}$.
The following figure is an example of $SF_n\lozenge K_1$ graphs.

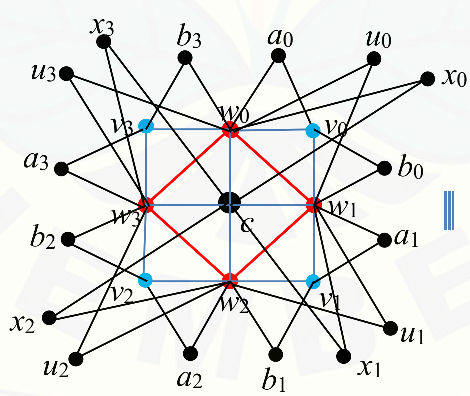


**FIGURE 3.** $SF_4\lozenge K_1$

The following theorem is the local adjacency metric dimension of sunflower graphs with pendant points.

**Theorem 3.** *Let* $G\cong SF_n\lozenge K_1$ *with* $|V(G)|=6n+1$, *then* $\dim_{A,l}(G)=n$ *for* $n\geq 3$.
**Proof.** Choose $S=\{w_0,w_1,\ldots,w_{n-1}\}\subseteq V(G)$. We will show that $S$ is a local adjacency resolving set of $G$. The local adjacency representations of vertices from $V(G)-S$ are as follows:

$$r_A(c\,|\,S)=(1,1,\ldots,1)$$
$$r_A(a_j\,|\,S)=(2,\ldots,2,\underbrace{1}_{(j+1)-term},2,\ldots,2)=r(x_j\,|\,S);\quad j\in\{1,2,\ldots,n-1\},\text{ but } a_{j+1}\nsim x_{j+1}$$
$$r_A(b_j\,|\,S)=(2,\ldots,2,\underbrace{1}_{(j+2)-term},2,\ldots,2);\quad j\in\{1,\ldots,n-1\}.$$
$$r_A(u_j\,|\,S)=(2,\ldots,2,\underbrace{1}_{(j+1)-term},\underbrace{1}_{(j+2)-term},2,\ldots,2)=r(v_j\,|\,S);\quad j\in\{1,2,\ldots,n-1\},\text{ but } u_j\nsim v_{j+1}$$

As we see that all of the adjacency representation of adjacent vertices are distinct. So, $S=\{w_0,w_1,\ldots,w_{n-1}\}$ is a local adjacency resolving set for $G$. The cardinality of $S$, $|S|=n$ is minimum, because if $|S|<n$ certainly there are $x\neq y\in V(G)-S$ such that $r(x\,|\,S)=r(y\,|\,S)$. Let $S_1=\{w_0,w_1,\ldots,w_{n-2}\},|S_1|=n-1<n$ then $r(a_{n-1}\,|\,S_1)=(2,2,\ldots,2,1)=r(v_{n-1}\,|\,S_1)$. Therefore, $\dim_{A,l}(G)=n$. □

**Local Adjacency Metric Dimension of Friendship Graphs with Pendant Points.**

A friendship graph $f_n$ is a graph obtained from a wheel graph $W_n\cong K_1+C_n$ by deleting alternate edges of the cycle $C_n$. In the other word, friendship graph $f_n$ is collection of $n$ triangles with a common point [6]. A friendship graph with pendant points denoted by $f_n\Diamond K_1$, that is a graph obtained from edge-corona of a friendship graph $f_n$ and a trivial graph $K_1$. Let $G\cong f_n\Diamond K_1$ with a set of vertices $V(G)=\{c\}\cup\{v_{ij}\,|\,i=1,2,\ldots,n;j=1,2\}\cup\{a_{ik}\,|\,i=1,2,\ldots,n;k=1,2,3\}$. As illustration, we can see FIGURE 4.

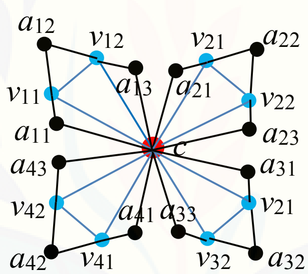


**FIGURE 4.** $f_4\Diamond K_1$

**Theorem 4.** *Let $G\cong f_n\Diamond K_1$ with $|V(G)|=5n+1$, then $\dim_{A,l}(G)=n$ for $n\geq 3$.*

**Proof.** Choose $S=\{a_{i2}\,|\,i=1,2,\ldots,n\}\subseteq V(G)$. We will show that $S$ is a local adjacency resolving set of $G$. The adjacency representations of vertices from $V(G)-S$ are as follows:

$$r_A(c\,|\,S)=(1,1,\ldots,1)$$
$$r_A(a_{i1}\,|\,S)=(2,2,\ldots,2)=r(a_{i3}\,|\,S);\quad i\in\{1,2,\ldots,n\},\text{ but } a_{i1}\nsim a_{i3}$$
$$r_A(v_{i1}\,|\,S)=(2,\ldots,2,\underbrace{1}_{i-term},2,\ldots,2)=r_A(v_{i2}\,|\,S);\quad i\in\{2,\ldots,n\},\text{ but } v_{i1}\nsim v_{i2}.$$

All of the adjacency representations of adjacent vertices are distinct. So, $S=\{a_{i2}\,|\,i=1,2,\ldots,n\}$ is a local adjacency resolving set for $G$. The cardinality of $S$, $|S|=n$ is minimum, because if $|S|<n$ certainly there are $x\neq y\in V(G)-S$ such that $r(x\,|\,S)=r(y\,|\,S)$. Let $S_1=\{a_{i2}\,|\,i=1,2,\ldots,n-1\}$, $|S_1|=n-1<n$ then $r(a_{n1}\,|\,S_1)=(2,2,\ldots,2)=r(v_{n1}\,|\,S_1)=r(a_{n2}\,|\,S_1)=r(v_{n2}\,|\,S_1)=r(a_{n3}\,|\,S_1)$, and $a_{n1}\sim v_{n1}\sim a_{n2}\sim v_{n2}\sim a_{n3}$. So, $\dim_{A,l}(G)=n$. □

**Local Adjacency Metric Dimension of Wheel Graphs with Pendant Points.**

**Theorem 5.** *Let* $G \cong W_n \lozenge K_1$ *with* $|V(G)| = 3n+1$, *then* $dim_{A,l}(G) = \left\lfloor \frac{n+9}{6} \right\rfloor$ *for* $n \geq 4$.

Buczkowski et.al. in [11] have mentioned that the metric dimension of the wheel graph, $W_n = K_1 + C_n$, is

$$\dim(W_n) = \begin{cases} 3 & , for\ n = 3,6 \\ \left\lfloor \frac{2n+2}{5} \right\rfloor & , otherwise. \end{cases}$$

where $K_1$ is a trivial graph and $C_n$ is a cycle graph of order $n$.

From the results have discussed above, we can conclude that $\dim_{A,l}(W_n \lozenge K_1) \leq \dim(W_n)$ and $\dim_{A,l}(W_n \lozenge K_1) \leq \dim_{A,l}(G \lozenge K_1)$ with $G$ be a wheel related graph.

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