

Research Article

Total Vertex Irregularity Strength of the Disjoint Union of Sun Graphs

Slamin,¹ Dafik,² and Wyse Winnona²

¹ Information System Study Program, University of Jember, Jember 68121, Indonesia

² Mathematics Education Study Program, University of Jember, Jember 68121, Indonesia

Correspondence should be addressed to Slamin, slamin@unej.ac.id

Received 11 January 2011; Accepted 31 March 2011

Academic Editor: R. Yuster

Copyright © 2012 Slamin et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

A vertex irregular total k -labeling of a graph G with vertex set V and edge set E is an assignment of positive integer labels $\{1, 2, \dots, k\}$ to both vertices and edges so that the weights calculated at vertices are distinct. The total vertex irregularity strength of G , denoted by $\text{tvs}(G)$ is the minimum value of the largest label k over all such irregular assignment. In this paper, we consider the total vertex irregularity strengths of disjoint union of s isomorphic sun graphs, $\text{tvs}(sM_n)$, disjoint union of s consecutive nonisomorphic sun graphs, $\text{tvs}(\bigcup_{i=1}^s M_{i+2})$, and disjoint union of any two nonisomorphic sun graphs $\text{tvs}(M_k \cup M_n)$.

1. Introduction

Let G be a finite, simple, and undirected graph with vertex set V and edge set E . A *vertex irregular total k -labeling* on a graph G is an assignment of integer labels $\{1, 2, \dots, k\}$ to both vertices and edges such that the weights calculated at vertices are distinct. The *weight* of a vertex $v \in V$ in G is defined as the sum of the label of v and the labels of all the edges incident with v , that is,

$$wt(v) = \lambda(v) + \sum_{uv \in E} \lambda(uv). \quad (1.1)$$

The notion of the vertex irregular total k -labeling was introduced by Bača et al. [1]. The *total vertex irregularity strength* of G , denoted by $\text{tvs}(G)$, is the minimum value of the largest label k over all such irregular assignments.

The total vertex irregular strengths for various classes of graphs have been determined. For instances, Bača et al. [1] proved that if a tree T with n pendant vertices and no vertices of degree 2, then $\lceil (n+1)/2 \rceil \leq \text{tvs}(T) \leq n$. Additionally, they gave a lower bound and an upper bound on total vertex irregular strength for any graph G with v vertices and e edges, minimum degree δ and maximum degree Δ , $\lceil (|V| + \delta)/(\Delta + 1) \rceil \leq \text{tvs}(G) \leq |V| + \Delta - 2\delta + 1$. In the same paper, they gave the total vertex irregular strengths of cycles, stars, and complete graphs, that is, $\text{tvs}(C_n) = \lceil (n+2)/3 \rceil$, $\text{tvs}(K_{1,n}) = \lceil (n+1)/2 \rceil$ and $\text{tvs}(K_n) = 2$.

Furthermore, the total vertex irregularity strength of complete bipartite graphs $K_{m,n}$ for some m and n had been found by Wijaya et al. [2], namely, $\text{tvs}(K_{2,n}) = \lceil (n+2)/3 \rceil$ for $n > 3$, $\text{tvs}(K_{n,n}) = 3$ for $n \geq 3$, $\text{tvs}(K_{n,n+1}) = 3$ for $n \geq 3$, $\text{tvs}(K_{n,n+2}) = 3$ for $n \geq 4$, and $\text{tvs}(K_{n,an}) = \lceil n(a+1)/(n+1) \rceil$ for all n and $a > 1$. Besides, they gave the lower bound on $\text{tvs}(K_{m,n})$ for $m < n$, that is, $\text{tvs}(K_{m,n}) \geq \max\{\lceil (m+n)/(m+1) \rceil, \lceil (2m+n-1)/n \rceil\}$. Wijaya and Slamir [3] found the values of total vertex irregularity strength of wheels W_n , fans F_n , suns M_n and friendship graphs f_n by showing that $\text{tvs}(W_n) = \lceil (n+3)/4 \rceil$, $\text{tvs}(F_n) = \lceil (n+2)/4 \rceil$, $\text{tvs}(M_n) = \lceil (n+1)/2 \rceil$, $\text{tvs}(f_n) = \lceil (2n+2)/3 \rceil$.

Ahmad et al. [4] had determined total vertex irregularity strength of Halin graph. Whereas the total vertex irregularity strength of trees, several types of trees and disjoint union of t copies of path had been determined by Nurdin et al. [5–7]. Ahmad and Bača [8] investigated the total vertex irregularity strength of Jahangir graphs $J_{n,2}$ and proved that $\text{tvs}(J_{n,2}) = \lceil (n+1)/2 \rceil$, for $n \geq 4$ and conjectured that for $n \geq 3$ and $m \geq 3$,

$$\text{tvs}(J_{m,n}) \geq \max\left\{\left\lceil \frac{n(m-1)+2}{3} \right\rceil, \left\lceil \frac{nm+2}{4} \right\rceil\right\}. \quad (1.2)$$

They also proved that for the circulant graph, $\text{tvs}(C_n(1,2)) = \lceil (n+4)/5 \rceil$, and conjectured that for the circulant graph $C_n(a_1, a_2, \dots, a_m)$ with degree at least 5, $1 \leq a_i \leq \lfloor n/2 \rfloor$, $\text{tvs}(C_n(a_1, a_2, \dots, a_m)) = \lceil (n+r)/(1+r) \rceil$.

A *sun graph* M_n is defined as the graph obtained from a cycle C_n by adding a pendant edge to every vertex in the cycle. In this paper, we determine the total vertex irregularity strength of disjoint union of the isomorphic sun graphs $\text{tvs}(sM_n)$, disjoint union of consecutive nonisomorphic sun graphs $\text{tvs}(\bigcup_{i=1}^s M_{i+2})$ and disjoint union of two nonisomorphic sun graphs $\text{tvs}(M_k \cup M_n)$, as described in the following section.

2. Main Results

We start this section with a lemma on the lower bound of total vertex irregularity strength of disjoint union of any sun graphs as follows.

Lemma 2.1. *The total vertex irregularity strength of disjoint union of any sun graphs is $\text{tvs}(\bigcup_{i=1}^s M_{n_i}) \geq \lceil ((\sum_{i=1}^s n_i) + 1)/2 \rceil$, for $s \geq 1$, $n_{i+1} \geq n_i$, and $1 \leq i \leq s$.*

Proof. The disjoint union of the isomorphic sun graphs $\bigcup_{i=1}^s M_{n_i}$ has $\sum_{i=1}^s n_i$ vertices $u_{i,j}$ of degree 1 and $\sum_{i=1}^s n_i$ vertices $v_{i,j}$ of degree 3. Note that the smallest weight of vertices of $\bigcup_{i=1}^s M_{n_i}$ must be 2. It follows that the largest weight of $\sum_{i=1}^s n_i$ vertices of degree 1 is at least $(\sum_{i=1}^s n_i) + 1$ and of $\sum_{i=1}^s n_i$ vertices of degree 3 is at least $2(\sum_{i=1}^s n_i) + 1$. As a consequence, at least one vertex $u_{i,j}$ or one edge incident with $u_{i,j}$ has label at least $\lceil ((\sum_{i=1}^s n_i) + 1)/2 \rceil$.

Moreover, at least one vertex $v_{i,j}$ or one edge incident with $v_{i,j}$ has label at least $\lceil (2(\sum_{i=1}^s n_i) + 1)/4 \rceil$. Then

$$\text{tvs}\left(\bigcup_{i=1}^s M_{n_i}\right) \geq \max\left\{\left\lceil \frac{(\sum_{i=1}^s n_i) + 1}{2} \right\rceil, \left\lceil \frac{2(\sum_{i=1}^s n_i) + 1}{4} \right\rceil\right\}. \quad (2.1)$$

Because of

$$\left\lceil \frac{(\sum_{i=1}^s n_i) + 1}{2} \right\rceil = \left\lceil \frac{2(\sum_{i=1}^s n_i) + 1}{4} \right\rceil, \quad (2.2)$$

then

$$\text{tvs}\left(\bigcup_{i=1}^s M_{n_i}\right) \geq \left\lceil \frac{(\sum_{i=1}^s n_i) + 1}{2} \right\rceil. \quad (2.3)$$

□

We now present a theorem on the total vertex irregularity strength of disjoint union of the isomorphic sun graphs $\text{tvs}(sM_n)$ as follows.

Theorem 2.2. *The total vertex irregularity strength of the disjoint union of isomorphic sun graphs is $\text{tvs}(sM_n) = \lceil (sn + 1)/2 \rceil$, for $s \geq 1$ and $n \geq 3$.*

Proof. Using Lemma 2.1, we have $\text{tvs}(sM_n) \geq \lceil (sn + 1)/2 \rceil$. To show that $\text{tvs}(sM_n) \leq \lceil (sn + 1)/2 \rceil$, we label the vertices and edges of sM_n as a total vertex irregular labeling. Suppose the disjoint union of the isomorphic sun graphs sM_n has the set of vertices

$$V(sM_n) = \{u_{i,j} \mid 1 \leq i \leq s, 1 \leq j \leq n\} \cup \{v_{i,j} \mid 1 \leq i \leq s, 1 \leq j \leq n\} \quad (2.4)$$

and the set of edges

$$E(sM_n) = \{u_{i,j}v_{i,j} \mid 1 \leq i \leq s, 1 \leq j \leq n\} \cup \{v_{i,j}v_{i,j+1} \mid 1 \leq i \leq s, 1 \leq j \leq n\}. \quad (2.5)$$

The labels of the edges and the vertices of sM_n are described in the following formulas:

$$\lambda(u_{i,j}) = \begin{cases} 1 & \text{for } i = 1, 2, \dots, \left\lfloor \frac{s-1}{2} \right\rfloor; j = 1, 2, \dots, n \\ 1 + j + (i-1)n & \text{and } i = \left\lfloor \frac{s+1}{2} \right\rfloor, j = 1, 2, \dots, \left(\left\lfloor \frac{sn+1}{2} \right\rfloor - n \left\lfloor \frac{s-1}{2} \right\rfloor \right) \\ -\left\lfloor \frac{sn+1}{2} \right\rfloor & \text{for other } i, j, \end{cases}$$

$$\begin{aligned}
\lambda(v_{i,j}) &= \begin{cases} 1 & \text{for } i = 1, 2, \dots, \left\lfloor \frac{s-1}{2} \right\rfloor; j = 1, 2, \dots, n \\ & \text{and } i = \left\lfloor \frac{s+1}{2} \right\rfloor, j = 1, 2, \dots, \left(\left\lfloor \frac{sn+1}{2} \right\rfloor - n \left\lfloor \frac{s-1}{2} \right\rfloor \right) \\ 1 + j + (i-1)n & \\ - \left\lfloor \frac{sn+1}{2} \right\rfloor & \text{for other } i, j, \end{cases} \\
\lambda(u_{i,j}v_{i,j}) &= \begin{cases} j + (i-1)n & \text{for } i = 1, 2, \dots, \left\lfloor \frac{s-1}{2} \right\rfloor; j = 1, 2, \dots, n \\ & \text{and } i = \left\lfloor \frac{s+1}{2} \right\rfloor, j = 1, 2, \dots, \left(\left\lfloor \frac{sn+1}{2} \right\rfloor - n \left\lfloor \frac{s-1}{2} \right\rfloor \right) \\ \left\lfloor \frac{sn+1}{2} \right\rfloor & \text{for other } i, j, \end{cases} \\
\lambda(v_{i,j}v_{i,j+1}) &= \left\lfloor \frac{sn+1}{2} \right\rfloor, \quad \text{for } i = 1, 2, \dots, s, j = 1, 2, \dots, n.
\end{aligned} \tag{2.6}$$

The weights of the vertices $u_{i,j}$ and $v_{i,j}$ of sM_n are

$$\begin{aligned}
wt(u_{i,j}) &= 1 + j + (i-1)n, \quad \text{for } i = 1, 2, \dots, s, j = 1, 2, \dots, n, \\
wt(v_{i,j}) &= 1 + j + (i-1)n + 2 \left\lfloor \frac{sn+1}{2} \right\rfloor, \quad \text{for } i = 1, 2, \dots, s, j = 1, 2, \dots, n.
\end{aligned} \tag{2.7}$$

It is easy to see that the weights calculated at vertices are distinct. So, the labeling is vertex irregular total. Therefore $\text{tvs}(sM_n) = \lfloor (sn+1)/2 \rfloor$ for $s \geq 1$ and $n \geq 3$. \square

Figure 1 illustrates the total vertex irregular labeling of the disjoint union 5 copies sun graphs M_5 .

If we substitute $s = 1$ into the theorem above, we obtain a result that has been proved by Wijaya and Slamir [3] as follows.

Corollary 2.3. *The total vertex irregularity strength of sun graph $\text{tvs}(M_n) = \lfloor (n+1)/2 \rfloor$, for $s = 1$ and $n \geq 3$.*

The following theorem shows the total vertex irregularity strength of disjoint union of nonisomorphic sun graphs with consecutive number of pendants.

Theorem 2.4. *The total vertex irregularity strength of disjoint union of consecutive nonisomorphic sun graphs is $\text{tvs}(\bigcup_{i=1}^s M_{i+2}) = \lfloor (s(s+5))/4 \rfloor$, for $s \geq 1$.*

Proof. Using Lemma 2.1, we have $\text{tvs}(\bigcup_{i=1}^s M_{i+2}) \geq \lfloor (s(s+5)+2)/4 \rfloor$. To show that $\text{tvs}(\bigcup_{i=1}^s M_{i+2}) \leq \lfloor (s(s+5)+2)/4 \rfloor$, we label the vertices and edges of $\bigcup_{i=1}^s M_{i+2}$ as a total

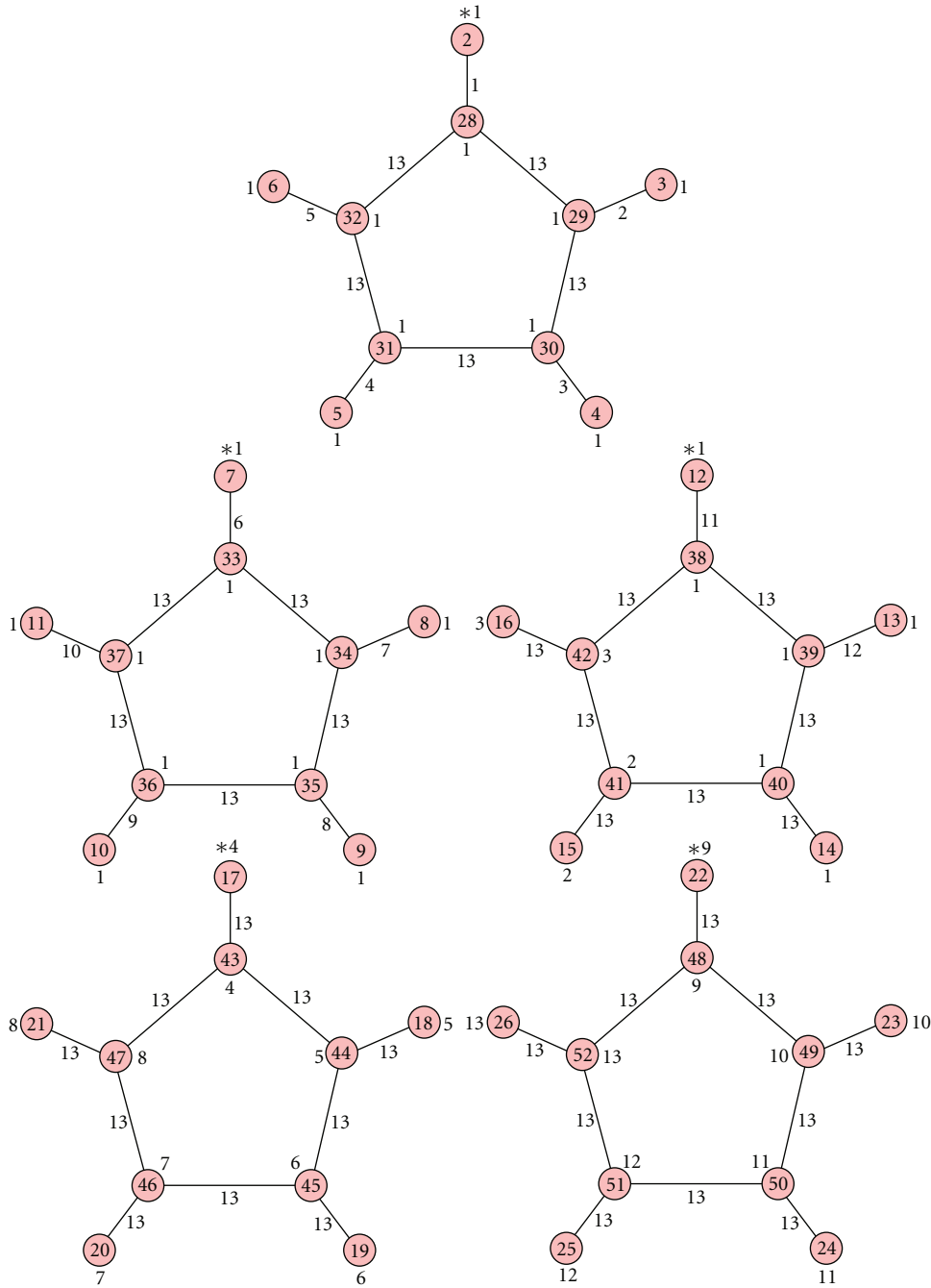


Figure 1: Vertex irregular total 13 labelings of $5M_5$.

vertex irregular labeling. Suppose the disjoint union of the nonisomorphic sun graphs with consecutive number of pendants $\cup_{i=1}^s M_{i+2}$ has the set of vertices

$$V(\cup_{i=1}^s M_{i+2}) = \{u_{1,1}, u_{1,2}, u_{1,3}, u_{2,1}, u_{2,2}, u_{2,3}, u_{2,4}, \dots, u_{s,1}, u_{s,2}, \dots, u_{s,s+2}, v_{1,1}, v_{1,2}, v_{1,3}, v_{2,1}, v_{2,2}, v_{2,3}, v_{2,4}, \dots, v_{s,1}, v_{s,2}, \dots, v_{s,s+2}\} \quad (2.8)$$

and the set of edges

$$E\left(\bigcup_{i=1}^s M_{i+2}\right) = \{u_{1,1}v_{1,1}, \dots, u_{1,3}v_{1,3}, u_{2,1}v_{2,1}, \dots, u_{2,4}v_{2,4}, \dots, u_{s,1}v_{s,1}, u_{s,2}v_{s,2}, \dots, u_{s,s+2}v_{s,s+2}\} \\ \cup \{v_{1,1}v_{1,2}, \dots, v_{1,3}v_{1,1}, v_{2,1}v_{2,2}, \dots, v_{2,4}v_{2,1}, \dots, v_{s,1}v_{s,2}, \dots, v_{s,s+2}v_{s,1}\}. \quad (2.9)$$

The labels of the edges and the vertices of $\bigcup_{i=1}^s M_{i+2}$ are described in the following formulas:

$$\lambda(u_{i,j}) = \begin{cases} 1 & \text{for } i = 1, 2, \dots, \left(\left\lfloor \frac{2s}{3} \right\rfloor - 1\right), j = 1, 2, \dots, i + 2, \\ & i = \left\lfloor \frac{2s}{3} \right\rfloor; \\ & j = 1, 2, \dots, \left(\left\lfloor \frac{s(s+5)+2}{4} \right\rfloor - \frac{([2s/3]-1)([2s/3]+4)}{2}\right) \\ 1 + j + \left(\frac{i^2 + 3i - 4}{2}\right) & \\ - \left\lfloor \frac{s(s+5)+2}{4} \right\rfloor & \text{for other } i, j, \end{cases}$$

$$\lambda(v_{i,j}) = \begin{cases} 1 & \text{for } i = 1, 2, \dots, \left(\left\lfloor \frac{2s}{3} \right\rfloor - 1\right), j = 1, 2, \dots, i + 2, \\ & i = \left\lfloor \frac{2s}{3} \right\rfloor; \\ & j = 1, 2, \dots, \left(\left\lfloor \frac{s(s+5)+2}{4} \right\rfloor - \frac{([2s/3]-1)([2s/3]+4)}{2}\right) \\ 1 + j + \left(\frac{i^2 + 3i - 4}{2}\right) & \\ - \left\lfloor \frac{s(s+5)+2}{4} \right\rfloor & \text{for other } i, j, \end{cases}$$

$$\lambda(u_{i,j}v_{i,j}) = \begin{cases} j + \left(\frac{i^2 + 3i - 4}{2}\right) & \text{for } i = 1, 2, \dots, \left(\left\lfloor \frac{2s}{3} \right\rfloor - 1\right), j = 1, 2, \dots, i + 2, \\ & i = \left\lfloor \frac{2s}{3} \right\rfloor; \\ & j = 1, 2, \dots, \left(\left\lfloor \frac{s(s+5)+2}{4} \right\rfloor - \frac{([2s/3]-1)([2s/3]+4)}{2}\right) \\ \left\lfloor \frac{s(s+5)+2}{4} \right\rfloor & \text{for other } i, j, \end{cases}$$

$$\lambda(v_{i,j}v_{i,j+1}) = \left\lfloor \frac{s(s+5)+2}{4} \right\rfloor, \quad \text{for } i = 1, 2, \dots, s, j = 1, 2, \dots, i + 2. \quad (2.10)$$

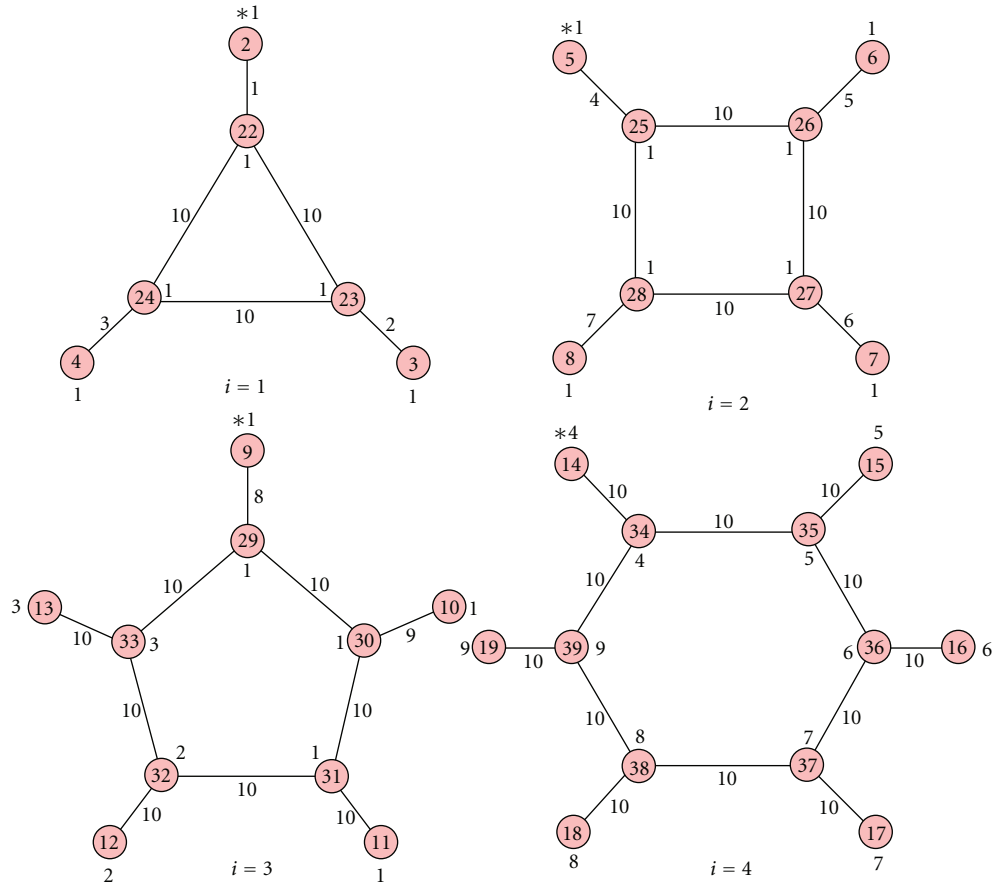


Figure 2: Vertex irregular total 10 labelings of $M_3 \cup M_4 \cup M_5 \cup M_6$.

The weights of the vertices $u_{i,j}$ and $v_{i,j}$ of $\bigcup_{i=1}^s M_{i+2}$ are

$$wt(u_{i,j}) = \frac{i^2 + 3i + 2j - 2}{2}, \quad \text{for } i = 1, 2, \dots, s, \quad j = 1, 2, \dots, i + 2, \tag{2.11}$$

$$wt(v_{i,j}) = \frac{i^2 + 3i + 2j - 2}{2} + 2 \left\lceil \frac{s(s+5) + 2}{4} \right\rceil \quad \text{for } i = 1, 2, \dots, s, \quad j = 1, 2, \dots, i + 2.$$

It is easy to see that the weights calculated at vertices are distinct. So, the labeling is vertex irregular total. Therefore $tv_s(\bigcup_{i=1}^s M_{i+2}) = \lceil (s(s+5) + 2)/4 \rceil$ for $s \geq 1$ dan $n \geq 3$. \square

Figure 2 illustrates the vertex irregular total 10 labelings of the disjoint union 4 consecutive nonisomorphic sun graphs $M_3 \cup M_4 \cup M_5 \cup M_6$.

Finally, we conclude this section with a result on the total vertex irregularity strength of disjoint union of two nonisomorphic sun graphs as follows.

Theorem 2.5. *The total vertex irregularity strength of disjoint union of two nonisomorphic sun graphs is $tv_s(M_k \cup M_n) = \lceil (k + n + 1)/2 \rceil$, for $n > k \geq 3$.*

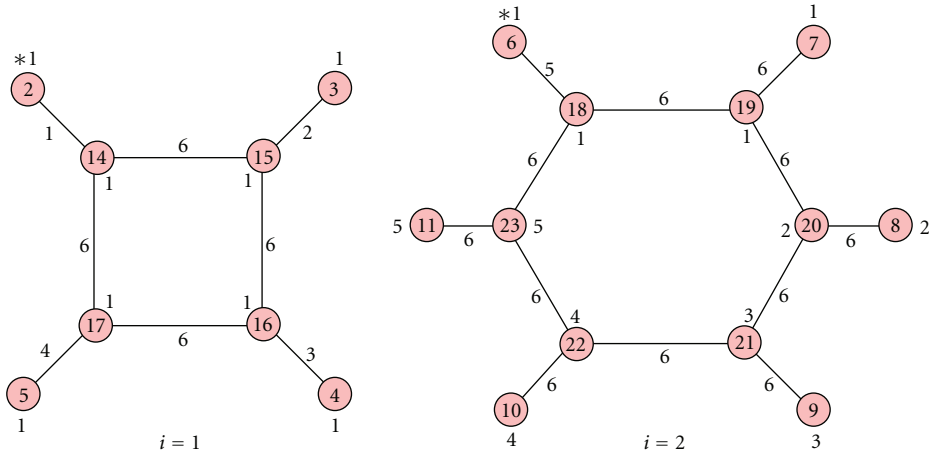


Figure 3: Vertex irregular total 6 labelings of $M_4 \cup M_6$.

Proof. Using Lemma 2.1, we have $\text{tvs}(\bigcup_{i=1}^s M_{i+2}) \geq \lceil (k+n+1)/2 \rceil$. To show that $\text{tvs}(M_k \cup M_n) \leq \lceil (k+n+1)/2 \rceil$, we label the vertices and edges of $M_k \cup M_n$ as a vertex irregular total k -labeling. Suppose the disjoint union of the nonisomorphic sun graphs with different pendant $M_k \cup M_n$ has the set of vertices

$$V(sM_n) = \{u_{1,1}, u_{1,2}, \dots, u_{1,n}, u_{2,1}, u_{2,2}, \dots, u_{2,n}, v_{1,1}, v_{1,2}, \dots, v_{1,n}, v_{2,1}, v_{2,2}, \dots, v_{2,n}\} \quad (2.12)$$

and the set of edges

$$E(sM_n) = \{u_{1,1}v_{1,1}, u_{1,2}v_{1,2}, \dots, u_{1,n}v_{1,n}, u_{2,1}v_{2,1}, u_{2,2}v_{2,2}, \dots, u_{2,n}v_{2,n}\} \\ \cup \{v_{1,1}v_{1,2}, v_{1,2}v_{1,3}, \dots, v_{1,n}v_{1,1}, v_{2,1}v_{2,2}, v_{2,2}v_{2,3}, \dots, v_{2,n}v_{2,1}\}. \quad (2.13)$$

The labels of the edges and the vertices of $M_k \cup M_n$ are described in the following formulas:

$$\lambda(u_{i,j}) = \begin{cases} 1 & \text{for } i = 1; j = 1, 2, \dots, k, \\ & i = 2; j = 1, 2, \dots, \left(\left\lceil \frac{k+n+1}{2} \right\rceil - 1\right) \\ 1 + j + (i-1)k \\ + \left(\left\lceil \frac{k+n+1}{2} \right\rceil\right) & \text{for other } i, j, \end{cases}$$

$$\lambda(v_{i,j}) = \begin{cases} 1 & \text{for } i = 1; j = 1, 2, \dots, k, \\ & i = 2; j = 1, 2, \dots, \left(\left\lceil \frac{k+n+1}{2} \right\rceil - 1\right) \\ 1 + j + (i-1)k \\ + \left(\left\lceil \frac{k+n+1}{2} \right\rceil\right) & \text{for other } i, j, \end{cases}$$

$$\lambda(u_{i,j}v_{i,j}) = \begin{cases} j + (i-1)k & \text{for } i = 1; j = 1, 2, \dots, k, \\ i = 2; j = 1, 2, \dots, \left(\left\lfloor \frac{k+n+1}{2} \right\rfloor - 1\right) \\ \left\lfloor \frac{k+n+1}{2} \right\rfloor & \text{for other } i, j, \end{cases}$$

$$\lambda(v_{i,j}v_{i,j+1}) = \left\lfloor \frac{k+n+1}{2} \right\rfloor, \text{ for } i = 1; j = 1, 2, \dots, k, i = 2; j = 1, 2, \dots, n. \quad (2.14)$$

The weights of the vertices $u_{i,j}$ and $v_{i,j}$ of $M_k \cup M_n$ are

$$wt(u_{i,j}) = 1 + j + (i-1)k, \quad \text{for } i = 1, 2, j = 1, 2, \dots, n,$$

$$wt(v_{i,j}) = 1 + j + (i-1)k + 2\left(\left\lfloor \frac{k+n+1}{2} \right\rfloor\right), \quad \text{for } i = 1, 2, j = 1, 2, \dots, n. \quad (2.15)$$

It is easy to see that the weights calculated at vertices are distinct. So, the labeling is vertex irregular total. Therefore $tv_s(M_k \cup M_n) = \lfloor (k+n+1)/2 \rfloor$ for $n > k \geq 3$. \square

Figure 3 illustrates the vertex irregular total 6 labelings of the disjoint union of 2 nonisomorphic sun graphs $M_4 \cup M_6$.

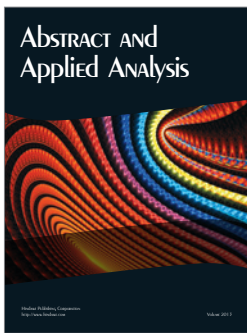
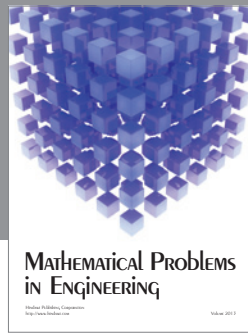
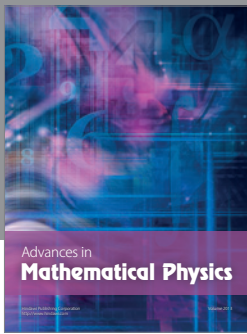
3. Conclusion

We conclude this paper with the following conjecture for the direction of further research in this area.

Conjecture 1. The total vertex irregularity strength of disjoint union of any sun graphs is $tv_s(\bigcup_{i=1}^s M_{n_i}) = \lfloor ((\sum_{i=1}^s n_i) + 1)/2 \rfloor$, for $s \geq 1$, $n_{i+1} \geq n_i$ and $1 \leq i \leq s$.

References

- [1] M. Bača, S. Jendrol', M. Miller, and J. Ryan, "On irregular total labellings," *Discrete Mathematics*, vol. 307, no. 11-12, pp. 1378–1388, 2007.
- [2] K. Wijaya, Slamain, Surahmat, and S. Jendrol', "Total vertex irregular labeling of complete bipartite graphs," *Journal of Combinatorial Mathematics and Combinatorial Computing*, vol. 55, pp. 129–136, 2005.
- [3] K. Wijaya and Slamain, "Total vertex irregular labelings of wheels, fans, suns and friendship graphs," *Journal of Combinatorial Mathematics and Combinatorial Computing*, vol. 65, pp. 103–112, 2008.
- [4] A. Ahmad, Nurdin, and E. T. Baskoro, "On total irregularity strength of generalized Halin graph," to appear in *Ars Combinatoria*.
- [5] Nurdin, E. T. Baskoro, A. N. M. Salman, and N. N. Gaos, "On total vertex-irregular labellings for several types of trees," *Utilitas Mathematica*, vol. 83, pp. 277–290, 2010.
- [6] Nurdin, E. T. Baskoro, A. N. M. Salman, and N. N. Gaos, "On the total vertex irregularity strength of trees," *Discrete Mathematics*, vol. 310, no. 21, pp. 3043–3048, 2010.
- [7] Nurdin, A. N. M. Salman, N. N. Gaos, and E. T. Baskoro, "On the total vertex-irregular strength of a disjoint union of t copies of a path," *Journal of Combinatorial Mathematics and Combinatorial Computing*, vol. 71, pp. 227–233, 2009.
- [8] A. Ahmad and M. Bača, "On vertex irregular total labelings," to appear in *Ars Combinatoria*.



Hindawi

Submit your manuscripts at
<http://www.hindawi.com>

