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INTERIOR POINT METHOD FOR SOLVING LINEAR PROGRAMMING WITH INTERVAL COEFFICIENTS USING AFFINE SCALING

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Abstract

Linear programming with interval coefficients was developed to overcome cases in classical linear programming where the coefficient value is unknown and must be estimated. This paper discusses the affine scaling algorithm which is one variant of the interior point method to solve linear programming with interval coefficients. The

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process is to change linear programming with interval coefficients into two classic linear programming models with special characteristics, namely, the best optimum problem and the worst optimum problem. Furthermore, these two problems have solved using affine scaling algorithm.

1. Introduction

Linear programming is one of the decision making tools widely used to solve the real world problems. Linear programming is an optimization problem that satisfies assumptions of proportionality, additivity, divisibility and certainty [1]. In certainty assumption, all the coefficients and variables in linear programming are known. However, in real situation, sometimes the values of coefficients are not known with certainty. Linear programming problems in general can be solved by the simplex method But there are other methods also, for example, the interior point method discussed by several researchers [1-6]. The interior point method has several variants including Khachiyan algorithm, Karmarkar algorithm and affine scaling algorithm. All the three algorithms use a numerical approach to solve linear programming problems. The Khachiyan algorithm was first completed in 1979, the Karmarkar algorithm in 1984, while an affine scaling algorithm became popular in the 1990s [2].

Interval linear programming is the development of classical linear programs supported by the concept of interval analysis and the theory developed by Moore [8]. The purpose of this development is to solve the case, that is when the data value is unknown, but the data is in interval where the infimum and supremum values are known. The special characteristics of interval linear programming problem were the coefficients of objective function and constraint functions in the form of interval. Research on linear programming with interval coefficients has been done by Chinneck and Ramadan [9]. Linear programming with interval coefficients converted into two classic linear programming models with special characteristics, which are called the *best optimum problem* and the *worst optimum problem*. Molai et al. [10-13] only describe interval in constraint functions. Aliyev [14] presents applications from linear programming with interval coefficients in the field of advertising. The researchers [9-14] used the simplex method to solve linear programming with interval coefficients.

This paper proposes an affine scaling algorithm which is one of variants of interior point method, and compares with the simplex method in terms of width of interval. The completion step uses the method in [9].

This paper is organized as follows: We demonstrate some preliminaries of interval arithmetic and formula of the linear programming with interval coefficients in Section 2. Section 3 is devoted to the solution of linear programming with interval coefficients using affine scaling. Numerical examples are given in Section 4. Finally, we allocate the Section 5 to conclusions.

2. Preliminaries

In this section, we review some of the concepts needed, such as arithmetic interval and formula of linear programming with interval coefficients. For more details, we refer to [1, 8, 9, 15].

2.1. Interval arithmetic

The basic definition and properties of interval number and interval arithmetic can be found in [8, 15]. Let R denote the set of all real numbers.

Definition 1. A closed real interval $\underline{x} = [x_I, x_S]$ is a real interval number which can be defined by

$$\underline{x} = [x_I, x_S] = \{x \in R \mid x_I \leq x \leq x_S;\ x_I, x_S \in R\}, \tag{1}$$

where x_I and x_S are called *infimum* and *supremum* of $\underline{x}$, respectively.

Definition 2. A real interval number $\underline{x} = [x_I, x_S]$ is called a *degenerate*, if $x_I = x_S$.

Definition 3. Let $I(R)$ be the set of all intervals on R. A real interval vector $\underline{\mathbf{x}} \in I(R^n)$ is a vector in the form $\underline{\mathbf{x}} = (\underline{x}_i)_{n\times 1}$, where $\underline{x}_i = [x_{iI}, x_{iS}] \in I(R)$, $I = 1, 2, \ldots, n$.

Definition 4. Let $\underline{x}, \underline{y} \in I(R)$, where $\underline{x} = [x_I, x_S]$ and $\underline{y} = [y_I, y_S]$. Then

(a) $\underline{x} + \underline{y} = [x_I + y_I, x_S + y_S]$ (addition),

(b) $\underline{x} - \underline{y} = [x_I - y_S, x_S - y_I]$ (subtraction),

(c) $\underline{x} \cdot \underline{y} = [\min\{x_I y_I, x_I y_S, x_S y_I, x_S y_S\}, \max\{x_I y_I, x_I y_S, x_S y_I, x_S y_S\}]$ (multiplication),

(d) $\underline{x}/\underline{y} = [x_I, x_S][1/y_S, 1/y_I]$, $0 \notin \underline{y}$ (division),

(e) $w(\underline{x}) = x_S - x_I$ (width of interval).

2.2. Linear programming with interval coefficients

The general form of linear programming (maximize) is defined as follows [1]:

Maximize

$$Z = \sum_{j=1}^{n} c_j x_j \tag{2}$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i, \quad i = 1, 2, \ldots, m, \tag{3}$$

$$x_j \ge 0, \quad j = 1, 2, \ldots, n. \tag{4}$$

The general form of linear programming with interval coefficients is defined as follows:

Maximize

$$\underline{Z} = \sum_{j=1}^{n} [c_{jI}, c_{jS}] x_j \tag{5}$$

subject to

$$\sum_{j=1}^{n} [a_{ijI}, a_{ijS}] x_j \le [b_{iI}, b_{iS}], \quad i = 1, 2, \dots, m, \tag{6}$$

$$x_j \ge 0, \quad j = 1, 2, \dots, n, \tag{7}$$

where $x_j \in R^n$, $[c_{jI}, c_{jS}]$, $[a_{ijI}, a_{ijS}] \in I(R)$.

Problem (5)-(7) is solved by changing into two classic linear programming problems with special characteristics, namely, the best optimum problem and the worst optimum problem. The best optimum problem has the best properties of objective function and maximum feasible area on the constraint functions, whereas the worst optimum problem has the worst properties of objective function and minimum feasible area on the constraint functions. Chinneck and Ramadan [9] provide a theorem to determine the best optimum problem and the worst optimum problem in linear programming with the interval coefficients given by the following theorem:

Theorem 1 [9]**.** *Assume the interval inequality*

$$\sum_{j=1}^{n} [a_{jI}, a_{jS}] x_j \le [b_I, b_S], \quad j = 1, 2, \dots, n,$$

where $x_j \geq 0$. *Then*:

(a) *Maximum feasible area satisfies the following inequality*:

$$\sum_{j=1}^{n} a_{jI} x_j \leq b_S.$$

(b) *Minimum feasible area satisfies the following inequality*:

$$\sum_{j=1}^{n} a_{jS} x_j \leq b_I.$$

Theorem 2 [9]**.** *Suppose an objective function*

$$\underline{Z} = \sum_{j=1}^{n} [c_{jI}, c_{jS}] x_j, \quad j = 1, 2, \dots, n,$$

for $x_j \geq 0$. *Then*

$$\sum_{j=1}^{n} c_{jS} x_j \geq \sum_{j=1}^{n} c_{jI} x_j,$$

where $\sum_{j=1}^{n} c_{jS} x_j$ *is the best version of the objective function and* $\sum_{j=1}^{n} c_{jI} x_j$ *is the worst version of the objective function*.

3. Solving Linear Programming with Interval Coefficients Using Affine Scaling

In order to solve a linear programming model with the interior point method, it has to be determined previously an initial interior point in feasible area. A system of linear equations and linear inequalities is said to satisfy an interior point conditions if there is a set of feasible solutions on the system [16]. The affine scaling algorithm is a variant of the interior point method that can be used to solve linear programming with interval coefficients. The linear programming model used in the affine scaling algorithm has to be in the canonical form [1].

Steps of interior point method for solving linear programming with interval coefficients using affine scaling algorithm are declared as follows.

Algorithm 1

Step 1. Given a linear programming problem with interval coefficients in (5)-(7).

Step 2. Use Theorems 1 and 2 for transforming the linear programming with interval coefficients in (5)-(7) into two classic linear programming models with special characteristics, namely:

(a) The best optimum problem is

Maximize

$$Z_S = \sum_{j=1}^{n} c_{jS} x_j \tag{8}$$

subject to

$$\sum_{j=1}^{n} a_{ijI} x_j \le b_S \quad \text{and} \quad x_j \ge 0. \tag{9}$$

(b) The worst optimum problem is

Maximize

$$Z_I = \sum_{j=1}^{n} c_{jI} x_j \tag{10}$$

subject to

$$\sum_{j=1}^{n} a_{ijS} x_j \le b_I \quad \text{and} \quad x_j \ge 0. \tag{11}$$

Step 3. Use Algorithm 2 (affine scaling algorithm) to solve the best optimum problem and the worst optimum problem.

Algorithm 2 (**affine scaling algorithm**)

(1) Choose an initial interior point $\tilde{X}^0 = (x_1, x_2, \ldots, x_{n+m})$. The interior point $\tilde{X}^0$ should satisfy all constraints. Then evaluate the objective function

$$Z_0 = c^T \tilde{X}^0.$$

Define a diagonal matrix $\mathbf{D}_{i+1} = diag(\tilde{X}^0)$,

$$\mathbf{D}_{i+1} = \begin{bmatrix} x_1 & 0 & 0 & & 0 \\ 0 & x_2 & 0 & & 0 \\ 0 & 0 & x_3 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & x_{n+m} \end{bmatrix}.$$

(2) Calculate $\mathbf{A}_{i+1} = \mathbf{AD}_{i+1}$ and $\mathbf{C}_{i+1} = \mathbf{D}_{i+1}\mathbf{C}$.

(3)(a) Calculate the projected matrix

$$\mathbf{P}_{i+1} = \mathbf{I} - \mathbf{A}_{i+1}^T(\mathbf{A}_{i+1}\mathbf{A}_{i+1}^T)^{-1}\mathbf{A}_{i+1},$$

where $\mathbf{I}$ is an identity matrix.

(b) Calculate the projected gradient

$$\mathbf{C}_{P_{i+1}} = \mathbf{P}_{i+1}\mathbf{C}_{i+1}.$$

(4) Calculate $\mathbf{V}_{i+1} = |\min(\mathbf{C}_{P_{i+1}})|$ and

$$\mathbf{M}_{i+1} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} + \frac{\alpha}{\mathbf{V}_{i+1}} \mathbf{C}_{P_{i+1}},$$

where $\mathbf{M}_{i+1}$ is an $(n + m) \times 1$ matrix and $\alpha \in (0, 1)$.

(5) Calculate the next interior point

$$\tilde{X}^{i+1} = \mathbf{D}_{i+1}\mathbf{M}_{i+1}.$$

(6) Calculate the objective function

$$Z_{i+1} = c^T \tilde{X}^{i+1}.$$

(a) If $Z_{i+1} > Z_i$, then the iteration should be continued. Go to Step 1, where $\tilde{X}^{i+1}$ is the next interior point.

(b) If $Z_{i+1} \leq Z_i$, then the optimal solution was reached. Here, Z_i is the optimal solution.

Step 4. The optimum value of the linear programming with interval coefficients is obtained by combining the optimum value from the worst optimum and the best optimum problem; $Z = [z_I, z_S]$.

4. Numerical Examples

In this section, two examples of linear programming with interval coefficients are given. The first example has been solved in Chinneck and Ramadan [9], and the second example has been solved in Aliyev [14]. We compare the results.

Example 1.

Maximize

$$Z = 4x_1 + [8, 12]x_2 \tag{12}$$

subject to

$$6x_1 + [4.25, 5.75]x_2 \leq 30, \tag{13}$$

$$[0.95, 1.05]x_1 \leq 3, \tag{14}$$

$$x_2 \leq [3.6, 4.4], \tag{15}$$

$$x_1, x_2 \geq 0. \tag{16}$$

According to [9], for the solution of the model, the worst optimum problems are $z_1 = 35$, $x_1 = 1.55$ and $x_2 = 3.6$, the best optimum problems are $z_S = 60.3333$, $x_1 = 1.8833$ and $x_2 = 4.4$, and the optimum value is $Z = [z_I, z_S] = [35, 60.3333]$ with interval width $w(Z) = 25.3333$.

We apply Algorithm 1 to change linear programming with interval coefficients model (12)-(16) into two classic linear programming models with special characteristics, that is, the best optimum and the worst optimum problems.

(a) The best optimum problem

Maximize

$$z_S = 4x_1 + 12x_2$$

subject to

$$6x_1 + 4.25x_2 \leq 30,$$

$$0.95x_1 \leq 3,$$

$$x_2 \leq 4.4,$$

$$x_1,\ x_2 \geq 0.$$

By using Algorithm 2 and $\alpha = 0.95$ [1], the best optimum values are $x_1 \cong 1.883333$, $x_2 \cong 4.400000$ and $z_S \cong 60.3333334$ with the iteration results as shown in Table 1.

Table 1. The best optimum value from affine scaling algorithm

Iteration	x_i	z_i	Optimality test	Decision
0	(2, 3)	44.0000000	Z_0	
1	(1.666684, 4.329999)	58.6267390	$Z_1 > Z_0$	Continue
2	(1.889353, 4.372708)	60.0299137	$Z_2 > Z_1$	Continue
3	(1.873683, 4.398635)	60.2783580	$Z_3 > Z_2$	Continue
4	(1.883498, 4.399017)	60.3222020	$Z_4 > Z_3$	Continue
5	(1.882956, 4.399950)	60.3312351	$Z_5 > Z_4$	Continue
6	(1.883338, 4.399963)	60.3329175	$Z_6 > Z_5$	Continue
7	(1.883318, 4.399998)	60.3332536	$Z_7 > Z_6$	Continue
8	(1.883333, 4.399998)	60.3333175	$Z_8 > Z_7$	Continue
9	(1.883332, 4.399999)	60.3333303	$Z_9 > Z_8$	Continue
10	(1.883333, 4.400000)	60.3333335	$Z_{10} > Z_9$	Continue
11	(1.883333, 4.400000)	60.3333334	$Z_{11} \leq Z_{10}$	Stop

(b) The worst optimum problem

Maximize

$$z_I = 4x_1 + 8x_2$$

subject to

$$6x_1 + 5.75x_2 \leq 30,$$

$$1.05x_1 \leq 3,$$

$$x_2 \leq 3.6,$$

$$x_1, x_2 \geq 0.$$

By using Algorithm 2 and $\alpha = 0.95$ [1], the worst optimum values are $x_1 \cong 1.550003$, $x_2 \cong 3.599999$ and $z_I \cong 35.000014$ with the iteration results as shown in Table 2.

Table 2. The worst optimum value from affine scaling algorithm iteration

Iteration	x_i	z_i	Optimality test	Decision
0	(2, 2)	24.000000	Z_0	
1	(1.942044, 3.134387)	32.843283	$Z_1 > Z_0$	Continue
2	(1.527426, 3.576719)	34.723460	$Z_2 > Z_1$	Continue
3	(1.558583, 3.588701)	34.943947	$Z_3 > Z_2$	Continue
4	(1.548703, 3.599435)	34.990296	$Z_4 > Z_3$	Continue
5	(1.550284, 3.599606)	34.997995	$Z_5 > Z_4$	Continue
6	(1.549946, 3.599980)	34.999629	$Z_6 > Z_5$	Continue
7	(1.550010, 3.599985)	34.999926	$Z_7 > Z_6$	Continue
8	(1.549997, 3.599999)	34.999986	$Z_8 > Z_7$	Continue
9	(1.550000, 3.599999)	34.999997	$Z_9 > Z_8$	Continue
10	(1.549999, 3.599999)	34.999998	$Z_{10} > Z_9$	Continue
11	(1.549999, 3.600000)	35.000004	$Z_{11} > Z_{10}$	Continue
12	(1.550000, 3.600002)	35.000022	$Z_{12} > Z_{11}$	Continue
13	(1.550003, 3.599999)	35.000014	$Z_{13} \leq Z_{12}$	Stop

(c) The optimum value of the linear programming with interval coefficients is obtained with combining the optimum value from the worst and the best optimum problem, that is, $Z = [z_I, z_S] = [35.000014, 60.3333334]$ with interval width $w(Z) = 25.3333194$. This solution gives the same value as obtained by Chinneck and Ramadan [9].

Example 2. A company produced new product and the manager has to determine money to spend in advertising outlet. There are two outlets: TV and newsmagazine. The problem is to determine targets for each market segment and minimize the expenses to these targets. There are two segments. The costs for each minute of TV and news magazine advertisement are shown in Table 3 (in million).

Table 3. Cost advertisement for each minute (in million)

Outlet	Segment 1	Segment 2	Cost
TV	[4, 6]	[2.5, 3.5]	[400, 500]
Magazines	[1.5, 2.5]	[2.5, 3.5]	[350, 400]
Target	[20, 25]	[18, 20]	

The formulated problem is as follows:

Minimize

$$Z = [400, 500]x_1 + [350, 450]x_2$$

subject to

$$[4, 6]x_1 + [1.5, 2.5]x_2 \geq [20, 25],$$

$$[2.5, 3.5]x_1 + [2.5, 3.5]x_2 \geq [18, 20],$$

$$x_1, x_2 \geq 0.$$

According to [14], for the solution of the model, the best optimum problems are $z_I = 1922.86$, $x_1 = 2.06$ and $x_2 = 3.14$, the worst optimum problems are $z_S = 4140$, $x_1 = 5.4$ and $x_2 = 3.2$, and the optimum value is $Z = [z_I, z_S] = [1922.86, 4140]$ with interval width $w(Z) = 2217$. Furthermore, in this paper, a solution is taken to minimize advertising costs. Taking $x_1 = 2.06$, $x_2 = 3.14$, the optimum value is $Z = [z_I, z_S] = [1922.86, 2443]$ with interval width $w(Z) = 520.14$.

We present only the maximization problem as any minimization problem can be converted into maximization problem. A simple procedure to convert a minimization problem to maximization problem and vice versa is simply to multiply the objective function of minimization problem by –1 converting it into a maximization problem and vice versa.

Maximize

$$-Z = [-500, -400]x_1 + [-450, -350]x_2 \tag{17}$$

subject to

$$[-6, -4]x_1 + [-2.5, -1.5]x_2 \leq [-25, -20], \tag{18}$$

$$[-3.5, -2.5]x_1 + [-3.5, -2.5]x_2 \leq [-20, -80], \tag{19}$$

$$x_1, x_2 \geq 0. \tag{20}$$

We apply Algorithm 1 to change linear programming with interval coefficients model (17)-(20) into two classic linear programming models with special characteristics: the best optimum problem and the worst optimum problem and apply Algorithm 2, the results are shown in Table 4.

Table 4. The best optimum and the worst optimum problem

The best optimum problem	The worst optimum problem
Maximize	Maximize
$-z_S = -400x_1 - 350x_2$	$-z_I = -500x_1 - 450x_2$
subject to	Subject to
$-6x_1 - 2.5x_2 \leq -20,$	$-4x_1 - 1.5x_2 \leq -25,$
$-3.5x_1 - 3.5x_2 \leq -18,$	$-2.5x_1 - 2.5x_2 \leq -20,$
$x_1, x_2 \geq 0$	$x_1, x_2 \geq 0$
$x_1 \cong 2.041, \quad x_2 \cong 3.102$	$x_1 \cong 5.2, \quad x_2 \cong 2.8$
$-z_S \cong -1902.041$	$-z_I \cong -3860.002$

The optimum value of the linear programming with interval coefficients is obtained by combining the optimum value from the worst and the best optimum problems, that is, $-Z \cong [-z_I, -z_S] = [-3860.002, -1902.041]$ or $Z \cong [1902.041, 3860.002]$ with the width of the interval $w(Z) = 1957.961$. Solution to minimize advertising costs taken $x_1 \cong 2.041$, $x_2 \cong 3.102$, the optimum value is $Z \cong [z_I, z_S][1902.041, 2416.327]$ with interval width $w(Z) = 514.286$. This solution provides values that are almost the same as those obtained by Aliyev [14] but with shorter width interval. This shows an increasingly definite value.

5. Conclusions

This paper presents the use of affine scaling algorithm. The affine scaling algorithm is a variant of the interior point method that can be used to solve linear programming with interval coefficients. Linear programming with interval coefficients can be solved with converting the problem into two classic linear programming models called the *best optimum problem* and the *worst optimum problem*. Furthermore, two problems were solved by an affine scaling algorithm. The optimum interval value is obtained from a combination of these two problems. Use of affine scaling algorithm has been compared with the simplex method. It has been obtained that the affine scale algorithm is more efficient. Shorter/smaller width intervals indicate that the optimal value approaches a fixed value. This makes it easy for manager/decision maker to make decisions from some uncertain situation in the real world.

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