# Bound of Distance Domination Number of Graph and Edge Comb Product Graph

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**Abstract.** Let $G = (V, E)$ be a simple, nontrivial, finite, connected and undirected graph. For an integer $1 \leq k \leq diam(G)$, a distance $k$-dominating set of a connected graph $G$ is a set $S$ of vertices of $G$ such that every vertex of $V(G)\backslash S$ is at distance at most $k$ from some vertex of $S$. The $k$-domination number, denoted by $\gamma_k(G)$, of $G$ is the minimum cardinality of a $k$-dominating set of $G$. In this paper, we improve the lower bound on the distance domination number of $G$ regarding to the diameter and minimum degree as well as the upper bound regarding to the order and minimum $k$ distance neighbourhood. In addition, we determine the bound of distance domination number of edge comb product graph.

**Keywords:** distance domination, diameter, edge comb product graph.

## 1. Introduction

Let $G = (V(G), E(G))$ be a simple, finite, connected and undirected graph. Graph terminology related with $k$-dominating number are distance, diameter, neighbourhood of vertex, closed neighbourhood of vertex, $k$-distance neighbourhood of vertex, and $k$-distance closed neighbourhood of vertex. The distance $d(u, v)$ from a vertex $u$ to a vertex $v$ in a connected graph $G$, or simply $d(u, v)$, is the minimum of the lengths of the $u - v$ paths in $G$, for detail definition see [4]. A neighbour of a vertex $v$ in $G$ is a vertex adjacent to $v$. The open neighbourhood of $v$, denoted by $N(v)$, is the set of all neighbours of $v$ in $G$, while the closed neighbourhood of $v$ is the set $N[v] = N(v) \cup \{v\}$. The closed $k$-neighbourhood of $v$, denoted by $N_k[v]$, is defined as the set of all vertices within distance $k$ from $v$ in G; that is, $N_k[v] = \{u|d(u, v) \leq k\}$ [13]. When $k = 1$, $N_k[v] = N[v]$. The degree of a vertex $v$ in $G$, denoted by $d(v)$, is the number of neighbours of $v$ in $G$, that is $|N(v)|$. The minimum and maximum degree among all the vertices of $G$ are denoted by $\delta(G)$ and $\Delta(G)$, respectively.

Domination in graphs is well-known research interest. The study has been done by several authors such as [2], [6], [8], [10], [11], [12], [14],[15], [16]. A dominating set in a graph $G$ is a set $S$ of vertices of $G$ such that every vertex in $V(G) \setminus S$ is adjacent to at least one vertex in S. The domination number of $G$, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of $G$. For more details about the notion of domination in graphs, see [4] and [5].

In 1975, Meir and Moon [3] introduced the concept of a distance $k$-dominating set (or or $k$-covering) in a graph. A set $S$ is a $k$-dominating set of $G$ if every vertex is within distance $k$ from some vertex of S; that is, for every vertex $v$ of $G$, we have $d(v, S) \leq k$. The $k$-domination number of $G$, denoted by $\gamma_k(G)$, is the minimum cardinality of a $k$-dominating set of $G$. When $k = 1$, the 1-domination number of $G$ is precisely the domination number of $G$; that is, $\gamma_1(G) = \gamma(G)$. The literature on the subject of distance domination in graphs up to the year 1997 can be found in the book chapter [19]. Distance domination is now widely studied, for example [7], [9], [13], [17], [20], and [21].

The $k$-domination number of G is in the class of $NP$-hard problem see [18]. Because of its computational complexity, graph theorists have studied upper and lower bounds on $\gamma_k(G)$ in terms of simple graph parameters like order, size, and degree. Previous results on the bound of $\gamma_k(G)$ is credited to Meir and Moon, Tian and Xu, Henning and Lichiardopol. In 1975, Meir and Moon [3] established an upper bound for the k-domination number of a tree in terms of its order. They proved that for $k \geq 1$, if $T$ is a tree of order $n \geq k+1$, then $\gamma_k(T) \leq \frac{n}{k+1}$. Since $\gamma_k(G) \leq \gamma_k(T)$, then $\gamma_k(G) \leq \frac{n}{k+1}$ see [3]. Tian and Xu [7] improved the Meir-Moon upper bound and showed that for $k \geq 1$, if $G$ is a connected graph of order $n \geq k+1$ with maximum degree $\Delta$, then $\gamma_k(G) \leq \frac{1}{k(n-\Delta+k-1)}$. The Tian-Xu bound was further improved by Henning and Lichiardopol [9] by showing that for $k \geq 2$, if $G$ is a connected graph with minimum degree $\delta \geq 2$ and maximum degree $\Delta$ and of order $n \geq \Delta + k - 1$, then $\gamma_k(G) \leq \frac{n+\delta-\Delta}{\delta+k-1}$. For lower bound of $\gamma_k(G)$, Randy proved that $\gamma_k(G) \geq \frac{d+1}{2k+1}$ for $d$ is diameter of $G$. He also investigated $\gamma_k(G)$ for direct product of graph, where $\gamma_k(G \times H) \leq \gamma_k(G) + \gamma_k(H) - 1$ [13].

In this paper we improve the lower bound on distance domination number of $G$ regarding to the diameter and minimum degree as well as the upper bound regarding to the order and minimum $k$ distance neighborhood. In addition, we determine the bound of distance domination number of graph resulting from operation, namely, edge comb product graph. Edge comb product graph is a varian of comb product graph. For definition of comb product graph operation see [1].

## 2. Results

Prior to show the main result, we first show the upper bound of simple connected graph. We start with the following theorem in [17] explains a propperty of $k$ distance dominating set.

**Theorem 1.** *[17] Let $k \geq 1$ and $S$ be a distance $k$-dominating set of a graph $G$. Then $S$ is a minimal distance $k$-dominating set of $G$ if and only if each $d \in S$ has at least one of the following two properties hold.*

*(i) There exist a vertex $v \in V(G) - S$ such that $N_k(v) \cap S = \{d\}$.*

*(ii) The vertex $d$ is at distance at least $k+1$ from every other vertex $d$ of $S$ in $G$.*

We use the minimum cardinality of $k$ distance neighbourhood of a vertex on $G$, $|X|$, as parameter of our upper bound. As example for $k = 1$, it is easy to obtain that, $|X| = 1 + \delta(G)$. But, for $k \geq 2$ we obtain $|X|$ by evaluating $k$ distance neighbourhood of each vertex in $G$.

**Lemma 1.** *Let $S$ be distance $k$-dominating set and $C = N_k[u] \cap N_k[v], \forall u, v \in S$, then $|C| < n - |S|$.*

*Proof.* Since $S$ is distance $k$ dominating set of $G$ then by Theorem 1, the following two cases is hold.

(i) For all $u \in S$, $u$ have special neighbour at distance at most $k$ that are not dominated by another vertex in $S$. The maximum order of $C$ is $n = |V(G)|$. Since every two vertices in $G$ has its own special neigbour at most $k$ distance and each vertex in $G$ has at least one special neigbour. Then, there exist at least $|S|$ element that not belongs to $C$. So, $|C| < n - |S|$.

(ii) Every two vertices $u, v \in S$, $u \notin N_k[v]$. In other word, any two vertices in $S$ are not adjacent one to another at distance at most $k$. This mean, all vertices in $S$ are not belongs to $C$. So, $|C| < n - |S|$.

□

**Theorem 2.** *Let $|X|$ be minimum cardinality of $k$ distance neighbourhood of a vertex on $G$, the following holds*

$$\gamma_k(G) < \frac{2n}{|X|+1}.$$

*Proof.* Let $S$ be the minimum of distance $k$ dominating set, $|S| = \gamma_k(G)$, and $|V(G)| = n$. We proof the theorem by using the following counting argument.

$$\begin{aligned} &\Sigma_{v\in S}(|N_k[v]| - |X|) \geq 0 \\ \Leftrightarrow\ &\Sigma_{v\in S}|N_k[v]| - \Sigma_{v\in S}|X| \geq 0 \\ \Leftrightarrow\ &\Sigma_{v\in S}|N_k[v]| - |S||X| \geq 0 \\ \Leftrightarrow\ &|S||X| \leq \Sigma_{v\in S}|N_k[v]|. \end{aligned}$$

Since, $\Sigma_{v\in S}|N_k[v]| = n + |C|$, where $C = N_k[u] \cap N_k[v], \forall u, v \in S$. We can obtain

$$|S||X| \leq n + |C|. \tag{1}$$

By Lemma 1 we get

$$|C| < n - |S|. \tag{2}$$

By combining Equation 1 and 2 we can get $\gamma_k(G) < \frac{2n}{|X|+1}$. □

This upper bound is tight for some cases, for example, if we take $G$ be regular graph, such as prism, with order $n, \delta = \Delta = 3$ and $k = 2$. By using Henning and Lichiardopol bound we can get $\gamma_k(G) \leq n/4$. By using our bound we get $\gamma_k(G) < 2n/9$ where $|X| = 8$ for $n \geq 5$.

The trivial lower bound for distance domination number of regular graph is $\gamma_k(G) \geq \frac{(d-2)n}{d(d-1)^k-2}$. The lower bound is derived by considering the maximum number of vertices a vertex may dominated. For graph is any simple connected graph with maximum degree $\Delta$, $\gamma_k(G) \geq \frac{(\Delta-2)n}{\Delta(\Delta-1)^k-2}$. We derive distance $k$-domination number of $G$ based on its minimum degree and diameter. This result has similar strategy as in proof of R. Davila *et al.* [13] result, but we add minimum degree parameter as improvement.

**Theorem 3.** *If $G$ is simple connected graph with minimum degree, $\delta \geq 2$, and diameter, $diam(G) = d$, $k \leq d$. Then*

$$\gamma_k(G) \geq \frac{\left\lceil \frac{d-1}{3} \right\rceil (\delta - 2) + d + 1}{\left\lceil \frac{2k-1}{3} \right\rceil (\delta - 2) + (2k+1)}.$$

*Proof.* Suppose that $P$ is diameter path of $G$ and let $P = \{u_i | i = 1, 2, \ldots, d, d+1\}$. Since $\delta$ is minimum degree of $G$, then $\forall u_i \in P, i = 2, 3, \ldots, d, |N(u_i)| \geq \delta$. Supposed that $P'$ is subgraph of $G$ that is obtained by $P \cup \{w_j | w_j \in N(u_i), u_i \in V(P) \backslash \{u_1, u_{d+1}\}, 1 \leq j \leq \delta - 2\}$. Note that it is not necessary for any two vertices $v_i, v_j \in V(P), N(v_i) \cap N(v_j) = \emptyset$ for $i, j \in \{2, 3, \ldots, d\}$. The maximum number of vertices in $V(P)$ that share its neighbourhood is three. In contrary suppose that there are four vertices in $V(P)$ that share at least one of its neighbourhood. without loss of generality, suppose that for some $w \in V(G), w \in N(v_2) \cap N(v_3) \cap N(v_4) \cap N(v_5)$. Then there is a path $v_1 v_2 w v_5 v_6 \ldots v_d v_{d+1}$ with lenght is equal to $d - 1$. This situation contradiction with our asumption that $d(v_1, d_{d+1}) = d$ where $d$ is diameter. If we include the minimum neighbourhood of every vertices in $P$ a part of $v_1$ and $v_{d+1}$, we can obtain a subgraph $P'$ with $V(P') = V(P) \cup N(v_i), v_i \in V(P), i = 2, 3, \ldots d$. Since three vertices of $\{v_2, v_3, \ldots, v_d\}$ can share the same neighbour, then the minimum cardinality of $V(P')$ is

$$|V(P')| = \left\lceil \frac{d-1}{3} \right\rceil (\delta - 2) + d + 1.$$

The maximum value of $N_k[s], s \in S$, is obtained by putting $s$ in the middle of diameter path $P$. The maximum number of vertices that can be dominated by $s$ in $P$ is $2k+1$. Suppose that $N_k[s, V(P)] = \{v_1, v_2, \ldots v_{2k+1}\}$ is closed $k$ distance neighbourhood of $s$ on $V(P)$, then the maximum number of vertices that can be dominated by $s$ in $P'$ are

$$|N_k[s]| \leq |N_k[s, V(P)]| + |\cup_{i=2}^{2k} N(u), u \in N_k[s, V(P)]|.$$

Since at most three vertices in $N_k[s, V(P)]$ can share the same neoghbour, then $|\cup_{i=2}^{2k} N(u), u \in N_k[s, V(P)]| = \left\lceil \frac{2k-1}{3} \right\rceil (\delta - 2)$. Therefore,

$$|N_k[s]| \leq (2k+1) + \left\lceil \frac{2k-1}{3} \right\rceil (\delta - 2).$$

Let $S$ be a minimum $k$-dominating set of G. Thus, $|S| = \gamma_k(G)$. Each vertex of $S$ $k$-dominates at most $(2k+1) + \left\lceil \frac{2k-1}{3} \right\rceil (\delta - 2)$ vertices of $P'$, and so $S$ $k$-dominates at most $|S|((2k+1) + \left\lceil \frac{2k-1}{3} \right\rceil (\delta - 2))$ vertices of $P'$. However, since $S$ is a $k$-dominating set of $G$, every vertex of $P'$ is $k$ dominated the set $S$, and so $S$ $k$-dominates $|V(P')| = \left\lceil \frac{d-1}{3} \right\rceil (\delta - 2) + d + 1$ vertices of $P'$. Therefore, $|S|((2k+1) + \left\lceil \frac{2k-1}{3} \right\rceil (\delta - 2)) \geq \left\lceil \frac{d-1}{3} \right\rceil (\delta - 2) + d + 1$, or, equivalently, $\gamma_k(G) \geq \frac{\left\lceil \frac{d-1}{3} \right\rceil (\delta-2)+d+1}{\left\lceil \frac{2k-1}{3} \right\rceil (\delta-2)+(2k+1)}$. $\square$

For $\delta = 2$, then $\gamma_k(G) \geq \frac{d+1}{2k+1}$. Our lower bound is greater than $\frac{d+1}{2k+1}$ when the value $d > 2k$. We can verify thus condition by the following algebraic operation

$$d > 2k \Leftrightarrow 2d > 4k \Leftrightarrow 2kd - 2k + d - 1 > 2kd + 2k - d - 1 \Leftrightarrow (2k+1)(d-1) > (2k-1)(d+1)$$

$$\cdots \Leftrightarrow (2k+1)\frac{(d-1)}{3} > \frac{(2k-1)}{3}(d+1)$$

$$\Rightarrow (2k+1) \left\lceil \frac{(d-1)}{3} \right\rceil > \left\lceil \frac{(2k-1)}{3} \right\rceil (d+1)$$

$$\Leftrightarrow (2k+1) \left\lceil \frac{(d-1)}{3} \right\rceil (\delta - 2) > \left\lceil \frac{(2k-1)}{3} \right\rceil (\delta - 2)(d+1)$$

$$\Leftrightarrow (2k+1) \left\lceil \frac{(d-1)}{3} \right\rceil (\delta - 2) + (d+1)(2k+1) > \left\lceil \frac{(2k-1)}{3} \right\rceil (\delta - 2)(d+1) + (d+1)(2k+1)$$

$$\Leftrightarrow (2k+1) \left( \left\lceil \frac{(d-1)}{3} \right\rceil (\delta - 2) + (d+1) \right) > (d+1) \left( \left\lceil \frac{(2k-1)}{3} \right\rceil (\delta - 2) + (2k+1) \right)$$

$$\Leftrightarrow \frac{\left\lceil \frac{(d-1)}{3} \right\rceil (\delta - 2) + (d+1)}{\left\lceil \frac{(2k-1)}{3} \right\rceil (\delta - 2) + (2k+1)} > \frac{d+1}{2k+1}.$$

We end this section by determining the distance $k$-domination number of an edge comb product graph. The edge comb product between $G$ and $H$, denoted by $G \trianglelefteq H$, is a graph obtained by taking one copy of $G$ and $|E(G)|$ copies of $H$ and grafting the $i$-th copy of $H$ at the $i$-th edges of $G$. The cardinality of edge and vertex of $G \trianglelefteq H$ are defined as follow.

**Definition 1.** *Let $G = (V(G), E(H))$ and $H = (V(H), E(H))$. The edge comb product between $H$ onto $G$ with linking edge $yy' \in E(H)$, denoted by $G \trianglelefteq H$, is graph with vertex set*

$$V(G \trianglelefteq H) = V(G) \cup ((V(H)\backslash\{y, y'\}) \times E(G))$$

*and*

$$\begin{aligned} E(G \trianglelefteq H) =& E(G) \cup \{(y_1e)(y_2e); y_1y_2 \in E(H); y_1, y_2 \notin \{y, y'\}; e \in E(G)\} \\ & \cup \{(x)(y''e); e = ww' \in E(G);\ \textit{if } x = w, \textit{then } yy'' \in E(H)\backslash\{yy'\} \textit{ or} \\ & \textit{if } x = w', \textit{then } y'y'' \in E(H)\backslash\{yy'\}\}. \end{aligned}$$

By simple calculation we can verify that

$$\begin{aligned} |E(G \trianglelefteq H)| &= |E(G)| + |E(G)|(|E(H)| - (|N(y)| + |N(y')| - 1)) + |E(G)|(|N(y)| - 1 + |N(y')| - 1) \\ &= |E(G)||E(H)| \end{aligned}$$

and $|V(G \trianglelefteq H)| = |V(G)| + |E(G)|(|V(H)| - 2)$. we will determine lower and upper bound $\gamma_k(G \trianglelefteq H)$ as a function with $\gamma_k(H)$ as independent variable.

**Theorem 4.** *If $G$ and $H$ be a simple connected graph, then $\gamma_k(G \trianglelefteq H) \leq \gamma_k(H)|E(G)|$.*

*Proof.* Vertex set of graph operation $G \trianglelefteq H$ can be written as $\bigcup_i^{|E(G)|} V(H_i)$, where $H_i$ is graph that isomorphic to $H$ that replace some edge on $G$. If we set

$$\begin{aligned} \gamma_k(G \trianglelefteq H) &= \Sigma_i^{|E(G)|}\gamma_k(H_i) \\ &= \gamma_k(H)|E(G)|, \end{aligned}$$

then $\forall v \in H_i, 1 \leq i \leq |E(G)|$ vertex $v$ is dominated at $k$ distance by some element of $\gamma_k(H_i)$, so $\gamma_k(G \trianglelefteq H) \leq \gamma_k(H)|E(G)|$. □

**Theorem 5.** *For any simple connected graph $G$ and regular graph $H$. If $\gamma_k(H) \geq 2$ and $|V(H)|$ divisible by $|N_k[u]|, u \in V(H)$, then $\gamma_k(G \trianglelefteq H) \geq (\gamma_k(H) - 2)|E(G)| + |V(G)|$.*

*Proof.* Supposed that $|V(G)| = n, |E(G)| = n', |V(H)| = m,$ and $|E(G)| = m'$. First we count how many vertices that can be dominated by $S_k$. We denote $|X|$ as the maximum number of vertices that can be dominated by $S_k$ (the set of distance $k$-dominaiton number).

$$\begin{aligned} |X| \leq& \Sigma_{v\in V(G)}(d(v)(|N_k[u]| - 1)) + |V(G)| + (\gamma_k(G) - 2)|N_k[u]|n' \\ =& 2n'(|N_k[u]| - 1) + n + \left(\left\lceil \frac{m}{|N_k[u]|} \right\rceil - 2\right) n'|N_k[u]| \\ =& 2n'(|N_k[u]| - 1) + n + \left\lceil \frac{m}{|N_k[u]|} \right\rceil |N_k[u]|n' - 2n'|N_k[u]| \\ =& \left\lceil \frac{m}{|N_k[u]|} \right\rceil |N_k[u]|n' + n - 2n'. \end{aligned}$$

Since $V(H)$ divisible by $|N_k[u]|$, then

$$|X| \leq mn' + n - 2n' = |V(G \trianglelefteq H)|.$$

By contradiction we will show that

$$\gamma(G \trianglelefteq H) < (\gamma_k(H) - 2)|E(G)| + |V(G)|$$

is imposible. If $V(G \trianglelefteq H)$ is written as $\bigcup_i^{|E(G)|} V(H_i)$, where $H_i$ is isomorphic graph to $H$ that replace some edge on $G$ and $\gamma(G \trianglelefteq H) < (\gamma_k(H) - 2)|E(G)| + |V(G)|$, then there is exist $v \in S(H_i), v \notin S(G \trianglelefteq H)$. Note that the notation $S(G)$ means $k$ distance dominating set of $G$. We have two cases:
*Case 1*, vertex $v$ is not sharing vertex of any two subgraph $H_i$ and $H_j$. We will count the number of vertices that can be reach by $S$. We denote the maximum number of vertices that can be dominated by $S$ as $|X'|$.

$$\begin{aligned}
|X'| &= \Sigma_{v \in V(G)} d(v)(|N_k[u]| - 1) + |V(G)| + ((\gamma_k(G) - 2) - 1)|N_k[u]| \\
&= \Sigma_{v \in V(G)} (d(v)(|N_k[u]| - 1)) + |V(G)| + (\gamma_k(G) - 2)|N_k[u]| - |N_k[u]| \\
&= |X| - |N_k[u]| \\
&\leq mn' + n - 2n' - |N_k[u]| \\
&< |V(G \trianglelefteq H)|
\end{aligned}$$

*Case 2*, vertex $v$ is the element of sharing vertex of some subgraph $H_i$ and $H_j$.

$$\begin{aligned}
|X'| &= (\Sigma_{v \in V(G)} d(v) - \delta)(|N_k[u]| - 1) + |V(G) - 1| + (\gamma_k(G) - 2)|)n'|N_k[u]| \\
&= \Sigma_{v \in V(G)} (d(v)(|N_k[u]| - 1)) + |V(G)| + (\gamma_k(G) - 2)(|N_k[u]|) - (1 + \delta(|N_k[u]| - 1)) \\
&\leq |X| - (1 + \delta(|N_k[u]| - 1)) \\
&< |X| - |N_k[u]| \\
&< |V(G \trianglelefteq H)|
\end{aligned}$$

All two cases above shows a contradiction, so $\gamma_k(G) \geq (\gamma_k(H) - 2)|E(H)| + |V(G)|$. It completes the proof. □

## 3. Conclusion

The result of this paper is related to new upper and lower bound for general graph and edge comb product graph operation. Our upper bound for general graph is tight especially if $G$ regular. For lower bound, we find that by including minimum degree and diameter as parameters, we can get better lower bound compared with if we consider only diameter as parameter. For edge comb product graph, we find that the value of its lower and upper bound of distance domination number is depend on the value of distance domination number of second graph in edge comb product operation of graphs.

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