International Journal of Health, Education and Social (IJHES)

E-ISSN : 2450-5173
P-ISSN : 2455-1266
www.ijhes.com

The International Journal of Health, Education and Social (IJHES

Register/Submission Login

International Journal of Health, Education and Social (IJHES)

E-ISSN: 2410-5171
P-ISSN: 2415-1246

About | About the Journal | Current | Archives | Editorial Team | Why Us | For Author | Contact

Search

Home / Editorial Team

Director

Dr. Kevin Abraham

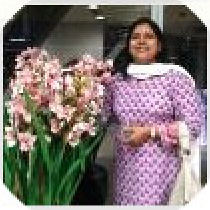
Multidisciplinary Research Center Institution (MRCI)

Chief Editor

Dr. Sahlan

Editor International Journal of Health, Education and Social (Ijhes)

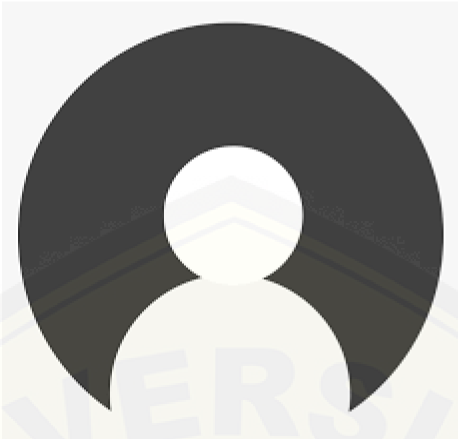
Editorial Advisors / Reviewer Panel

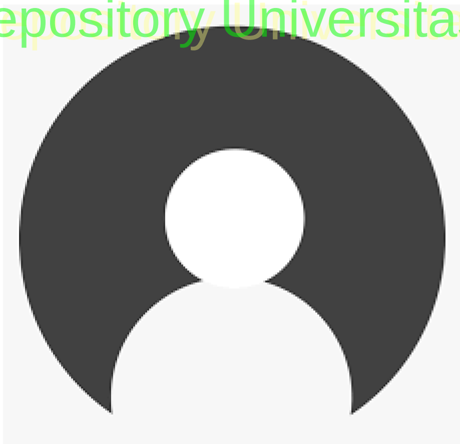


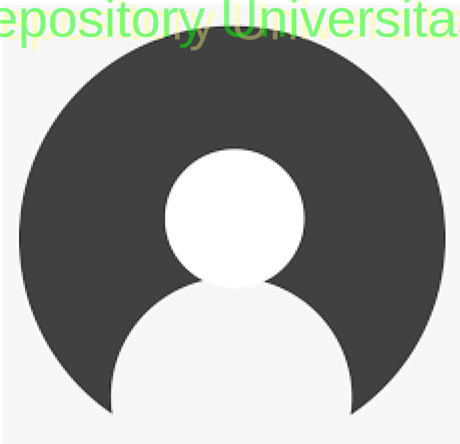
Kavita

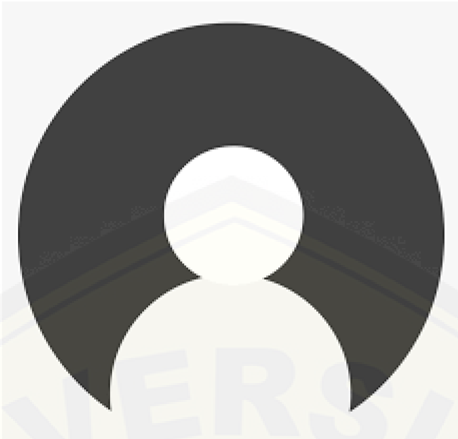
DESIGNATION: Assistant Professor

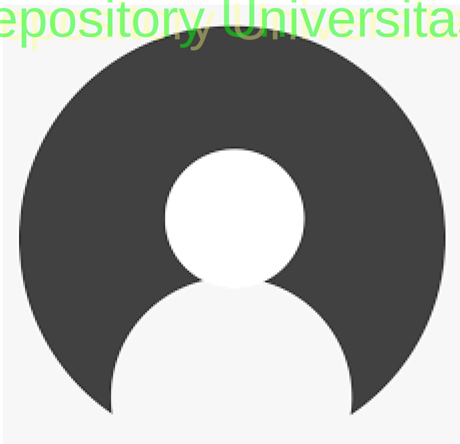
HIGHEST QUALIFICATION: Ph.D











| Google | | Scopus |

Dr. Ikhsan bin Othman

Senior Lecturer

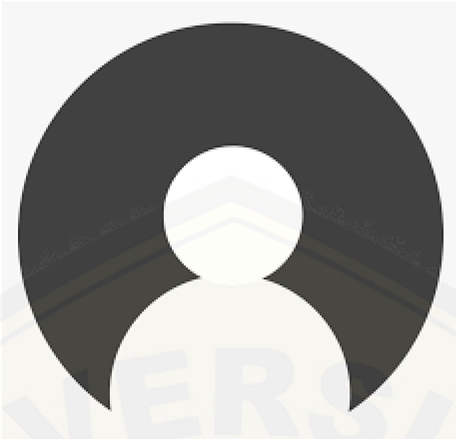
Malaysia

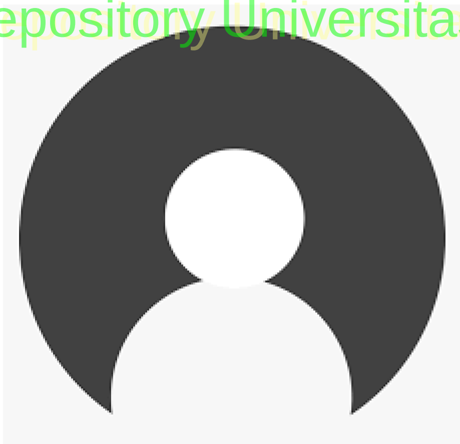
| Google | | Scopus |

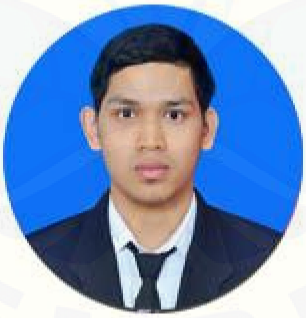
Dr. Ibrahim Suliman Ahmed Mukhtar

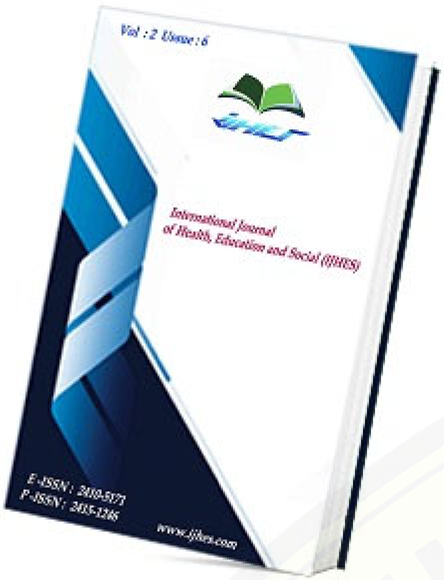
Islamic University Malaysia

Malaysia









This journal publishes research papers in the fields of health, education, management, social and religion, technology, culture, peace and conflict, library and information science, public administration, psychology, philosophy, sociology, women's studies, religious studies, social welfare, anthropology, linguistics, education, economics, international relations, law, development studies, population studies, political science, management, marketing, finance, banking, accounting, human resource management, international business, hotels and tourism, entrepreneurship development, business ethics

DOI: https://doi.org/10.1234/ijhes.v2i11

Published: 2019-11-30

Articles

On the Metric-Location-Domination Number of Some Exponential Graphs

A. Fauzul Fikri, Slamin, Dafik, Kusbudiono

1-4

Download This File

Computer Network And Its Consequences –A Literature Survey

Umang Rastogi

5-19

File PDF

Download This File

[Vol : 2 Issue : 11| November, 2019) E-ISSN : 2410-5171 | P-ISSN 2415-1246

The International Journal of Health, Education and Social (IJHES)

On the Metric-Location-Domination Number of Some Exponential Graphs

Fauzul Fikri[1] , Slamin[2] , Dafik[3] , Kusbudiono[4]

[1]Mathematics Depart. University of Jember, Indonesia, ahmadfauzulfikri@gmail.com

[2] Information System Depart. University of Jember, Indonesia, slamin@unej.ac.id

[3]Mathematics Edu. Depart. University of Jember, Indonesia, d.dafik@unej.ac.id

[4]Mathematics Depart. University of Jember, Indonesia, kusbudiono@unej.ac.id

***Abstract*-If u and v are vertices in a simple and undirected graph. A distance $d(u,v)$ is the shortest path between u and v in G. For an ordered set $W = \{w_1, w_2, ..., w_k\}$ of vertices and a vertex $v \in G$, the representation of v with respect to W is the ordered k-tuple $r(v|W) = \{d(v,w_1), d(v,w_2), ..., d(v,w_k)\}$. A dominating set W in a connected graph G is a metric-locating-dominating set (MLD), if $r(v|W)$ for $v \in V(G)$ are distinct. The metric-location-domination number of G denotes $\gamma_M(G)$ is the minimum cardinality of an MLD-set in G. The purpose of this paper is to find the metric-location-domination number of some exponential graphs. By an exponential graph, we mean a graph formed by combining two graphs G and H, where each edge of graph G is replaced by the graph H, denote by G^H. We have determined the metric-location-domination number of some exponential graph, namely $P_n{}^{Bt_m}$ and $P_n{}^{W_m}$.**

2010 Mathematics Subject Classification: *05C78*

Index Terms: *metric-locating-dominating, metric-location-domination number, exponential graph.*

I. Introduction

Graph theory is one part of mathematics were very useful to help solve a problem in life. If a problem in life represented in graph it will be easier understandable and simpler, so it is easier to solve. For example, shortest path in transport problems that can represented in the form of graphs (Chartrand dan Lesniak [3]).

A graph G is a finite nonempty set of $V(G) = \{v_1, v_2, \ldots, v_n\}$ called vertex and $E(G) = \{e_1, e_2, \ldots, e_n\}$ is every pair sets of $V(G)$ called edge. Member of the vertices $V(G)$ (cardinality), denoted by $|V(G)|$, called the order of the graph G. A graph G called a simple graph if each side of graph G connecting two different vertices and each of the two vertices different in the graph G is only connected by one edge. A Graph G connected if any two vertices u and v in the graph G there is always a path that sequence of vertex and edge which connects u to v.

The distance between two vertices u and v $d_G(u,v)$ in a connected graph G is the length of the shortest path from u to v in G.Representations of v to W in G is a vector with k tuple $r(v|W) = (d(v,w_1), d(v,w_2), ..., d(v,w_k))$ to its components is the distance of v to all vertices in W. The set W is called a locating set if $r(u|W) = r(v|W)$ then $u = v$ for $u, v \in G$. Furthermore, a set $W \subseteq V(G)$ is a dominating set of G if every vertex in $V(G) - W$ is adjacent to a vertex of W. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G. A dominating set of cardinality $\gamma(G)$ is called a $\gamma(G)$-set. The literature on this subject has been surveyed and detailed in the two books by Haynes et,al [8] and [9].

The concepts of a locating set and a dominating set merged by defining the metric-locating-dominating set, denoted by an MLD- set, in a connected graph G to be a set of vertices of G that is both a dominating set and a locating set in G (Henning and Oellermann [7]). Henning and Oellermann [7] define the metric-location-domination number $\gamma_M(G)$ of G to be the minimum cardinality of an MLD-set in G. An MLD-set in G of cardinality $\gamma_M(G)$ is called a $\gamma_M(G)$-set.

The study and development of the concept MLD-set continued among others by Iswadi cite Isw11a, cite Isw11b, C 'a ceres et al cite Cac13, and Palupi cite Pal13 concept MLD-set still studied and developed. Such research become a reference to search for

[Vol : 2 Issue : 11| November, 2019) E-ISSN : 2410-5171 | P-ISSN 2415-1246

The International Journal of Health, Education and Social (IJHES)

metric location domination number of other graph. By an exponential graph, we mean a graph formed by combining two graphs G and H, where each edge of graph G is replaced by the graph H, denote by G^H. a $P_n{}^{Bt_m}$ graph and $P_n{}^{W_m}$ graph is the one that has not been studied its metric location domination number.

II. The Result

Suppose G is a graph with vertices v and edge e, where each edge e of a graph G is an exponential graph of another graph, for example a graph H. By an exponential graph, we mean a graph formed by combining two graphs G and H, where each edge of graph G is replaced by the graph H, denote by G^H.

A $P_n{}^{Bt_m}$ graph where $n \geq 2, m \geq 2$ is an exponential graph with the vertices and the edge respectively,

$$\begin{aligned} V &= \{x_i|1 \leq i \leq n\} \bigcup \{x_{i,j}|1 \leq i \leq n-1, \\ &\quad 1 \leq j \leq m\} \\ E &= \{x_ix_{i+1}|1 \leq i \leq n-1\} \bigcup \{x_ix_{i,j}| \\ &\quad 1 \leq i \leq n-1, 1 \leq j \leq m\} \bigcup \\ &\quad \{x_{i+1}x_{i,j}|1 \leq i \leq n-1, 1 \leq j \leq m\}, \end{aligned}$$

and have $|V| = mn - m + n$ and $|E| = 2mn - 2m - 1$.

Teorema 1. *If $P_n{}^{Bt_m}$ is the exponential graph where $n \geq 2, m \geq 2$ then,*

1) $\gamma_M(P_n{}^{Bt_m}) = mn - m, n = 2,3$ *and* $m \geq 2$
2) $\gamma_M(P_n{}^{Bt_m}) = mn - m - n + 1 + \lfloor \frac{n}{3} \rfloor, n \geq 4$ *and* $m \geq 2$.

Proof. Metric location domination number of $P_n{}^{Bt_m}$ graph consists of two cases, that is $n = 2,3$ and another n $(n \geq 4)$.

Case 1. Let $n = 2,3$ dan $m \geq 2$. If we choose $W = \{x_{i+1}|1 \leq i \leq n-1\} \bigcup \{x_{i,j}|1 \leq i \leq n-1, 1 \leq j \leq m-1\}$. Then W is dominator because of every vertex $V(P_n{}^{Bt_m}) - W$ adjacent with a vertex from W. Obtained representation of any vertices of the $P_n{}^{Bt_m}$ graph with respect to W are distinct, namely,

$$\begin{aligned} r(x_i|W) &= \{(a_{i,k}), 1 \leq i \leq n, 1 \leq k \leq \\ &\quad mn - m\} \\ r(x_{i,j}|W) &= \{(a_{i,j,k}), 1 \leq i \leq n, 1 \leq j \leq m, \\ &\quad 1 \leq k \leq mn - m\} \end{aligned}$$

where (i) $(a_{i,k}) = |k + 1 - i|$ for $1 \leq i \leq n, 1 \leq k \leq n-1$; (ii) $(a_{i,k}) = \left\lceil \frac{k+1-n}{m-1} \right\rceil - i + 1$ for $1 \leq i \leq \left\lceil \frac{k+1-n}{m-1} \right\rceil, n-1 < k \leq mn - m$; (iii) $(a_{i,k}) = i - \left\lceil \frac{k+1-n}{m-1} \right\rceil$ for $\left\lceil \frac{k+1-n}{m-1} \right\rceil > i \geq n, n-1 < k \leq mn - m$. Furthermore, (i) $(a_{i,j,k}) = k + 1 - i$ for $1 \leq i < k+1, 1 \leq j \leq m, 1 \leq k \leq (n-1)$; (ii) $(a_{i,j,k}) = i - k$ for $k+1 \leq i \leq n, 1 \leq j \leq m, 1 \leq k \leq (n-1)$; (iii) $(a_{i,j,k}) = 0$ for $i = \left\lceil \frac{k+1-n}{m-1} \right\rceil, j \cong (k+1-n) \bmod (m-1), n-1 < k \leq mn - m$; (iv) $(a_{i,j,k}) = 2$ for $i, j,$ and k otherwise

Consequently W is MLD-set with $nm - m$ elements. In the case $n = 2,3$ not found a MLD-set with elements less than $mn - m$. Thus $\gamma_M(P_n{}^{Bt_m}) = mn - m$, for $n = 2,3$, and $m \geq 2$.

Case 2. Suppose n otherwise $(n \geq 4)$ and $m \geq 2$. For $n \cong 5 (\text{mod } 3), n \cong 6 (\text{mod } 3)$, if we choose $W = \{x_{3i-1}|1 \leq i \leq \lceil \frac{n}{3} \rceil\} \bigcup \{x_{i,j}|1 \leq i \leq n-1, 1 \leq j \leq m-1\}$. Then W is dominator because of every vertex $V(P_n{}^{Bt_m}) - W$ adjacent with a vertex from W. Obtained representation of any vertices of the $P_n{}^{Bt_m}$ graph with respect to W are distinct, namely

$$\begin{aligned} r(x_i|W) &= \{(a_{i,k}), 1 \leq i \leq n, 1 \leq k \leq \\ &\quad mn - m - n + 1 + \lfloor \frac{n}{2} \rfloor\} \\ r(x_{i,j}|W) &= \{(a_{i,j,k}), 1 \leq i \leq n, 1 \leq j \leq m, \\ &\quad 1 \leq k \leq mn - m - n + 1 + \lfloor \frac{n}{2} \rfloor\} \end{aligned}$$

Where (i) $(a_{i,k}) = |3k - 1 - i|$ for $1 \leq i \leq n, 1 \leq k \leq \lceil \frac{n}{3} \rceil$; (ii) $(a_{i,k}) = \left\lceil \frac{k - \lceil \frac{n}{3} \rceil}{m-1} \right\rceil - i + 1$ for $1 \leq i \leq \left\lceil \frac{k - \lceil \frac{n}{3} \rceil}{m-1} \right\rceil, k > \lceil \frac{n}{3} \rceil$; (iii) $(a_{i,k}) = i - \left\lceil \frac{k - \lceil \frac{n}{3} \rceil}{m-1} \right\rceil$, for $\left\lceil \frac{k - \lceil \frac{n}{3} \rceil}{m-1} \right\rceil > i \geq n, k > \lceil \frac{n}{3} \rceil$. Furthermore (i) $(a_{i,j,k}) = 3k - 1 - i$ for $1 \leq i < 3k, 1 \leq j \leq m, 1 \leq k \leq \lceil \frac{n}{3} \rceil$; (ii) $(a_{i,j,k}) = i - 3k + 2$ for $3k \leq i \leq n, 1 \leq j \leq m, 1 \leq k \leq \lceil \frac{n}{3} \rceil$; (iii) $(a_{i,j,k}) = i = \left\lceil \frac{k - \lceil \frac{n}{3} \rceil}{m-1} \right\rceil, j \cong (k - \lceil \frac{n}{3} \rceil)(\bmod (m-1)), k > \lceil \frac{n}{3} \rceil$; (iv) $(a_{i,j,k}) = 2$ for $i = \left\lceil \frac{k - \lceil \frac{n}{3} \rceil}{m-1} \right\rceil, j \ncong (k - \lceil \frac{n}{3} \rceil)(\bmod (m-1)), k > \lceil \frac{n}{3} \rceil$; (v)$(a_{i,j,k}) = \left| i - \left\lceil \frac{k - \lceil \frac{n}{3} \rceil}{m-1} \right\rceil \right|$ for $\left| i - \left\lceil \frac{k - \lceil \frac{n}{3} \rceil}{m-1} \right\rceil \right|, i \neq \left\lceil \frac{k - \lceil \frac{n}{3} \rceil}{m-1} \right\rceil, 1 \leq j \leq m, k > \lceil \frac{n}{3} \rceil$.

Representation of $V(P_n{}^{Bt_m})$ with respect to W are distinct. So we have W is MLD-set with $mn - n - m + 1 + \lfloor \frac{n}{2} \rfloor$ elements. in the case $n \geq$ not found a MLD-set with less elements. Thus $\gamma_M(P_n{}^{Bt_m}) = m - n - m + 1 + \lfloor \frac{n}{2} \rfloor$, for $n \geq 4$, and $m \geq 2$

[Vol : 2 Issue : 11| November, 2019) E-ISSN : 2410-5171 | P-ISSN 2415-1246
The International Journal of Health, Education and Social
(IJHES)

From **Case 1.** and **Case 2.** proved that $\gamma_M({P_n}^{Bt_m}) = mn - m$, for $n = 2, 3$, and $m \geq 2$, otherwise for $n \geq 4, m \geq 2$ we have $\gamma_M({P_n}^{Bt_m}) = mn - m - n + 1 + \lfloor \frac{n}{3} \rfloor$. □

Furthermore, a ${P_n}^{W_m}$ graph where $n \geq 2, m \geq 3$ is an exponential graph with the vertices and the edge respectively,

$$\begin{aligned} V &= \{x_{i,j} | 1 \leq i \leq n-1, 1 \leq j \leq m-1\} \bigcup \\ &\quad \{x_{n,m}\} \bigcup \{y_i | 1 \leq i \leq n-1\} \\ E &= \{x_{i,1}x_{i+1,1} | 1 \leq i \leq n-2\} \bigcup \\ &\quad \{x_{n-1,1}x_{n,m}\} \bigcup \\ &\quad \{x_{i,j}x_{i,j+1} | 1 \leq i \leq n-1, 1 \leq j \leq m-2\} \bigcup \\ &\quad \{x_{i+1,1}x_{i,m-1} | 1 \leq i \leq n-2\} \bigcup \\ &\quad \{x_{n,m}x_{n-1,m-1}\} \bigcup \\ &\quad \{y_i x_{i,j} | 1 \leq i \leq n-1, 1 \leq j \leq m-1\} \bigcup \\ &\quad \{y_i x_{i+1,1} | 1 \leq i \leq n-2\} \end{aligned}$$

and have $|V| = mn - m + 1$ and $|E| = 2mn - 2m$.

Teorema 2. *If ${P_n}^{W_m}$ is the exponential graph where $n \geq 2, m \geq 3$ then,*

1) $\gamma_M({P_n}^{W_m}) = n - 1$ *for $n \geq 2$ and $m = 3, 4$;*
2) $\gamma_M({P_n}^{W_m}) = n - 1 + 2\lfloor \frac{m-3}{2} \rfloor$ *for $n = 2$ and $m \geq 5$;*
3) $\gamma_M({P_n}^{W_m}) = n - 1 + 2\lfloor \frac{m-3}{2} \rfloor + (n-3)\lfloor \frac{m-4}{2} \rfloor$ *for $n \geq 3$ and $m \geq 5$.*

Proof. Metric location domination number of ${P_n}^{W_m}$ graph consists of three cases,

Case 1. For $n \geq 2$ and $m = 3, 4$. If we choose $W = \{y_i | 1 \leq i \leq n-1\}$, then W is dominator because of every vertex $V({P_n}^{W_m}) - W$ adjacent with a vertex from W. Obtained representation of any vertices of the ${P_n}^{W_m}$ graph with respect to W are distinct. Consequently W is MLD-set with $n-1$ elements. In the case $n \geq 2$ and $m = 3, 4$ not found a MLD-set with less elements. Thus $\gamma_M({P_n}^{W_m}) = n - 1$.

Case 2. For $n = 2$ and $m \geq 5$. If we choose $W = \{y_i | 1 \leq i \leq n-1\} \bigcup \{x_{1,2j} | 1 \leq j \leq \lfloor \frac{m-3}{2} \rfloor\} \bigcup \{x_{n-1,2j+1} | 1 \leq j \leq \lfloor \frac{m-3}{2} \rfloor\}$, then W is dominator because of every vertex $V({P_n}^{W_m}) - W$ adjacent with a vertex from W. Obtained representation of any vertices of the ${P_n}^{W_m}$ graph with respect to W are distinct. Consequently W is MLD-set with $n - 1 + 2\lfloor \frac{m-3}{2} \rfloor$ elements. In the case $n = 2$ and $m \geq 5$ not found a MLD-set with less elements. Thus $\gamma_M({P_n}^{W_m}) = n - 1 + 2\lfloor \frac{m-3}{2} \rfloor$.

Case 3. For $n \geq 3$ and $m \geq 5$. If we choose $W = \{y_i | 1 \leq i \leq n-1\} \bigcup \{x_{1,2j} | 1 \leq j \leq \lfloor \frac{m-3}{2} \rfloor\} \bigcup \{x_{i,2j+1} | 2 \leq i \leq n-2, 1 \leq j \leq \lfloor \frac{m-4}{2} \rfloor\} \bigcup \{x_{n-1,2j+1} | 1 \leq j \leq \lfloor \frac{m-3}{2} \rfloor\}$, then W is dominator because of every vertex $V({P_n}^{W_m}) - W$ adjacent with a vertex from W. Obtained representation of any vertices of the ${P_n}^{W_m}$ graph with respect to W are distinct. Consequently W is MLD-set with $n - 1 + 2\lfloor \frac{m-3}{2} \rfloor + (n-3)\lfloor \frac{m-4}{2} \rfloor$ elements. In the case $n \geq 3$ dan $m \geq 5$ not found a MLD-set with less elements. Thus $\gamma_M({P_n}^{W_m}) = n - 1 + 2\lfloor \frac{m-3}{2} \rfloor + (n-3)\lfloor \frac{m-4}{2} \rfloor$. From **Case 1.**, **Case 2.**, and **Case 3.** proved that for $n \geq 2, m = 3, 4$ then $\gamma_M({P_n}^{W_m}) = n - 1$. For $n = 2, m \geq 5$ then $\gamma_M({P_n}^{W_m}) = n - 1 + 2\lfloor \frac{m-3}{2} \rfloor$. For $n \geq 3, m \geq 5$ then $\gamma_M({P_n}^{W_m}) = n - 1 + 2\lfloor \frac{m-3}{2} \rfloor + (n-3)\lfloor \frac{m-4}{2} \rfloor$. □

III. Conclusion

We have shown the existence of Metric-Location-Domination Number of some exponential graph, namely ${P_n}^{Bt_m}$, ${P_n}^{W_m}$ and ${C_n}^{Bt_m}$ graphs. We can prove that (i) $\gamma_M({P_n}^{Bt_m}) = mn - m$, for $n = 2, 3; m \geq 2$ and $\gamma_M({P_n}^{Bt_m}) = mn - m - n + 1 + \lfloor \frac{n}{3} \rfloor$, for $n \geq 4, m \geq 2$. Thus, we propose the following open problems. (ii) $\gamma_M({P_n}^{W_m}) = n - 1$ for $n \geq 2$ and $m = 3, 4$; $\gamma_M({P_n}^{W_m}) = n - 1 + 2\lfloor \frac{m-3}{2} \rfloor$ for $n = 2$ and $m \geq 5$; and $\gamma_M({P_n}^{W_m}) = n - 1 + 2\lfloor \frac{m-3}{2} \rfloor + (n-3)\lfloor \frac{m-4}{2} \rfloor$ for $n \geq 3$ and $m \geq 5$.

Open Problem 1. *Find the metric-location-domination number in the other graph.*

Open Problem 2. *Let P_n be a line graph and H be any graph. How many $\gamma_M({P_n}^{H})$.*

References

[1] Bondy, J. A. and U. S. R. Murty. 1976. *Graph Theory with Applications*. London: The MacMillan Press Ltd.
[2] Cáceres, J., C. Hernando, M. Mora, I.M. Pelayo, M.L. Puertas. 2013. *Locating-Dominating Codes: Bounds and Extremal Cardinalities*. Appl. Math. Comput. 220:38-45.
[3] Chartrand, G. and L. Lesniak. 1996. *Graphs and Digraphs*, 3rd ed. California: Chapman and Hall.

[Vol : 2 Issue : 11| November, 2019) E-ISSN : 2410-5171 | P-ISSN 2415-1246

The International Journal of Health, Education and Social
(IJHES)

[4] Chartrand, G. and O. R. Oellermann. 1993. *Applied and Algorithmic Graph Theory, International Series in Pure and Applied Mathematics*. California: McGraw- Hill Inc.

[5] Chartrand, G., L. Eroh, M. A. Johnson, and O. R. Oellermann. 2000. *Resolvabi- lity in Graphs and the Metric Dimension of a Graph*. Discrete Appl. Math. 105, 99-113.

[6] Harary, F., dan R. A. Melter. 1976. *On the Metric Dimension of a Graph*. Ars Combin. 2, 191-195.

[7] Henning, M.A., and O.R. Oellermann. 2004. *Meteric-Locating-Dominating Sets in Graphs*. Ars Combinatorica. 73, 129-141.

[8] Haynes, T.W., Hedetniemi, S.T., and Slater, P.J. 1998a. *Funda- mentals of Domination in Graphs*. New York: Marcel Dekker.

[9] Haynes, T.W., Hedetniemi, S.T., and Slater, P.J. 1998b. *Dom- ination in Graphs: Advanced Topics*. New York: Marcel Dekker.

[10] Iswadi, H. 2011a. *Batas Atas Bilangan Dominasi Lokasi Metrik dari Graf Hasil Operasi Korona*. Prosiding Seminar Matematika dan Pendidikan Matema- tika Universitas Jember, 1 (1).

[11] Iswadi, H. 2011b. *Bilangan Dominasi Lokasi Metrik dari Graf Hasil Operasi Korona*. Prosiding Seminar Nasional Matematika Unand 2011. pp. 22-29. ISSN 978-602-19249-0-7

[12] Palupi, R., L. Susilowati, N. Estuningsih and H. Iswadi. 2013. *Bilangan Dominasi Lokasi Metrik pada Graf Kisi*. Jurnal Matematika UNAIR. 2 (1), 54-61.

[13] Slater, P.J. *Leaves of trees*. 1976. Ars Combin. 2, 549-559.