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Preface

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PREFACE

The International Conference on Mathematics and its Applications (ICoMathApp) 2020

The International Conference on Mathematics and its Applications (ICoMathApp) 2020 is the first annual conference hosted by the Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Negeri Malang. This conference was held virtually on September 30, 2020 using the platform Zoom. The aim of the conference is to provide a platform to the researchers, experts, and practitioners from academia, governments, NGOs, research institutes, and industries to meet and share cutting-edge progress in the fields of algebra, analysis, applied mathematics, combinatorics, computational sciences, geometry, and statistics. The ICoMathApp 2020 theme "Strengthening Researches on Mathematics for the Challenge of Global Society" is expected to give more contribution from the mathematical aspect as a response to the emerging of Covid-19 pandemic. This conference consists of a plenary and parallel session. The plenary session focuses on comprehensive reviews, concepts and perspectives. Specialized talks on recent developments are presented in the parallel session.

My highest appreciation for the four keynote speakers, Assoc. Prof. Arifah Bahar from Universiti Teknologi Malaysia, Malaysia; Prof. Purwanto, Ph.D from Universitas Negeri Malang, Indonesia; Prof. Hadi Susanto from Khalifa University, Abu Dhabi, & University of Essex, United Kingdom; and Assoc. Prof. Andrea Semaničová-Feňovčíková, Ph.D from Technical University of Kosice, Slovak Republic. My highest gratitude also goes to the invited speakers, Prof. Dr. Basuki Widodo, M.Sc from Institut Teknologi Sepuluh Nopember, Indonesia; Prof. Dr.rer.nat, Indah Emilia Wijayanti, S.Si., M.Si. from Universitas Gadjah Mada, Indonesia; Prof. Dr. Toto Nusantara, M.Si., from Universitas Negeri Malang, Indonesia; Dr. Swasono Rahardjo, M.Si, from Universitas Negeri Malang, Indonesia; and Dr. Desi Rahmadani, S.Si., M.Si. from Universitas Negeri Malang, Indonesia.

We would like to express our gratitude to all authors of contributed papers for participating excellently and eagerly. We hope that all participants in the ICoMathApp 2020 get a lot of insights and knowledge from this conference. We also would like to thank the reviewers for their positive contribution to maintain the quality of the articles presented in this conference. I want to also thank the committee members for their hard work, commitment, and dedication in organizing this conference. Our sincere gratitude also goes to the Journal of Physics: Conference Series IOP Publishing editors and coordinator for their helpful cooperation during the preparation of the proceedings.

Mochammad Hafiizh, Ph.D

Chairman

International Conference on Mathematics and its Application 2020

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On inclusive distance vertex irregularity strength of small identical copies of star graphs

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On inclusive distance vertex irregularity strength of small identical copies of star graphs

F Susanto, C N Betistiyan, I Halikin and K Wijaya*

Graph, Combinatorics, and Algebra Research Group, Department of Mathematics FMIPA, Universitas Jember, Jl. Kalimantan 37 Jember 68121, Indonesia

*Corresponding author: kristiana.fmipa@unej.ac.id

E-mail: `faisalsusanto16@gmail.com`, `catrin.tian@yahoo.com`, `ikhsan.fmipa@unej.ac.id`

Abstract. For a simple graph G, an inclusive distance vertex irregular k-labeling of G is a mapping $\lambda : V(G) \to \{1, 2, \ldots, k\}$ such that all the vertex-weights are pairwise distinct, where the weight of a vertex v, denoted by $wt(v)$, is the sum of labels of vertices in the close neighborhood of the vertex v. The minimum k for which the graph G has an inclusive distance vertex irregular k-labeling is called the inclusive distance vertex irregularity strength of G, $\widehat{\text{dis}}(G)$. Here we introduce a new lower bound for $\widehat{\text{dis}}(G)$ and determine the exact value of the inclusive distance vertex irregularity strength for identical copies of star graphs, especially $2S_n$ and $3S_n$.

1. Introduction

Let G be a finite, undirected graph with neither loops nor multiple edges. We denote by $V(G)$ and $E(G)$ the set of vertices and edges of G, respectively. By a *labeling* we mean any mapping that carries some elements (vertices, edges, or both) of a graph to the set of integers (usually positive integers) called *labels*. Such a labeling is called a *vertex* (resp. an *edge*) *labeling* if the domain is the vertex-set (resp. the edge-set). Moreover, if the domain is $V(G) \cup E(G)$ then it is called a *total labeling*. There are many types of graph labeling techniques that have been established. The most recent complete surveys on labelings is given by Gallian [6].

In [10], Slamin introduced distance vertex irregular labelings as a combination of distance based-labelings [1, 7] and irregularity strengths of graphs [5]. A vertex k-labeling $\lambda : V(G) \to \{1, 2, \ldots, k\}$ is said to be a *distance vertex irregular k-labeling* of G if the weights of all vertices are all distinct where the weight of a vertex v of G is $wt(v) = \sum_{u \in N(v)} \lambda(u)$, where $N(v)$ denotes the open neighborhood of v, i.e., the set of vertices adjacent to v in G. The minimum k for which G has a distance vertex irregular k-labeling is called the *distance vertex irregularity strength* of G, $\text{dis}(G)$. There are some results on distance vertex irregularity strengths that have been found for both connected graphs [4, 8, 9, 10] and disconnected graphs [11].

Later on, Bača *et al.* [2] developed a variation of distance vertex irregular labelings namely the inclusive distance vertex irregular labelings of graphs. Such a labeling λ is called an *inclusive distance vertex irregular k-labeling* of G if there are no two vertices having the same weight, where the weight of a vertex v is now calculated by summing all the vertices in the close neighborhood of v, i.e., $wt(v) = \sum_{u \in N[v]} \lambda(u)$. The *inclusive distance vertex irregularity strength* of G is

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the smallest k such that G has an inclusive distance vertex irregular k-labeling and is denoted by $\widehat{\text{dis}}(G)$. There are some results on the inclusive distance vertex irregularity strengths that have been found, including, for example, paths, cycles, complete and complete bipartite graphs, special type of complete tripartite graphs, fans and wheels, caterpillars, join products $G \oplus K_1$ [2, 3], and triangular ladders [12].

In [3], there is also investigated a generalization of the non-inclusive and inclusive distance vertex irregular labelings. The most recent result on this parameter was due to Utami *at al.* [13].

The following theorem showing a lower bound for the inclusive distance vertex irregularity strengths was developed by Bong *et al.* [3].

Theorem 1. [3] *Let G be a graph with order n, maximum degree Δ, and minimum degree δ. Then*

$$\widehat{\text{dis}}(G) \geq \left\lceil \frac{\delta + n}{\Delta + 1} \right\rceil.$$

In this paper we introduce a new lower bound for the inclusive distance irregularity strengths and prove the exact values on this parameter for small identical copies of star graphs.

2. Main Results

We introduce a new lower bound for the inclusive distance vertex irregularity strength of graphs that generalizes the lower bound in Theorem 1.

Theorem 2. *Let G be a graph with maximum degree Δ and minimum degree δ. Let n_i be the number of vertices of degree i in G for every $\delta \leq i \leq \Delta$. Then*

$$\widehat{\text{dis}}(G) \geq \max_{\delta \leq i \leq \Delta} \left\{ \left\lceil \frac{\delta + \sum_{j=\delta}^{i} n_j}{i + 1} \right\rceil \right\}.$$

Proof. Suppose that k is the largest vertex label under an inclusive distance vertex irregular labeling of a graph G. The value of k will be minimum if the vertex weights are induced on vertices such that the vertex with smaller degree receive smaller weight. For each i, the largest among the weights of n_i vertices having degree i must be at least $\delta + \sum_{j=\delta}^{i} n_j$, and this weight is obtained from the sum of $i + 1$ integers. Therefore

$$k \geq \widehat{\text{dis}} \geq \max_{\delta \leq i \leq \Delta} \left\{ \left\lceil \frac{\delta + \sum_{j=\delta}^{i} n_j}{i + 1} \right\rceil \right\}.$$

□

3. Identical Copies of Stars

In this part, we present our main results dealing with the inclusive distance vertex irregularity strength for the identical copies of stars. For positive integers $m \geq 2$ and $n \geq 3$, let mS_n be the identical copies of stars with

$$V(mS_n) = \{u_j : 1 \leq j \leq m\} \cup \{v_{i,j} : 1 \leq i \leq n, 1 \leq j \leq m\}$$

such that

$$E(mS_n) = \{u_j v_{i,j} : 1 \leq i \leq n, 1 \leq j \leq m\}.$$

Here we consider the graph mS_n for $m = 2, 3$ only.

Theorem 3. *Let n be a positive integer, $n \geq 3$. Then $\widehat{\text{dis}}(2S_n) = n + 1$.*

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Proof. We have $\widehat{\text{dis}}(2S_n) \geq \max\{\lceil (2n+1)/2 \rceil, \lceil (2n+3)/(n+1) \rceil\} = n+1$ by Theorem 2. To prove that $\widehat{\text{dis}}(2S_n) \leq n+1$ it suffices to show that there exists an optimal inclusive distance vertex irregular $(n+1)$-labeling of $2S_n$. For $n = 3$, a corresponding inclusive distance vertex irregular 4-labeling of $2S_3$ is described in Figure 1.

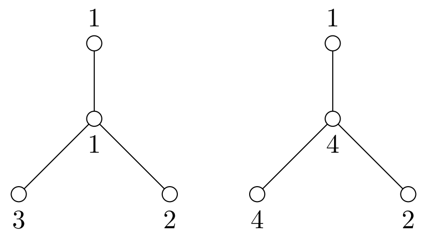

Figure 1. An inclusive distance vertex irregular 4-labeling of $2S_3$.

Let $n \geq 4$. Let λ be a labeling on the vertices of $2S_n$ defined as follows.

$$\lambda(u_j) = \begin{cases} 1 & \text{for } j = 1, \\ n & \text{for } j = 2, \end{cases}$$

$$\lambda(v_{i,j}) = i + j - 1 \quad \text{for } 1 \leq i \leq n \text{ and } 1 \leq j \leq 2.$$

Clearly, there is no label greater than $n+1$. For the the vertex-weights we have the following.

$$wt(u_j) = \begin{cases} 1 + \frac{n(n+1)}{2} & \text{for } j = 1, \\ 2n + \frac{n(n+1)}{2} & \text{for } j = 2, \end{cases}$$

$$wt(v_{i,j}) = (j-1)n + i + 1 \quad \text{for } 1 \leq i \leq n \text{ and } 1 \leq j \leq 2.$$

It is not difficult to see that the vertex weights are all distinct and therefore the labeling λ is the desired inclusive distance vertex irregular $(n+1)$-labeling of $2S_n$. The proof is complete. □

Theorem 4. *Let n be a positive integer, $n \geq 3$. Then*

$$\widehat{\text{dis}}(3S_n) = \begin{cases} 6 & \textit{for } n = 3, \\ \lceil \frac{3n+1}{2} \rceil & \textit{for } n \geq 4. \end{cases}$$

Proof. We distinguish our proof into two cases.

Case 1: $n = 3$. By Theorem 2, we obtain $\widehat{\text{dis}}(3S_3) \geq \max\{\lceil 10/2 \rceil, \lceil 13/4 \rceil\} = 5$. However, we will show that $\widehat{\text{dis}}(3S_3) \geq 6$. Suppose to the contrary that $\widehat{\text{dis}}(3S_3) = 5$ and let λ be an inclusive distance vertex irregular 5-labeling of $3S_3$. Then the integers $2, 3, \ldots, 10$ must be realizable as the weights of all the 9 pendant vertices of $3S_3$. We suppose without loss of generality that $wt(v_{1,1}) = 2$ and $wt(v_{3,3}) = 10$ which mean that $\lambda(u_1) = \lambda(v_{1,1}) = 1$ and $\lambda(v_3) = \lambda(v_{3,3}) = 5$. Thus we have $\{\lambda(v_{1,2}), \lambda(v_{1,3})\} = \{4, 5\}$ (and hence $\{wt(v_{1,2}), wt(v_{1,3})\} = \{5, 6\}$), otherwise there will be two vertices having the same weight. Furthermore, the weights 3 and 4 must be lied on the two of the three vertices $v_{2,1}, v_{2,2}$ and $v_{2,3}$. Let say, $wt(v_{2,1}) = 3$ and $wt(v_{2,2}) = 4$. Then we have two subcases.

- $\lambda(u_2) = 1$. Then $\lambda(v_{2,1}) = 2$ and $\lambda(v_{2,2}) = 3$. Regardless the label of $v_{2,3}$, there will always be two vertices having the same weight, a contradiction.

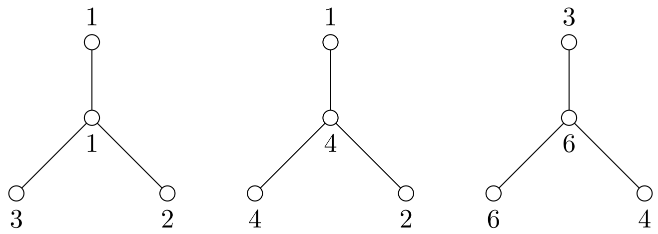

Figure 2. An inclusive distance vertex irregular 6-labeling of $3S_3$.

- $\lambda(u_2) = 2$. Then $\lambda(v_{2,1}) = 1$ and $\lambda(v_{2,2}) = 2$. Similarly, regardless the label of $v_{2,3}$, there will always be two vertices having the same weight, a contradiction.

From the aforementioned arguments, we can conclude that $\widehat{\mathrm{dis}}(3S_3) \geq 6$. A corresponding inclusive distance vertex irregular 6-labeling of $3S_3$ given in Figure 2 proves the equality.

Case 2: $n \geq 4$. According to Theorem 2, $\widehat{\mathrm{dis}}(3S_n) \geq \max\{\lceil (3n+1)/2 \rceil, \lceil (3n+4)/(n+1) \rceil\} = \lceil (3n+1)/2 \rceil$. To prove that $\widehat{\mathrm{dis}}(3S_n) \leq \lceil (3n+1)/2 \rceil$, it is enough for us to construct a corresponding inclusive distance vertex irregular $(\lceil (3n+1)/2 \rceil)$-labeling of $3S_n$. For $n = 4$ and $n = 5$, their corresponding inclusive distance vertex irregular labelings are shown in Figure 3 and 4, respectively.

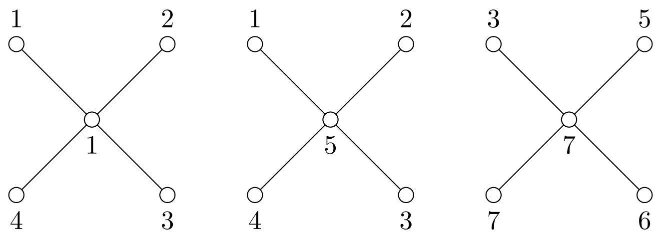

Figure 3. An inclusive distance vertex irregular 7-labeling of $3S_4$.

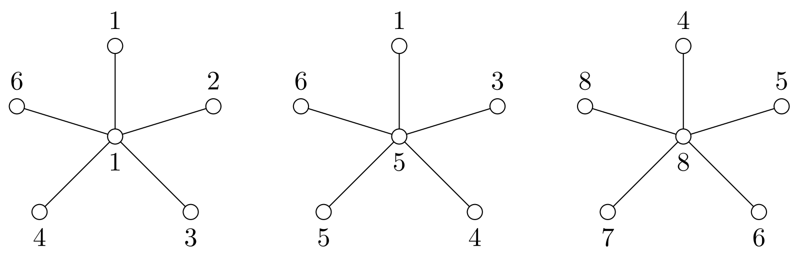

Figure 4. An inclusive distance vertex irregular 8-labeling of $3S_5$.

Now let $n \geq 6$ and let $\lambda : V(3S_n) \to \{1, 2, \ldots, \lceil (3n+1)/2 \rceil\}$ be a labeling on the vertices of the graph $3S_n$ defined such that

$$\lambda(u_j) = \begin{cases} 1 & \text{for } j = 1, \\ n+1 & \text{for } j = 2, \\ \left\lfloor \frac{3n+1}{2} \right\rfloor & \text{for } j = 3, \end{cases}$$

$$\lambda(v_{i,j}) = \begin{cases} i & \text{for } 1 \le i \le n \text{ and } 1 \le j \le 2, \\ \lceil \frac{n+1}{2} \rceil + i & \text{for } 1 \le i \le n \text{ and } j = 3. \end{cases}$$

Then we have the following vertex-weights.

$$wt(u_j) = \begin{cases} 1 + \frac{n(n+1)}{2} & \text{for } j = 1, \\ n+1+\frac{n(n+1)}{2} & \text{for } j = 2, \\ 2n+1+\frac{n(n+1)}{2} + (n-1)\lceil \frac{n+1}{2} \rceil & \text{for } j = 3, \end{cases}$$

$$wt(v_{i,j}) = \begin{cases} (j-1)n + i + 1 & \text{for } 1 \le i \le n \text{ and } 1 \le j \le 2, \\ 2n+i+1 & \text{for } 1 \le i \le n \text{ and } j = 3. \end{cases}$$

Thus the set of weights of all the pendant vertices is $\{2, 3, \ldots, 3n+1\}$. Furthermore, it is easy to see that the weights of all the three center vertices are distinct and also as $n \ge 6$, we have

$$1 + \frac{n(n+1)}{2} \ge 1 + \frac{7n}{2} > 3n+1,$$

meaning that there are no two vertices having the same weight. This completes the proof. □

4. Conclusion

In this paper we introduced a new lower bound for the inclusive distance vertex irregularity strength of arbitrary graphs and determined the exact value for identical copies stars mS_n particularly for $m = 2, 3$. We have also tried to find this parameter for $m \ge 4$ but unsuccessful. Therefore we propose the problem below.

Problem 1. *Determine the inclusive distance vertex irregularity strength of mS_n for $m \ge 4$ and $n \ge 3$.*

More general, the following problem is also considerable to be investigated.

Problem 2. *Determine the inclusive distance vertex irregularity strength of $\bigcup_{i=1}^{m} S_{n_i}$ for $m \ge 2$ and $n_i \ge 3$.*

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