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Numerical simulation of mass in core decay of radioactive substance Thorium-232 series

Yushardi¹, B Supriadi¹, D S Tresnowati¹ and Z R Ridlo¹

¹Physics Education Departement, University of Jember, Jember, Indonesia

Email: yusagk@gmail.com

Abstract. Radioactive elements are elements whose isotopes have unstable nuclei, so they have the possibility to emit radiation through the decay process. Thorium-232 is one of the most stable radioactive elements with isotopes that have abundance in nature 3 times more than Uranium. The purpose of this research is to find the mass of remaining atoms in the Th-232 series which experience very slow decay through the alpha and beta decay process to become a stable core Pb-208. The settlement of radioactive chain decays usually uses very complicated differential concepts. The matrix algebra method is a solution to facilitate the completion of radioactive chain decay which can be implemented computationally through matlab programming. Research that occurred on Th-232 with a mass of 7 gram for $1,34 \times 10^{10}$ years ago showed that the parent nuclide had the most decay residual mass compared to its derived nuclides, because the half-life of the derived nuclide was shorter than the decay time. The number of decay residual mass owned by the parent nuclide is 3,596 grams. The results of this research indicate that the mass of residual radioactive decay is influenced by the half-life of each nuclide and the length of decay.

1. Introduction

The universe is a collection of matters composed of elements. Element is a collection of atoms which is the smallest part and cannot be divided. Each atom is composed of nuclei (protons and neutrons) and electrons around the nucleus. The stability of an element can be seen from the number of protons and neutrons that make up the nucleus of an atom. About 270 elements with stable nuclei are found in nature, while about 4½ times that of stable elements are elements with unstable atomic nuclei [1]. The unstable atomic nucleus is caused by an excess of energy, thus energy release events occur in the form of radiation. Elements with unstable nuclei (radioactive elements) will undergo a process of spontaneous change from an unstable atomic nucleus (radionuclides) to a stable nucleus, known as radioactivity[2].

Thorium is one of the radioactive elements that can still be found in the universe, with an abundance of 3 times more than Uranium. There are 6 unstable isotopes owned by Thorium, namely ^{228}Th , ^{229}Th , ^{230}Th , ^{231}Th , ^{232}Th and ^{234}Th . Th-232 has the most stable isotope with a half-life equivalent to the age of the universe, or about 14,05 billion years so that it can be used as a date for age of the earth and can be alternative fuel for nuclear reactors [3]. Thorium-232 is a radioactive element that has an atomic number of 90 and a mass number of 232,0381 in the period 7 actinide series periodic table.

Thorium-232 will undergo a process of decay from an unstable atomic nucleus to be stable atomic nucleus namely Lead-208 through emitting alpha and beta rays. Alpha decay occurs when the nucleus



of an atom emits alpha particles in the form of a Helium nucleus (4_2He) which has 2 protons and 2 neutrons, while beta decay occurs when an atomic nucleus emits beta particles in the form of positrons (${}^0_{-1}e$) or electrons (${}^0_{+1}e$). Th-232 decay that occurs at any time unit is called radioactive activity, depending on the number of N nuclei and the decay constant λ [4]. Thorium-232 decay process can be written as follows:

$$\frac{dN}{N} = -\lambda dt \tag{1}$$

$$\int_{N_0}^N \frac{dN}{N} = -\lambda \int_{N_0}^N dt \tag{2}$$

$$N = N_0 e^{-\lambda t} \tag{3}$$

N is the number of atomic nuclei that decays each time, depending on the number of nuclei initially N_0 , the decay constant (disintegration) λ , and the intervals of decay t [5]. Equation (3) is the law of radioactive decay.

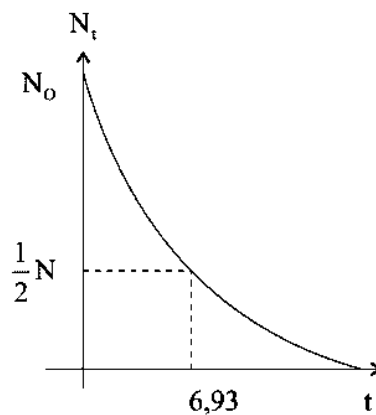


Figure 1. Graph of radioactive decay.

The number of nuclei can indicate the large mass in the nuclide. The mass of an atom's nucleus is very dependent on the number of protons, electrons, and neutrons that make up the atom. The relationship between atomic mass (m), atomic mole (Mr), and the number of nuclei (N) is obtained using the concept of Avogadro numbers as follows:

$$n = \frac{m}{Mr} \tag{4}$$

$$n = \frac{N}{N_A} \tag{5}$$

if equation (4) is equal to equation (5), then

$$m = Mr \times \frac{N}{N_A} \tag{6}$$

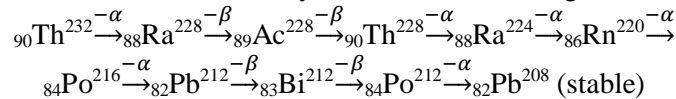
where m is the decayed atomic mass that affected by the number of remaining atoms N over the exponential time t . The Thorium-232 series with mass number A will experience a decay of $4n$ from the unstable nucleus Th-232 to the stable nucleus of Pb-208, meaning that each mass number A will always be divisible by 4. Each nuclide in the Thorium-232 series has a half-life as in Table 1 below [6].

Table 1. Half-life Th-232 Series.

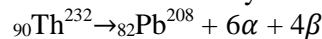
Decay to	Nuclide	Half-life ($t_{1/2}$)
1	${}_{90}Th^{232}$	$1,39 \times 10^{10}$ years
2	${}_{88}Ra^{228}$	6,7 years
3	${}_{89}Ac^{228}$	6,13 hours
4	${}_{90}Th^{228}$	1,91 years
5	${}_{88}Ra^{224}$	3,64 days

6	${}_{86}\text{Rn}^{220}$	51,5 seconds
7	${}_{84}\text{Po}^{216}$	0,16 seconds
8	${}_{82}\text{Pb}^{212}$	10,6 hours
9	${}_{83}\text{Bi}^{212}$	60,5 minutes
10	${}_{84}\text{Po}^{212}$	3×10^{-7} seconds
11	${}_{82}\text{Pb}^{208}$	-

The half-life is the time needed for the nucleus to decay into half of the original nucleus[7]. The Th-232 sequence undergoes 10 main branchless decay stages, 6 alpha decay (α) and 4 beta decay (β) which is then known as the Thorium chain decay, such as the following reaction :



The decay reaction in Thorium series can be written briefly as :



The chain decay process can be completed using the following differential concepts:

$$\frac{dN_1}{dt} = -\lambda_1 N_1 \tag{7}$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \tag{8}$$

$$\frac{dN_3}{dt} = \lambda_2 N_2 - \lambda_3 N_3 \tag{9}$$

$$\vdots = \quad \vdots - \quad \vdots$$

$$\frac{dN_n}{dt} = \lambda_{n-1} N_{n-1} - \lambda_n N_n \tag{10}$$

Equation (10) is the Bateman equation for serial radioactive decay that occurs in time t , with $n = 1, 2, 3, \dots, n$ is the number of decays that occur in radioactive sequence[8].

Highly accuracy is needed to solve chain of decay reaction because the solution is complex [9]. The advantage of this research is to provide solutions that can facilitate the completion of chain decay using the matrix algebra method. Based on this description, this research aimed to determine the amount of residual mass during decay that occurs in Thorium-232 series as an alternative nuclear fuel using the matrix algebraic method.

2. Method

This research is included in the type of theoretical study research. The method used in this research is the matrix algebra method. The matrix algebra method is a method that can easily solve a system of linear differential equation with n unknown functions using constant coefficients[10]. The stages carried out in this research include, (1) Writing the Bateman equation in the form of a matrix, (2) Writing the N' equation using eigen value solutions, (3) Writing each matrix component into the N' equation, (4) Limiting the number of decays in Thorium-232 series, (5) Performing the calculation N' , (6) Converting results of N' into Avogadro number equation to get the residual decay mass.

The radioactive decay chain using the matrix algebra method can be solved by the Bateman equation as a solution to the differential equation system[11]. Equation (10) can be written in the form of a matrix as follows :

$$N' = AN \tag{11}$$

Where each component of the vector can be broken down into:

$$N' = \begin{bmatrix} N'_1 \\ N'_2 \\ N'_3 \\ \vdots \\ N'_n \end{bmatrix}, \tag{12}$$

$$N = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ \vdots \\ N_n \end{bmatrix}, \tag{13}$$

$$A = \begin{bmatrix} -\lambda_1 & 0 & 0 & \dots & 0 \\ \lambda_1 & -\lambda_2 & 0 & \dots & 0 \\ 0 & \lambda_2 - \lambda_3 & \dots & \dots & 0 \\ \dots & \dots & \ddots & \ddots & \dots \\ 0 & 0 & \lambda_{n-1} & \dots & -\lambda_n \end{bmatrix} \tag{14}$$

N' is a column matrix that states the number of nuclei remaining decay, while N is the matrix of the initial atomic nucleus N_0 , and A is the decay constant matrix λ . The matrix solution for differential resolution under the condition N_0 can be analogized using the exponential matrix [12] as follows:

$$N' = e^{At}N_0 \tag{15}$$

Equation (15) is defined from Taylor's exponential expansion and equals equation (3) which is the law of radioactive decay equation.

$$e^{At} = 1 + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \dots \tag{16}$$

Radioactive decay is a system of differential homogeneous differential equations, so one of the solutions uses the eigenvalues as follows:

$$AV = VD \tag{17}$$

$$A = VDV^{-1} \tag{18}$$

by substituting equation (18) for equation (16), it is obtained:

$$e^{At} = Ve^{Dt}V^{-1} \tag{19}$$

so equation (15) becomes,

$$N' = Ve^{Dt}V^{-1}N_0 \tag{20}$$

Equation (20) is an equation similar to the Bateman equation and is a general equation of radioactive chain decay, where V is the eigenvector matrix, V^{-1} represents the inverse matrix of eigenvector V , and e^{Dt} is the negative exponential diagonal matrix of lambda.

$$e^{Dt} = \begin{bmatrix} e^{-\lambda_1 t} & 0 & 0 & 0 & 0 \\ 0 & e^{-\lambda_2 t} & 0 & 0 & 0 \\ 0 & 0 & e^{-\lambda_3 t} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & e^{-\lambda_n t} \end{bmatrix} \tag{21}$$

Eigenvector V value and inverse eigenvector V^{-1} can use the following equation :

$$V = \begin{bmatrix} 1 & 0 & 0 & 0 \\ S_{2,1} & 1 & 0 & 0 \\ S_{3,1} & S_{3,2} & 1 & 0 \\ \dots & \dots & \dots & \dots \\ S_{n,1} & S_{n,2} & S_{n,3} & S_{n,n-1} & 1 \end{bmatrix} \tag{22}$$

$$V^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ T_{2,1} & 1 & 0 & 0 \\ T_{3,1} & T_{3,2} & 1 & 0 \\ \dots & \dots & \dots & \dots \\ T_{n,1} & T_{n,2} & T_{n,3} & T_{n,n-1} & 1 \end{bmatrix} \tag{23}$$

According to L. Moral and A.F. Pacheco (2003) states that the values of $S_{x,y}$ and $T_{x,y}$ can be represented from the equation[13] as follows:

$$F_{q,r}^p = \frac{\lambda_r}{\lambda_q - \lambda_p} \tag{24}$$

So that each value of $S_{x,y}$ and $T_{x,y}$ is,

$$S_{x,y} = F_{x,x-1}^y F_{x-1,x-2}^y \dots F_{y+1,y}^y \tag{25}$$

$$T_{x,y} = F_{y,y}^x F_{y+1,y+1}^x \dots F_{x-1,x-1}^x \tag{26}$$

by substituting equations (21), (22), and (23) for equation (20), the value of N' with order $n = 3$ is obtained as follows:

$$\begin{bmatrix} N_1' \\ N_2' \\ N_3' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ S_{2,1} & 1 & 0 \\ S_{3,1} & S_{3,2} & 1 \end{bmatrix} \times \begin{bmatrix} e^{-\lambda_1 t} & 0 & 0 \\ 0 & e^{-\lambda_2 t} & 0 \\ 0 & 0 & e^{-\lambda_3 t} \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ T_{2,1} & 1 & 0 \\ T_{3,1} & T_{3,2} & 1 \end{bmatrix} \times \begin{bmatrix} N_1(0) \\ 0 \\ 0 \end{bmatrix} \tag{27}$$

through substitution of $S_{x,y}$ and $T_{x,y}$ into matrix (27), the final equation is obtained as follows :

$$N_1' = e^{-\lambda_1 t} \times N_1(0) \tag{28}$$

$$N_2' = (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \times \frac{\lambda_1}{\lambda_1 - \lambda_2} N_1(0) \tag{29}$$

$$N_3' = \lambda_1 \times \lambda_2 \left(\frac{e^{-\lambda_1 t}}{(\lambda_3 - \lambda_1)(\lambda_2 - \lambda_1)} + \frac{e^{-\lambda_2 t}}{(\lambda_3 - \lambda_2)(\lambda_1 - \lambda_2)} + \frac{e^{-\lambda_3 t}}{(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)} \right) \times N_1(0) \tag{30}$$

The value of N' can be used to find residual mass of decay in the Thorium-232 series by entering each value obtained in equation (6). The data used to obtain the residual mass of decay comes from previous research data, that is 7 grams of Thorium-232 with an initial atomic number of $1,817025519 \times 10^{22}$ for $1,34 \times 10^{10}$ years ago. The data will be simulated using the matrix algebra method assisted by the matlab program. The data generated can be validated using calculations with general concept of radioactive decay, which is differential equation.

3. Result and discussion

Research conducted on 7 grams showed that the parent nuclide has the most remaining decay mass compared to its derivative nuclides, because the half-life owned by derived nuclides in Thorium-232 series is very short compared to the decay time. Nuclides which have a half-life shorter than the decay time $t_{1/2} \ll t_{\text{decay}}$, then the nuclide will be more easily depleted or close to 0 ($m \rightarrow 0$). Seen in the Table 2, the parent nuclide Thorium-232 with a half-life of $1,39 \times 10^{10}$ years, has a maximum residual decay mass of 3,596 grams. The first derivative Radium-228 with the longest half-life of 6,7 years, has a residual mass of decay of $1,703 \times 10^{-9}$ grams. The second derivative Actinium-228 with a half-life of $6,988 \times 10^{-4}$ years, has a residual mass of decay of $1,774 \times 10^{-13}$ grams. The third derivative Thorium-228 with a half-life of 1,91 years, has a residual mass of decay of $4,78 \times 10^{-10}$ grams. The fourth derivative Radium-224 with a half-life of $9,97 \times 10^{-3}$ years, has a residual mass of decay of $2,49 \times 10^{-12}$ grams. The fifth derivative Radon-220 with a half-life of $1,63 \times 10^{-6}$ years, has a residual mass of decay of $3,96 \times 10^{-16}$ grams. The sixth derivative Polonium-216 with a half-life of $5,074 \times 10^{-9}$ years, has a residual mass of decay of $1,21 \times 10^{-18}$ grams. The seventh derivative Lead-212 with a half-life of $1,208 \times 10^{-3}$ years, has a residual mass of decay of $2,82 \times 10^{-13}$ grams. The eighth derivative Bismuth-212 with a half-life of $1,15 \times 10^{-4}$ years, has a residual mass of decay of $2,69 \times 10^{-14}$ grams. The ninth derivative Polonium-212 with the shortest half-life of $9,15 \times 10^{-15}$ years, has a residual mass of decay of $2,31 \times 10^{-24}$ grams. The tenth derivative or stable nuclide Lead-208 with a half-life of 0,693 years, has a residual mass of decay of $1,60 \times 10^{-10}$ grams.

Table 2. The Mass Product Residual Decay of Thorium-232.

Nuclide	Half-life (years)	mass(grams)
Thorium-232	$1,39 \times 10^{10}$	3,596
Radium-228	6,7	$1,702 \times 10^{-9}$
Actinium-228	$6,988 \times 10^{-4}$	$1,774 \times 10^{-13}$
Thorium-228	1,91	$4,78 \times 10^{-10}$
Radium-224	$9,97 \times 10^{-3}$	$2,49 \times 10^{-12}$
Radon-220	$1,63 \times 10^{-6}$	$3,96 \times 10^{-16}$
Polonium-216	$5,074 \times 10^{-9}$	$1,21 \times 10^{-18}$

Lead-212	$1,208 \times 10^{-3}$	$2,82 \times 10^{-13}$
Bismut-212	$1,15 \times 10^{-4}$	$2,69 \times 10^{-14}$
Polonium-212	$9,51 \times 10^{-15}$	$2,31 \times 10^{-24}$
Lead-208	0,693	$1,60 \times 10^{-10}$

The amount of residual mass from radioactive decay in Thorium-232 series is not only shown in the calculated data, but also supported by visualization with Figure 2.

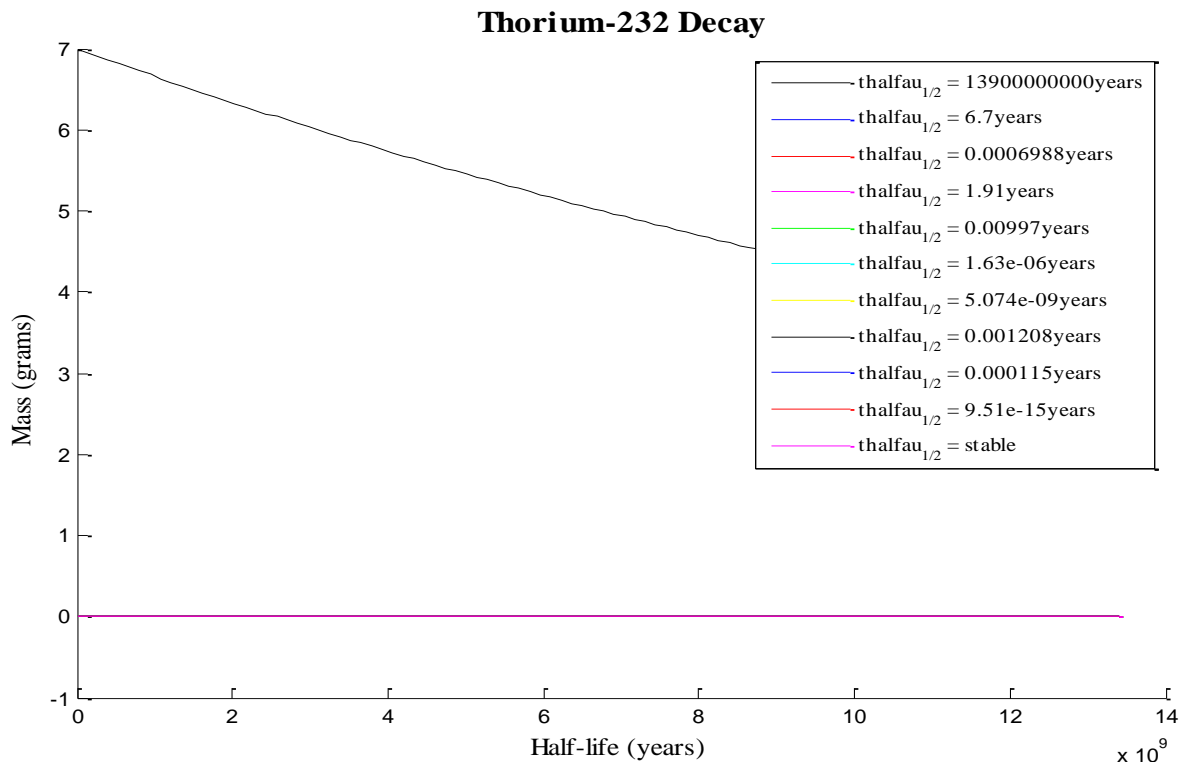


Figure 2. Graph of the relationship of mass to decay time in thorium-232.

Figure 2 shows the mass of nuclide in Thorium-232 series which initially had a mass of 7 grams will decay over an exponential time so that only half is left, namely the mass of parent nuclide at 3,596 grams, while the ten mass of derived nuclide is close to zero. Figure 2 above shows there is only one line belonging to the parent nuclide with black curved downward shape because the line is a graph with an exponential function, while the straight line at point 0 is the line belonging to the derived nuclide which are terminated by stable nuclide coloured Magenta. The residual mass of decay depends on the half-life of each nuclide in Thorium-232 series and the length of decay. The residual mass of decay is inversely proportional to decay time and is directly proportional to the half-life of each nuclide. The longer decay time, then more decays so that the remaining mass during decay is less. The longer half-life of the nuclide, so greater the residual mass produced during decay.

The radioactive chain decay that occurs in Thorium-232 series undergoes 10 major decay stages without branches namely 6 alpha decay (α) through the emitting of alpha rays in the form a Helium nucleus which reduces 2 protons and 2 neutron parent nuclides, and 4 beta decay (β) through conversion spontaneously neutrons become protons. The mass remaining during chain decay in Thorium-232 series is in accordance with the law of radioactive decay which states that the radioactive element will decay in half over an exponential period of time. Nuclides with shorter half-lives than decay times have less residual mass compared to nuclides that have longer half-lives.

The remaining mass during the Thorium-232 chain decay to become Lead-208 not only produces solid tangible elements. There is an element in the gaseous form produced in the Th-232 decay, namely Radon-220 with a half-life of 51,5 seconds which then decays into a radioactive element in solid form again until it becomes a stable element. The difference in simulation results with the data source obtained an error of 2% so that the data produced in the study is still at an allowable tolerance value of under 5%.

4. Conclusion

Based on the discussion it can be concluded that the residual mass generated in the chain decay using the matrix algebraic method at 7 grams of Thorium-232 for $1,34 \times 10^{10}$ years shows that the parent nuclide has a residual mass of 3,596 grams while the residual mass of the derived nucleus is close to zero. The residual mass of decay depends on the half-life of each nuclide in the radioactive sequence and the length of decay. The longer the decay time occurs, the more mass decays, and the longer the half-life of the nuclide, the more mass of decay residue is produced.

The results of this study are expected to be a benchmark for similar studies. The results obtained in this study are expected to be a source of information about alternative nuclear fuels left in the universe. It is hoped that this research will also get a response for subsequent researchers to continue with other methods and different variables.

Acknowledgment

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