On two conjectures concerning vertex-magic total labelings of generalized Petersen graphs

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Abstract

A vertex-magic total labeling of a graph with v vertices and e edges is defined as a one-to-one map taking the vertices and edges onto the integers $1, 2, \ldots, v+e$ with the property that the sum of the label on a vertex and the labels on its incident edges is a constant, independent of the choice of vertex.

In this paper we give a vertex-magic total labeling for the prism D_n for all $n \geq 3$; and a vertex-magic total labeling for the generalized Petersen graphs $P(n, m)$ for $n \geq 3$, $1 \leq m \leq \lfloor \frac{n-1}{2} \rfloor$, and n and m coprime.

1 Introduction

A *vertex-magic total labeling* of a graph $G = (V, E)$ with v vertices and e edges is an assignment of the integers from 1 to $v + e$ to the vertices and edges of G with the property that the sum of the label on a vertex and the labels on its incident edges is a constant, independent of the choice of vertex. More formally, a one-to-one map λ from $E \cup V$ onto the integers $\{1, 2, \cdots, e + v\}$ is a *vertex-magic total labeling* if there is a constant k so that for every vertex x,

$$\lambda(x) + \sum \lambda(xy) = k \tag{1}$$

where the sum is over all vertices y adjacent to x. The constant k is called the *magic constant* for λ. This notion was introduced in [5]. We note that not every graph has a vertex-magic total labeling. For the graph K_2, since $\lambda(x) \neq \lambda(y)$, then $\lambda(x) + \lambda(xy) \neq \lambda(y) + \lambda(xy)$, and so no vertex-magic total labeling is possible. Another example is $K_{m,n}$, for $n > m + 1$, see [5].