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Kristiana Wijaya, Edy Tri Baskoro, Asep Iqbal Taufik, Denny Riama Silaban

Let G, H, and F be simple graphs. The notation F ⟶ (G, H) means that any red-blue coloring of all edges of F contains a red copy of G or a blue copy of H. The graph F satisfying this property is called a Ramsey (G, H)-graph. A Ramsey (G, H)-graph is called minimal if for each edge e ∈ E(F), there exists...

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Ramsey Graphs for a Star on Three Vertices Versus a Cycle

Maya Nabila, Edy Tri Baskoro, Hilda Assiyatun

Let G, A, and B be simple graphs. The notation G → (A, B) means that for any red-blue coloring of the edges of G, there is a red copy of A or a blue copy of B in G. A graph G is called a Ramsey graph for (A, B) if G → (A, B). Additionally, if the graph G satisfies that G – e ↛ (A, B), for any e ∈ E(G),...

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On Ramsey (mK_2, P_4)-Minimal Graphs

Asep Iqbal Taufik, Denny Riama Silaban, Kristiana Wijaya

Let F, G, and H be simple graphs. The notation F → (G, H) means that any red-blue coloring of all edges of F will contain either a red copy of G or a blue copy of H. Graph F is a Ramsey (G, H)-minimal if F → (G, H) but for each e ∈ E(F), (F – e) ↛ (G, H). The set $\mathscr{R}$(G, H) consists of all Ramsey (G,...

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Spectrum of Unicyclic Graph

Budi Rahadjeng, Dwi Nur Yunianti, Raden Sulaiman, Agung Lukito

Let G be a simple graph with n vertices and let A(G) be the (0, 1)-adjacency matrix of G. The characteristic polynomial of the graph G with respect to the adjacency matrix A(G), denoted by χ(G, λ) is a determinant of (λI – A(G)), where I is the identity matrix. Suppose that λ1 ≥ λ2 ≥ ⋯ ≥ λn are the adjacency...

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Distinguishing Number of the Generalized Theta Graph

Andi Pujo Rahadi, Edy Tri Baskoro, Suhadi Wido Saputro

A generalized theta graph is a graph constructed from two distinct vertices by joining them with l (>=3) internally disjoint paths of lengths greater than one. The distinguishing number D(G) of a graph G is the least integer d such that G has a vertex labelling with d labels that is preserved only...

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Edge Magic Total Labeling of (*n, t*)-Kites

Inne Singgih

An edge magic total (EMT) labeling of a graph G = (V, E) is a bijection from the set of vertices and edges to a set of numbers defined by λ: V ∪ E → {1,2, ..., |V| + |E|} with the property that for every xy ∈ E, the weight of xy equals to a constant k, that is, λ(x) + λ(y) + λ(xy) = k for some integer...

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Further Result of *H*-Supermagic Labeling for Comb Product of Graphs

Ganesha Lapenangga P., Aryanto, Meksianis Z. Ndii

Let G = (V, E) and H = (V', E') be a connected graph. H-magic labeling of graph G is a bijective function f: V(G) ∪ E(G) → {1,2, ..., |V(G)| + |E(G)|} such that for every subgraph H'of G isomorphic to H, ∑v ∈ V(H') f(v) + ∑e ∈ E(H') f(e) = k. Moreover, it is H-supermagic labeling if f(V) = {1,2 ,..., |V|}....

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Labelling of Generalized Friendship, Windmill, and Torch Graphs with a Condition at Distance Two

Ikhsanul Halikin, Hafif Komarullah

A graph labelling with a condition at distance two was first introduced by Griggs and Robert. This labelling is also known as L(2,1)-labelling. Let G = (V, E) be a non-multiple graph, undirected, and connected. An L(2,1)-labelling on a graph is defined as a mapping from the vertex set V(G) to the set...

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On the Minimum Span of Cone, Tadpole, and Barbell Graphs

Hafif Komarullah, Ikhsanul Halikin, Kiswara Agung Santoso

Let G be a simple and connected graph with p vertices and q edges. An L(2,1)-labelling on the graph G is a function f: V(G) → {0,1, ..., k} such that every two vertices with a distance one receive labels that differ by at least two, and every two vertices at distance two receive labels that differ by at...

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L(2,1) Labeling of Lollipop and Pendulum Graphs

Kusbudiono, Irham Af'idatul Umam, Ikhsanul Halikin, Mohamat Fatekurohman

One of the topics in graph labeling is L(2,1) labeling which is an extensio[redacted] graph labeling. Definition of L(2,1) labeling is a function that maps the set[redacted] vertices in the graph to non-negative integers such that every two vertices u, v that have a distance one must have a label with a difference...

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Magic and Antimagic Decomposition of Amalgamation of Cycles

Sigit Pancahayani, Annisa Rahmita Soemarsono, Dieky Adzkiya, Musyarofah

Consider G = (V, E) as a finite, simple, connected graph with vertex set V and edge set E. G is said to be a decomposable graph if there exists a collection of subgraphs of G, say $\mathscr{H}$ = {Hi|1 $\leq$ i $\leq$n} such that for every i $\neq$ j, Hi is isomorphic to Hj, $\cup$ i=1n Hi = G and should satisfy that E(Hi) $\cap$ E(Hj)...

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A Minimum Coprime Number *for Amalgamation of Wheel*

Hafif Komarullah, Slamin, Kristiana Wijaya

Let G be a simple graph of order n. A coprime labeling of a graph G is a vertex labeling of G with distinct positive integers from the set {1, 2, ..., k} for some k $\geq$ n such that any adjacent labels are relatively prime. The minimum value of k for which G has a coprime labelling, denoted as $\mathfrak{pr}$(G), is...

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Rainbow Connection Number of Shackle Graphs

M. Ali Hasan, Risma Yulina Wulandari, A.N.M. Salman

Let G be a simple, finite and connected graph. For a natural number k, we define an edge coloring c: E(G) → {1,2,..., k} where two adjacent edges can be colored the same. A u – v path (a path connecting two vertices u and v in V(G)) is called a rainbow path if no two edges of path receive the same color....

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Local Antimagic Vertex Coloring of Corona Product Graphs $P_n \circ P_k$

Setiawan, Kiki Ariyanti Sugeng

Let G = (V, E) be a graph with vertex set V and edge set E. A bijection map f : E → {1,2, ..., |E|} is called a local antimagic labeling if, for any two adjacent vertices u and v, they have different vertex sums, i.e. w(u) ≠ w(v), where the vertex sum w(u) = Σe ∈ E(u) f(e), and E(u) is the set of edges...

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Local Antimagic Vertex Coloring of Gear Graph

Masdaria Natalina Br Silitonga, Kiki Ariyanti Sugeng

Let G = (V, E) be a graph that consist of a vertex set V and an edge set E. The local antimagic labeling f of a graph G with edge-set E is a bijection map from E to {1, 2, ..., |E|} such that w(u) ≠ w(v), where w(u) = ∑e ∈ E(u) f(e) and E(u) is the set of edges incident to u. In this labeling, every vertex...

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Implementations of Dijkstra Algorithm for Searching the Shortest Route of Ojek Online and a Fuzzy Inference System for Setting the Fare Based on Distance and Difficulty of Terrain (Case Study: in Semarang City, Indonesia)

Vani Natali Christie Sebayang, Isnaini Rosyida

Ojek Online is a motorcycle taxi that is usually used by people that need a short time for traveling. It is one of the easiest forms of transportation, but there are some obstacles in hilly areas such as Semarang City. The fare produced by online motorcycle taxis is sometimes not in accordance with the...

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Granular Dynamic Simulations of Depositing Materials: Two-Dimensional Approach

Mohamad Hasan

During the deposition process, many factors play a role in the dynamics of the system including materials' characteristics and media onto which the materials dropped. The stick-slip model has been applied to simulate the depositions of polydisperse granular materials. As the size of the materials varied,...

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Natanael Karjanto

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Agustina Pradjaningsih, Fatmawati, Herry Suprajitno

Linear programming is mathematical programming developed to deal with optimization problems involving linear equations in the objective and constraint functions. One of the basic assumptions in linear programming problems is the certainty assumption. Assumption of certainty shows that all coefficients...

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Generalization of Chaos Game on Polygon

Kosala D. Purnomo

The original chaos game has been applied to the triangular attractor points. With the rules for selecting attractor points randomly, the points generated in large iterations will form like a Sierpinski triangle. Several studies have developed it on the attractor points of quadrilaterals, pentagons, and...

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High Order Three-Steps Newton Raphson-like Schemes for Solving Nonlinear Equation Systems

Rizki Multazamil Fatahillah, M Ziaul Arif, Rusli Hidayat, Kusbudiono, Ikhsanul Halikin

This study proposes several new 3-steps schemes based on the Newton-Raphson method for solving non-linear equation systems. The proposed schemes are analysed and formulated based on the Newton-Raphson method and the Newton-cotes open form numerical integration method. In general, the schemes can be considered...

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Symmetry Functions with Respect to Any Point in R^n and Their Properties

Firdaus Ubaidillah

A function f : R → R is said to be an odd function if f(−x) = −f(x) for every x in R. The graph of an odd function is symmetric with respect to the origin, that is the point (0,0). The aims of this paper are to generalize odd functions on Rn and introduce symmetry functions with respect to any point...

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Root Water Uptake Process for Different Types of Soil in Unsteady Infiltration from Periodic Trapezoidal Channels

Millatuz Zahroh, Imam Solekhudin

This study involves a non-linear partial differential equation known as Richard's Equation. An unsteady infiltration from trapezoidal periodic irrigation channel with root-water uptake is considered as the problem. To solve the problem, A set of transformations, Kirchhoff transformation, dimensionless...

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Contact Tracking with Social Network Analysis Graph

Alvida Mustika Rukmi, Wildan Zakky, M. Lutfhi Shahab

In 2020, the world is facing a Covid-19 virus pandemic. The fields of epidemiology and networks are needed in dealing with its spread. Individual (contact) tracing is an important control measure in the spread of infectious diseases. The network of contacts describes the potential pathways for the spread...

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Learning Materials Development of Parametric Curves and Surfaces for Modeling the Objects Using Maple on Differential Geometry Courses

Kusno, Abduh Riski

Modeling industrial objects needs the formulas of curves and surfaces to construct a precise shape of real goods and simulate some process of form creations. For this reason, the study of the equations of curves and surfac [redacted] for objects modeling is essential for resulting in a required shape and fea [redacted]

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Hanging Rotera Modeling by Joining Deformation Result of Space Geometry Objects

Een Ubaningrum, Bagus Juliyanto, Ahmad Kamsyakawuni, Firdaus Ubaidillah

The hanging rotera is a small lamp covered by a glass lid with a light source from a burning candle or LED (Light Emitting Diode) candle and hung on a support pole that is hooked to the rotera connector. The purpose of this paper are to obtain a models of the various and symmetrical components of the...

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Kusno, Bagus Juliyanto, Kiswara Agung Santoso

Creative industries are a national strategic commodity to support international marketing. For this reason, modeling creative industry objects are essential for resulting in various shapes and features of the goods. This paper presents to develop learning content in modeling creative goods supported...

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SHINY OFFICE-R: A Web-based Data Mining Tool for Exploring and Visualizing Company Profiles

I Made Tirta, Mohamad Fatekurahman, Khairul Anam, Bayu Taruna Widjaja Putra

The profile of institutions or companies are often measured internally, nationally and internationally using several indicators that may be changed over time. We develop SHINY OFFICE-R a Web-GUI (Graphical User Interface) using R software to explore and visualize data on institution performance/ profile....

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Dennis Alexander, Sarini Abdullah

Preeclampsia (PE) often described as new-onset hypertension and proteinuria during the third trimester of pregnancy. PE, is one of the most feared complications of pregnancy because it can progress rapidly to serious complications, including death of both mother and fetus. It is important to get a better...

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Dinagusti Sianturi, Alvida Rukmi

The COVID-19 pandemic has impact in every sector of life. Studies of the impact of the COVID-19 pandemic on stock trading are also being develop[redacted] Indonesia regarding to the number of industries affected by the pandemic. This research aims to provide information about the results of the correlation...

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Classification of Bank Deposit Using Naïve Bayes Classifier (NBC) and *K*–Nearest Neighbor (*K*-NN)

Muhammad Hafidh Effendy, Dian Anggraeni, Yuliani Setia Dewi, Alfian Futuhul Hadi

Banks are financial institutions whose activities are to collect funds from the public in the form of deposits (saving deposit, demand deposit, and time deposit) and distribute them to the public in the form of credit or other forms. Deposits are an alternative for customers because the interest offered...

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Case Study: Three Popular Online News Sites in Indonesia

Ferry Wiranto, I Made Tirta

This research is part of data mining, a sub-section of information retrieval and text mining. This research focuses on finding an approach to getting relevant documents online news documents with a specific threshold value and improving computing performance to get relevant documents with large documents....

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Statistical Downscaling Technique Using Response Based Unit Segmentation-Partial Least Square (REBUS-PLS) for Monthly Rainfall Forecasting

Izdihar Salsabila, Alfian Futuhul Hadi, I Made Tirta, Yuliani Setia Dewi, Firdaus Ubaidillah, Dian Anggraeni

One of the newest forecasting techniques today is the Statistical Downscaling (SDs) technique. The SDs technique is a procedure for inferring high-resolution information from low-resolution variables. Forecasting rainfall using the SDs technique is to build a function that can predict the value of a...

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Nur Kamilah Sa'diyah, Ani Budi Astuti, Maria Bernadetha T. Mitakda

Poisson regression is one of the model to explain the functional relationship between response variable in the form of count and predictor variable. An important assumption in Poisson Regression analysis is equidispersion. In certain cases, where response variable consists of too many zeros, causing...

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Multiple Discriminant Analysis Altman Z-Score, Multiple Discriminant Analysis Stepwise and K-Means Cluster for Classification of Financial Distress Status in Manufacturing Companies Listed on the Indonesia Stock Exchange in 2019

Hazrina Ishmah, Solimun, Maria Bernadetha Theresia Mitakda

This study uses the MDA (Multiple Discriminant Analysis) Altman Z-Score to predict the status of financial distress in manufacturing companies listed on the Indonesia Stock Exchange in 2019. MDA Stepwise model is used to prove that the variables used in the MDA Altman Z-Score method are the best variables...

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Naive Bayes Classifier (NBC) for Forecasting Rainfall in Banyuwangi District Using Projection Pursuit Regression (PPR) Method

Ana Ulul Azmi, Alfian Futuhul Hadi, Yuliani Setia Dewi, I Made Tirta, Firdaus Ubaidillah, Dian Anggraeni

Rainfall is one of the climates that has a big influence on life, such as aviation, plantations, and agriculture. Remote areas like Banyuwangi Regency are most likely to lack information on weather and climate data. Rainfall information in the future is also very decisive for the community in carrying...

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Analysis of Factors Affecting Poverty Depth Index in Papua Province Using Panel Data Regression

Rufina Indriani, Erma Oktania Permatasari

One of the main problems in Papua Province is poverty, because the Pove[redacted] Depth Index (P1) in Papua Province is greater than other province, which [redacted] 7.17 in 2019. This value is bigger than the Poverty Depth Index in Indonesia which was only 1.55. This study will analyse the factors that affect the...

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Chandrika Desyana Putri, Ema Fahma Farikha, Alfian Futuhul Hadi, Yuliani Setia Dewi, I Made Tirta, Firdaus Ubaidillah, Dian Anggraeni

Information about rainfall is very necessary for the country of Indonesia which bears the title of an agricultural country. This is because the agricultural sector is very vulnerable to climate change, where rainfall is one indicator of climate change-related to crops. Therefore, an accurate rainfall...

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Weather Forecasting at BMKG Office Lumajang City Using Markov Chain Method

Ummi Masrurotul Jannah, Mohamat Fatekurohman

Weather forecasting is one of the important factors in everyday life, because it can affect the activities carried out by the community. Weather forecasting refers to a series of activities carried out to produce a set of information about weather conditions. One method that can be used to model these...

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Anjeli Lutfiani, Bety Hayat Susanti

Modification Detection Code (MDC) as an unkeyed hash function is designed to provide data integrity. Manipulation Detection Codes (MDC-2) is one of double-length (2n-bit) hash-values that requires 2 block cipher operations per block of hash input where the output size of the hash function is twice the...

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Primary Key Encryption Using Hill Cipher Chain (Case Study: STIE Mandala PMB Site)

Muhamat Abdul Rohim, Kiswara Agung Santoso, Alfian Futuhul Hadi

The condition of the world experiencing the COVID-19 pandemic has resulted in some daily activities limited by health protocols. The Indonesian government’s policy in the academic field has forced STIE Mandala Jember, as one of the private universities, to implement online-based new student admissions....

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A Modification of ECDSA to Avoid the Rho Method Attack

Amira Zahra, Kiki Ariyanti Sugeng

Elliptic Curve Digital Signature Algorithm (ECDSA) is a digital signature algorithm that utilizes an elliptic curve. ECDSA consists of three steps, w[redacted] are key generation, signature generation, and verification algorithm. ECDSA is used on Bitcoin transactions to generate the user's public key, private...

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Image Authentication Based on Magic Square

Kiswara Agung Santoso, Maulidyah Lailatun Najah, Moh. Hasan

Image is a digital media that is easy to change, so it is susceptible to being used for crime. Image changes may be affected by the unstable internet during transmission or deliberate manipulation of images for specific purposes. Hence, we need a tool to determine the authenticity of the image. One strategy...

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Pattern Recognition of Batik Madura Using Backpropagation Algorithm

Abduh Riski, Ega Bandawa Winata, Ahmad Kamsyakawuni

Since October 2, 2009, UNESCO has acknowledged batik as one of Indonesia's intellectual properties. Throughout the archipelago, distinct and diverse batik motifs have emerged and been produced with the passage of time; Madura batik is one of them. The Backpropagation Algorithm is used to recognize Madura...

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Hybrid Cat-Particle Swarm Optimization Algorithm on Bounded Knapsack Problem with Multiple Constraints

Kiswara Agung Santoso, Muhammad Bagus Kurniawan, Ahmad Kamsyakawuni, Abduh Riski

Optimization problems have become interesting problems to discuss, including the knapsack problem. There are many types and variations of knapsack problems. In this paper, the authors introduce a new hybrid metaheuristic algorithm to solve the modified bounded knapsack problem with multiple constraints...

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$L(2,1)$ Labeling of Lollipop and Pendulum Graphs

Kusbudiono*, Irham Af'idatul Umam, Ikhsanul Halikin, Mohamat Fatekurohman

Graph, Combinatorics, and Algebra Research Group, Department of Mathematics, FMIPA, Universitas Jember
**Corresponding author. Email: kusbudiono@unej.ac.id*

ABSTRACT

One of the topics in graph labeling is $L(2,1)$ labeling which is an extension of graph labeling. Definition of $L(2,1)$ labeling is a function that maps the set of vertices in the graph to non-negative integers such that every two vertices u, v that have a distance one must have a label with a difference at least two. Furthermore, every two vertices u, v that have a distance two must have a label with a difference at least one. This study discusses the $L(2,1)$ labeling on a lollipop graph $L_{m,n}$ with $m \geq 3$ and n positive integers. The purpose of this study is to determine the minimum span value from the $L(2,1)$ labeling on the lollipop graph $L_{m,n}$ and we can symbolize $\lambda_{2,1}(L_{m,n})$ and to determine the minimum span value from the $L(2,1)$ labeling on the pendulum graph. In addition, it also builds a simulation program for $L(2,1)$ labeling lollipop graphs up to tremendous values of m and n. In this paper, we obtained that the minimum span of a lollipop graph is $\lambda_{2,1}(L_{m,n}) = 2m - 2$, and the minimum span of a pendulum graph, let P_n^k with $k \geq 4$ and $n \geq 5$, is $k + 1$.

Keywords: *$L(2,1)$ Labeling, Lollipop graph, Pendulum graph.*

1. INTRODUCTION

Graph theory is a branch of mathematics that has undergone many developments. There are many topics studied in graph theory, one of which is graph labeling. A graph G expressed in $(V(G), E(G))$ is a pair of two sets. $V(G) = \{v_1, v_2, \dots, v_n\}$ is a non-empty set of elements called vertices and $E(G) = \{e_1, e_2, \dots, e_n\}$ is a possibly empty set of an unordered pair $\{v_1, v_2\}$ of two vertices $v_1, v_2 \in V(G)$, called the edge set of G [1]. In general, graph labeling discusses labeling in integers at graph points, graph edges, or both. In its development, we added some rules to graph labeling related to distance. One graph labeling based on this is labeling $L(h,k)$ [2].

Then, we will focus on $L(2,1)$ labeling as a function f which maps the vertex set $V(G)$ to non-negative integers such that if $d(u,v) = 1$, then $|f(u) - f(v)| \geq 1$ and if $d(u,v) = 2$, then $|f(u) - f(v)| \geq 1$ where $d(u,w)$ is the distance between vertices u and v. A number k such that an $L(2,1)$ labelling exist is called as span of $L(2.1)$-labelling if there is no label greater than k. The span of a graph G can be more than one, the minimum value of the span of a graph G is notated by $\lambda_{(2,1)}(G)$.

Some researchers have applied $L(2,1)$ labeling to several graphs and have obtained the minimum span of the graph. Some of these graphs include star graph [2]; cycle, path, and complete graph [3]; $K_{1,n}$-free graphs [4]; supercycle graph [5]; lotus, fan, wheel, and $K_1 \odot (P_n \cap F_n)$ graphs [6]; and Sierpinski graph [7]. In this study, we discuss the $L(2,1)$ labeling of the lollipop graph $L_{m,n}$ With $m \geq 3$, n positive integers and $L(2,1)$ labeling of the pendulum graph.

This study aims to determine the minimum span value of the $L(2,1)$ labeling on the lollipop graph. Determine the minimum span value of the $L(2,1)$ labeling on the pendulum graph. Build a simulation program that can handle $L(2,1)$ labeling on a lollipop graph up to tremendous values of m and n.

We will use three theorems as the basis for finding the minimum span value of the $L(2,1)$ labeling as follows.

Lemma 1.1 [4] If H is a subgraf of G, then $\lambda_{2,1}(H) \leq \lambda_{2,1}(G)$.

Lemma 1.2 [4] For any n natural numbers, $\lambda_{2,1}(K_n) = 2n - 2$.

Lemma 1.3 [4] For any n natural numbers, $\lambda_{2,1}(P_n) = 4$, with $n \geq 5$.

Lemma 1.4 [2] Let $S_{1,n}$ be a star, then $\lambda_{2,1}(S_{1,n}) = n + 1$.

2. RESULT AND DISCUSSION

We can obtain a lollipop graph by combining the complete graph K_m and the path graph P_n with a bridge (edge) so that the lollipop graph notation is $L_{m,n}$ for any natural number m and n [8]. The notation of the vertex set and the edge set of the lollipop graph is as follows, $V = \{u_i \mid 1 \le i \le m\} \cup \{v_j \mid 1 \le j \le n\}$ and $E = \{u_i u_k \mid 1 \le i \le m - 1 i + 1 \le k \le m\} \cup \{u_1 v_1\} \cup \{v_j v_{(j+1)} \mid 1 \le j \le n - 1\}$. Figure 1 illustrates the notation of a lollipop graph.

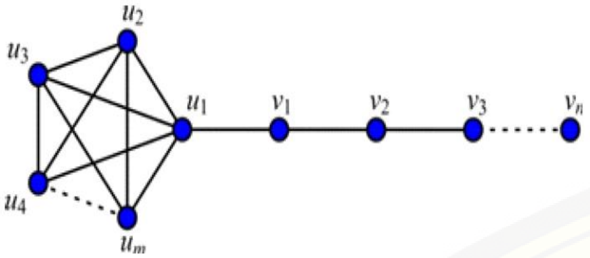

Figure 1 Lollipop graph notation $L_{m,n}$.

Theorem 2.1 For any m, n natural number, lollipop graph $L_{m,n}$ with $m \ge 3$ has the minimum span is on the $L(2,1)$ labeling is $2m - 2$.

Proof. To prove that the minimum span of a lollipop graph is $\lambda_{2,1}(L_{m,n}) = 2m - 2$, then we have to show that $\lambda_{2,1}(L_{m.n}) \ge 2m - 2$ and $\lambda_{2,1}(L_{m,n}) \le 2m - 2$.

Both a complete and path are subgraphs of a lollipop. Based on Lemma 1.2, a complete graph (K_m) has a minimum span value of $\lambda_{2,1}(K_m) = 2m - 2$, and according to Lemma 1.3, a path graph P_n has a minimum span of $\lambda_{2,1}(P_n) = 4$ for $n \ge 5$, otherwise $\lambda_{2,1}(P_n) < 4$. Thus $\lambda_{2,1}(K_m) \ge \lambda_{2,1}(P_n)$. Furthermore, based on the fact from Lemma 1.1 that $\lambda_{2,1}(L_{(m,n)}) \ge \lambda_{2,1}(K_m) = 2m - 2$ causes $\lambda_{2,1}(L_{(m,n)}) \ge 2m - 2$. To show that $\lambda_{2,1}(L_{(m,n)}) \le 2m - 2$, it is enough to do the labeling by,

$$f(u_i) = 2m - 2i; 1 \le i \le m$$

$$f(v_j) = \begin{cases} 1; j = 1 \\ 3; j = 2 \\ 2a; 3 \le j \le n; a \text{ is the remainder} \\ \quad \text{of } j - 3 \text{ divided by } m \end{cases}$$

In the lollipop graph $L_{m,n}$, the vertices formed from the complete graph $(u_i; 1 \le i \le m))$ have a distance of one. Then the vertices u_1 and v_1 also have a distance of one, and the last one where the distance between the two vertices is one, namely the vertices v_j and v_{j+1} with $1 \le j \le n - 1$. Next, the two vertices in the lollipop graph that have a distance of two are vertex u_i, $i \ne 1$ with v_1, u_1 with v_2, and v_j with v_{j+2}, $1 \le j \le n - 2$. From the labeling function f above, the labels given have met the labeling requirements of $L(2,1)$. Furthermore, based on the mapping rule of the function, the largest labeling is $2m - 2$. Thus, we can prove that $\lambda_{2,1}(L_{(m,n)}) \le 2m - 2$. So based on the above, the author concludes that the minimum span value for $L(2,1)$ labeling of Lollipop graph $L_{m,n}$ is $2m - 2$. ■

In this study, the author makes a labeling simulation program $L(2,1)$ on a lollipop graph $L_{m,n}$. We can see the display of the program in Figure 2.

The following are some of the results of labeling carried out using a simulation program.

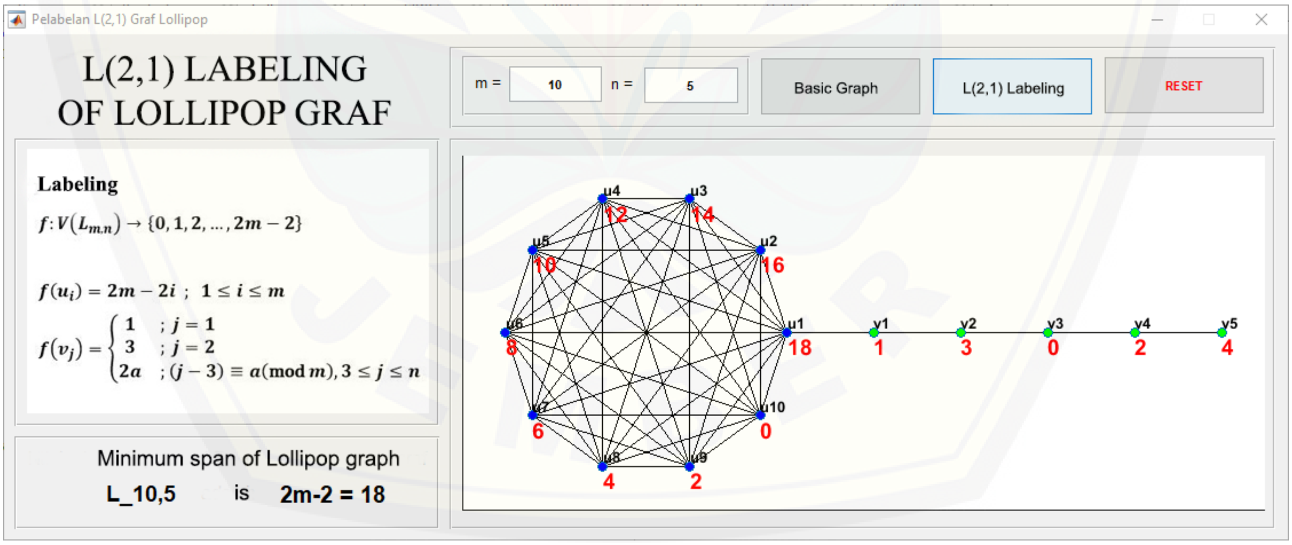

Figure 2 Simulation Program Display.

1. $L(2,1)$ Labeling of lollipop graph $L_{3,5}$

In Figure 3, the $L(2,1)$ labeling of the lollipop graph $L_{3,5}$ has the largest label value of four and the smallest label value of zero, so the labeling has a span of four. The span value follows Theorem 2.1, which is $2m - 2 = 2(3) - 2 = 4$.

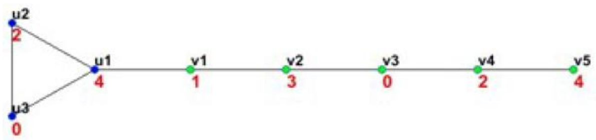

Figure 3 $L(2,1)$ labeling on the lollipop graph $L_{3,5}$.

2. $L(2,1)$ Labeling of lollipop graph $L_{4,8}$

In Figure 4, the $L(2,1)$ labeling of the lollipop graph $L_{4,8}$ has the largest label value of four and the smallest label value of zero, so the labeling has a span of four. The span value follows Theorem 2.1, which is $2m - 2 = 2(4) - 2 = 6$.

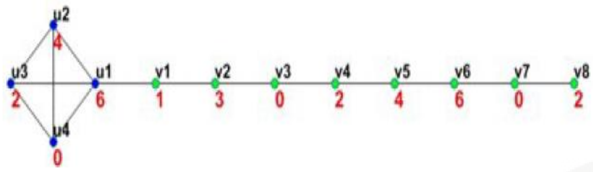

Figure 4 Labeling $L(2,1)$ of lollipop graph $L_{4,8}$.

3. $L(2,1)$ Labeling on the lollipop graph $L_{10,10}$

In Figure 5, the $L(2,1)$ labeling of the lollipop graph $L_{10,10}$ has the largest label value of four and the smallest label value of zero, so the labeling has a span of four. The span value follows Theorem 2.1, which is $2m - 2 = 2(10) - 2 = 18$.

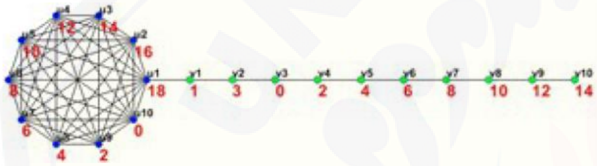

Figure 5 $L(2,1)$ Labeling on the lollipop graph $L_{10,10}$.

4. $L(2,1)$ Labeling on the lollipop graph $L_{20,10}$

In Figure 6, the $L(2,1)$ labeling of the lollipop graph $L_{20,10}$ has the largest label value of four and the smallest label value of zero, so the labeling has a span of four. The span value follows Theorem 2.1, which is $2m - 2 = 2(20) - 2 = 38$

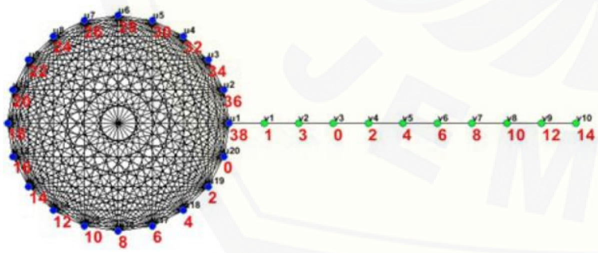

Figure 6 $L(2,1)$ Labeling on the lollipop graph $L_{20,10}$.

We defined pendulum graph P_n^k as a graph obtained by attaching a cycle graph C_n to every leaf of star graph S_n. Consider Figure 7 which illustrates graph P_n^k. Suppose that we notated vertices and edges of pendulum graph P_n^k as follows:

$V(P_n^k) = \{u_0\} \cup \{v_j^i;\ i \in [1,k] \text{ and } j \in [1,n]\}$

$E(P_n^k) = \{v_0 v_1^i, v_1^i v_n^i, v_j^i v_{j+1}^i;\ i \in [1,k] \text{ and } j \in [1,n]\}$

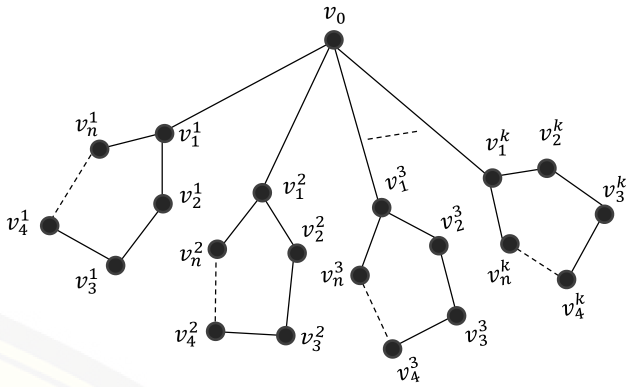

Figure 7 Pendulum graph P_n^k.

Theorem 2.2 Let P_n^k be a pendulum graph with $k \geq 4$ and $n \geq 5$, then $\lambda_{2,1}(P_n^k) = k + 1$.

Proof. Let P_n^k be a pendulum graph with $k \geq 4$ and $n \geq 5$. Since a star $S_{1,k}$ is a subgraph of pendulum graph P_n^k, then based on the Lemma 1.4, we get $\lambda_{2,1}(P_n^k) \geq \lambda_{2,1}(S_{1,k}) = k + 1$. Next, we will show that $\lambda_{2,1}(P_n^k) \leq k + 1$ by constructing the $L(2,1)$ labelling of pendulum graph P_n^k. For this labelling, we will consider three cases.

- Case 1: $n \equiv 0 \bmod 3$

$f(v_0) = 0$

$$f(v_j^1) = \begin{cases} 2; j = 1 \\ 4; j = n \\ 1; j = n-1 \\ 5; j \equiv 2 \bmod 3,\ j \neq n-1 \\ 0; j \equiv 0 \bmod 3,\ j \neq n \\ 3; j \equiv 1 \bmod 3,\ j \neq 1 \end{cases}$$

$$f(v_j^2) = \begin{cases} 3; j \equiv 1 \bmod 3 \\ 1; j \equiv 2 \bmod 3 \\ 5; j \equiv 0 \bmod 3 \end{cases}$$

$$f(v_j^3) = \begin{cases} 4; j = 1 \\ 1; j = n \\ 2; j = 2 \\ 5; j \equiv 0 \bmod 3, & j \neq n \\ 0; j \equiv 1 \bmod 3, & j \neq 1 \\ 3; j \equiv 2 \bmod 3, & j \neq 2 \end{cases}$$

$$f(v_j^i) = \begin{cases} i+1; j = 1 \bmod 3, i \geq 4 \\ 3; j \equiv 2 \bmod 3 \\ 1; j \equiv 0 \bmod 3 \end{cases}$$

- Case 2: $n \equiv 1 \bmod 3$

$f(v_0) = 0$

$$(v_j^1) = \begin{cases} 2; j \equiv 1 \, mod \, 3, \; j \neq n \\ 4; j \equiv 2 \, mod \, 3 \\ 0; j \equiv 0 \, mod \, 3 \\ 5; j = n \end{cases}$$

$$(v_j^2) = \begin{cases} 3; j = 1 \\ 5; j = 2, n-1 \\ 1; j = n \\ 2; j \equiv 0 \, mod \, 3, \; j \neq n-1 \\ 4; j \equiv 1 \, mod \, 3, j \neq n, j \neq 1 \\ 0; j \equiv 2 \, mod \, 3, j \neq 2 \end{cases}$$

$$(v_j^3) = \begin{cases} 5; j \equiv 0 \, mod \, 3, j \neq n \\ 3; j \equiv 1 \, mod \, 3, j \neq 1 \\ 0; j \equiv 2 \, mod \, 3, j \neq 2 \\ 4; j = 1 \\ 2; j = 2 \\ 1; j = n \end{cases}$$

$$f(v_j^i) = \begin{cases} i+1; j \equiv 1 \, mod \, 3, j \neq n, 4 \leq i \leq k \\ 2; j \equiv 2 \, mod \, 3 \\ 0; j \equiv 0 \, mod \, 3 \\ 3; j = n \end{cases}$$

- Case 3: $n \equiv 2 \, mod \, 3$

$f(v_0) = 0$

$$(v_j^1) = \begin{cases} 2; j \equiv 1 \, mod \, 3, j \neq n-1 \\ 4; j \equiv 2 \, mod \, 3, j \neq n \\ 0; j \equiv 0 \, mod \, 3 \\ 3; j = n-1 \\ 5; j = n \end{cases}$$

$$(v_j^2) = \begin{cases} 3; j = 1 \\ 1; j = n \\ 5; j = 2 \\ 2; j \equiv 0 \, mod \, 3, j \neq n, j \neq 2 \\ 4; j \equiv 1 \, mod \, 3, j \neq 1 \\ 0; j \equiv 2 \, mod \, 3 \end{cases}$$

$$(v_j^3) = \begin{cases} 4; j \equiv 1 \, mod \, 3, j \neq n-1 \\ 2; j \equiv 2 \, mod \, 3, j \neq n \\ 0; j \equiv 0 \, mod \, 3 \\ 3; j = n-1 \\ 1; j = n \end{cases}$$

$$(v_j^i) = \begin{cases} i+1; j \equiv 1 \, mod \, 3, j \neq n-1, 4 \leq i \leq k \\ 2; j \equiv 2 \, mod \, 3, j \neq n \\ 0; j \equiv 0 \, mod \, 3 \\ 3; j = n-1 \\ 1; j = n \end{cases}$$

In the same way as Theorem 4, it is easy to prove that every two vertices with distance one receive labels that differ by at least two, and every two vertices at a distance two receive labels that differ by at least one.■

3. CONCLUSION

Based on the results and discussion described in section 2. We conclude that the minimum span value of the $L(2,1)$ labeling) of the lollipop graph $L_{m,n}$ is $2m-2$ with $m \geq 3$ and n positive integers, and the minimum span value of the $L(2,1)$ labeling of the pendulum graph P_n^k is $k+1$ with $k \geq 4$ and $n \geq 5$. The minimum span value for a lollipop graph is the same as the minimum span value for a complete graph.

Further research suggests $L(2,1)$ labeling vertices and edges in several graphs or other graph operations

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