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Bounds on the number of isolates in sum graph labeling

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Abstract

A simple undirected graph H is called a *sum graph* if there is a labeling L of the vertices of H into distinct positive integers such that any two vertices u and v of H are adjacent if and only if there is a vertex w with label $L(w)=L(u)+L(v)$. The sum number $\sigma(G)$ of a graph $G=(V,E)$ is the least integer r such that the graph H consisting of G and r isolated vertices is a sum graph. It is clear that $\sigma(G)\leqslant|E|$. In this paper, we discuss general upper and lower bounds on the sum number. In particular, we prove that, over all graphs $G=(V,E)$ with fixed $|V|\geqslant 3$ and $|E|$, the average of $\sigma(G)$ is at least $|E|-3|V|(\log|V|)/[\log(\binom{|V|}{2}/|E|)]-|V|-1$. In other words, for most graphs, $\sigma(G)\in\Omega(|E|)$. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Sum graphs; Graph labeling; Sum number

1. Introduction

The notion of sum graphs was first introduced by Harary [7]. From a practical point of view, sum graph labeling can be used as a compressed representation of a graph, a data structure for representing the graph. Data compression is important not only for saving memory space but also for speeding up some graph algorithms when adapted to work with the compressed representation of the input graph (for example, see [5,10]).

There have been several papers determining or bounding the sum number of particular classes of graphs $G=(V,E)$ $(n=|V|,\ m=|E|)$:

- $\sigma(K_n)=2n-3$ for complete graphs K_n $(n\geqslant 4)$ [1],
- $\sigma(K_{p,q})\leqslant\lceil(3p+q-3)/2\rceil$ for complete bipartite graphs $K_{p,q}$ $(q\geqslant p\geqslant 2)$ [8],
- $\sigma(T)=1$ for trees T $(n\geqslant 2)$ [4],

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