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The local edge metric dimension of graph

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Abstract. In this paper, we introduce a new notion of graph theory study, namely a local edge metric dimension. It is a natural extension of metric dimension concept. $d_G(e, v) = min\{d(x, v), d(y, v)\}$ is the distance between the vertex v and the edge xy in graph G. A non empty set $S \subset V$ is an edge metric generator for G if for any two edges $e_1, e_2 \in E$ there is a vertex $k \in S$ such that $d_G(k, e_1 \neq d_G(k, e_2))$. The minimum cardinality of edge metric generator for G is called as edge metric dimension of G, denoted by $dim_E(G)$. The local edge metric dimension of G, denoted by $diml_E(G)$, is a local edge metric generator of G if $r(xk|S) \neq r(yk|S)$ for every pair xk, ky of adjacent edges of G. Our concern in this paper is investigating some results of local edge metric dimension on some graphs.

1. Introduction

The concept of Metric dimension was introduced in 1976. It raised from the concept of resolving set and minimum resolving set. The application of this study can be used in navigation system, chemistry, and optimization. In this regards, there has been some results carried out extensively in numerous types of metric dimension. The researchers have found and extend the types of metric dimension, such as partition dimension, star partition dimension, edge metric dimension, etc. In this Paper, we introduce a new notion of metric dimension which was introduced by Slater [12], Melter and Harrary [8, 9, 3], namely local edge metric dimension of graphs. We can see [1, 2, 3, 4, 5, 6, 7] for further terminology and definition of graph. Let $d_G(e, v) = min\{d(x, v), d(y, v)\}$ be a distance between the vertex v and the edge xy. A vertex $k \in V$ distinguishes two edges $e_1e_2 \in E$ if $(d_G(k, e_1) \neq d_G(k, e_2)$. A set S of vertices in G is an edge metric generator for G if every two edges of G have different representation respect to S. The edge metric dimension of G, denoted by $dim_E(G)$, is the minimum cardinality of edge metric generator for G. Some research related to edge metric dimension can be identified in [10, 11]. In this paper, we concern on local edge metric dimension of graph. In local edge metric dimension, the neighbourhood edges may not have the same representation. The local edge metric dimension of G, denoted by $diml_E(G)$, is the minimum cardinality of local edge metric generator of G if $r(vc|S) \neq r(cu|S)$ for every pair vc, cu of adjacent edges in G. In this paper, we focus on investigating the local edge metric dimension of some special graph.

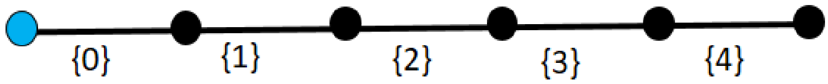

Figure 1. The local edge metric dimension of path graph order 6 and its representation respect to S

2. Main Results

This paper focus to find the exact value of the local edge metric dimension on some graphs. We begin this section with the result of local edge metric dimension of path graph in the following theorem.

Theorem 2.1 *The local edge metric dimension is* 1 *if only if* $G = P_n$ *for* G *is a connected with* n *vertices.*

Proof. In order to prove this theorem, we will divide the proof into two cases.

Case 1: If the local edge metric dimension is 1 then $G = P_n$. The graph P_n is a connected graph with $|V| = n$ and $|E| = n - 1$. The vertex set $V(P_n) = \{x_1, x_2, \ldots, x_n\}$ and edge set $E(P_n) = \{x_ix_{i+1}; 1 \leq i \leq n-1\}$. Choose $S = \{x_1\}$, where S is the local edge metric generator of G, thus $r(e|S) = d(e, S)$, $\forall e \in E(G)$. Since S is the local edge metric generator, thus $r(e|S)$ for every element of G consist of $1-$ tupple with the element of $r(e|S)$ less than n. It is easy to see that $r(e|S) = d(e, S)$, for $\forall e \in E(G)$ has different representation, thus $\exists x_i \in V(G)$ with $d(x_i, v_1) = n-1$. Since $d(x_i, v_1) = n-1$, there exist a path with length $n-1$ in G. Furthermore, Since there exist one path with the length $n-1$ in G, thus G is P_n. It concludes that If the local edge metric dimension is 1 then $G = P_n$

Case 2: If $G = P_n$, then the local edge metric dimension is 1

The graph P_n is a connected graph with $|V| = n$ and $|E| = n - 1$. The vertex set $V(P_n) = \{x_1, x_2, \ldots, x_n\}$ and edge set $E(P_n) = \{x_ix_{i+1}; 1 \leq i \leq n-1\}$. Choose x_1 as the initial vertex of P_n. Suppose that $S = x_1$, it is easy to see that x_1 is the local edge metric generator. Since $(x_1, x_2, \ldots, x_n)$ is a path, thus $d(x_ix_{i+1}|x_1) = i - 1$ for $2 \leq i \leq n$. Hence, $\forall e_1, e_2 \in E(G)$, $r(e_1|S) \neq r(e_2|S)$, where e_1, e_2 is an adjacent edges. Thus S is the local metric generator of P_n. Furthermore, we will prove that $S = v_1$ is the minimum cardinality of the local metric generator. Since there does not exist the local metric generator less than 1, thus $S = v_1$ is the minimum local metric generator. It implies that if $G = P_n$ then the local edge metric dimension is 1.

Based on case 1 and case 2, it can be concluded that the local edge metric dimension is 1 if only if $G = P_n$. The figure of the local edge metric dimension of path graph order 6 can be seen in Figure 1.

Theorem 2.2 *Let* L_n *be a ladder graph graph with* $n \geq 2$. *The local edge metric dimension of* L_n *is* 2.

Proof : The ladder graph L_n is a graph with $2n$ vertices. The vertex set $V(L_n) = \{x_1, x_2, \ldots, x_n\} \cup \{y_1, y_2, \ldots, y_n\}$ and edge set $E(C_n) = \{x_ix_{i+1} \cup y_iy_{i+1} \cup x_iy_i; 1 \leq i \leq n-1\}$. The cardinality of vertex set and edge set, respectively are $|V(L_n)| = 2n$ and $|E(C_n)| = 3n - 2$. Let we begin the proof by proving the upper bound of local edge metric dimension of ladder graph L_n. By choosing $S = \{x_1, y_1\}$ as the edge metric generator, we will have the representation of all edges in ladder graph respect to S in table 1. According to Table 1, it can be identified

Table 1. The representation of edges in Ladder graph L_n respect to local edge metric generator S

e	$r(e\|S)$	condition
x_ix_{i+1}	$(i-1,i)$	$2 \leq i \leq n-1$
y_ixy_{i+1}	$(i-1,i)$	$2 \leq i \leq n-1$
x_iy_i	$(i,i-1)$	$2 \leq i \leq n-1$

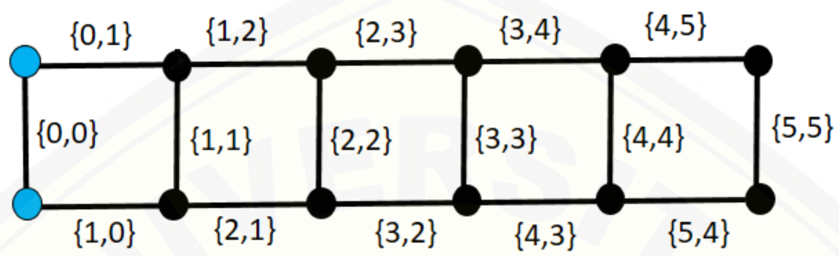

Figure 2. The local edge metric dimension of L_6

that the representation of all adjacent edges in ladder graph n with respect to S are distinct. Since the representation of all edges respect to S are distinct, it can be said that S is the local edge metric generator of ladder graph n with $|S| = 2$. Therefore, we have $diml_E(C_n) \leq 2$.

In the next step, we will prove $diml_E(L_n) \geq 2$. Assume that $diml_E(L_n) < 2$. We take $|S| = 1$ and $x_i \in S$ such that there are 1 vertices in L_n as the element of local edge metric generator. If we take 1 vertex as local edge metric generator in any vertices of L_n, there will be two possible conditions. These condition are as follow:

a) suppose we take 1 vertex in y_i, where $2 \leq i \leq n-1$ as local edge metric generator, there will be an adjacent edges such as y_iy_{i+1} and y_iy_{i-1} which has the same representation respect to S. The representation of y_iy_{i+1} and y_iy_{i-1} respect to S is (0). If we take y_1 as local edge metric generator, y_1y_2 and x_1y_1 will have the same representation. If we take y_n as local edge metric generator, $y_{n-2}y_n$ and x_ny_n will have the same representation. Thus, its contradiction with our definition of local edge metric dimension, where $r(vc|S) \neq r(cu|S)$ for every pair vc, cu of adjacent edges of G.

b) suppose we take 1 vertex in x_i, where $2 \leq i \leq n-1$ as local edge metric generator, there will be an adjacent edges such as x_ix_{i+1} and x_ix_{i-1} which has the same representation respect to S. The representation of x_ix_{i+1} and x_ix_{i-1} respect to S is (0). If we take x_1 as local edge metric generator, x_1x_2 and x_1y_1 will have the same representation. If we take x_n as local edge metric generator, $x_{n-2}x_n$ and x_nx_n will have the same representation. Thus, its contradiction with our definition of local edge metric dimension, where $r(vc|S) \neq r(cu|S)$ for every pair vc, cu of adjacent edges of G.

Based on the analysis above, It can be concluded that $diml_E(L_n) \geq 2$. Since, $diml_E(L_n) \geq 2$ and $diml_E(L_n) \leq 2$, thus $diml_E(L_n) = 2$. The figure 2 showed the $diml_E(L_6)$.

Theorem 2.3 *Let C_n be a cycle graph with $n \geq 2$. The local edge metric dimension of C_n is 2.*

The cycle C_n is a cyclic graph with n vertices. The vertex set $V(C_n) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(C_n) = \{v_1v_n, v_iv_{i+1}; 1 \leq i \leq n-1\}$. The cardinality of vertex set and edge set,

Table 2. The representation of edges in cycle graph C_n respect to local edge metric generator S, for n is odd

e	$r(e\|S)$	condition
v_1v_2	$(0,1)$	
$v_{n-1}v_n$	$(1,0)$	$n \geq 3$
v_1v_N	$(0,0)$	$n \geq 3$
v_iv_{i+1}	$(i-1,i)$	$2 \leq i \leq \frac{n-2}{2}$
v_iv_{i+1}	$(n-i,n-i-1)$	$\frac{n+1}{2} \leq i \leq n-2$

Table 3. The representation of edges in cycle graph C_n respect to local edge metric generator S, for n is even

e	$r(e\|S)$	condition
v_1v_2	$(0,1)$	
$v_{n-1}v_n$	$(1,0)$	$n \geq 3$
v_1v_N	$(0,0)$	$n \geq 3$
v_iv_{i+1}	$(i-1,i)$	$2 \leq i \leq \frac{n-2}{2}$
v_iv_{i+1}	$(n-i,n-i-11)$	$\frac{n}{2} \leq i \leq n-2$

respectively are $|V(C_n)| = n$ and $|E(C_n)| = n$. The proof divided into two cases as follows.

Case 1: For cycle graph order n with n is odd, we will prove that the local edge metric dimension of cycle graph is two.

First, we should identify the upper bound of local edge metric dimension of cycle graph C_n for n is odd by observing table 2. Suppose the edge metric generator is $S = v_1, v_n$. According to Table 2, it can be identified that the representation of all adjacent edges in C_n with respect to S are distinct. Based on that fact, S is the local edge metric generator of star graph order n. Thus, $diml_E(C_n) \leq 2$ for n is odd.

Furthermore, we propose the proof of lower bound of the local edge metric dimension of C_n. suppose we take v_i, there will be an adjacent edges such as v_iv_{i+1} and v_iv_{i-1} which has the same representation respect to S. The representation of v_iv_{i+1} and v_iv_{i-1} respect to S is (0). Thus, its contradiction with our definition of local edge metric dimension, where $r(vc|S) \neq r(cu|S)$ for every pair vc, cu of adjacent edges of G. It can be concluded that $diml_E(C_n) \geq 2$ for n is odd.

Based on that observation, we can say that $diml_E(C_n) = 2$ for n is odd.

Case 2:For cycle graph order n with n is even, it will be investigated that $diml_E(C_n) = 2$.

First, we should identify the upper bound of local edge metric dimension of cycle graph C_n for n is even. By choosing the edge metric generator of C_n is $S = v_1, v_n$, the representation of edges in cycle graph C_n respect to local edge metric generator S, for n is even can be seen in 3. Based on Table 3, all adjacent edges representation of C_n with respect to S are distinct, so S is the local edge metric generator of S_n. Thus, $diml_E(C_n) \leq 2$.

Furthermore, it will be proven that the lower bound of local edge metric dimension of cycle graph order n is 2 or $diml_E(C_n) \geq 2$ for n is even. Assume that $diml_E(C_n) < 2$. We take $|S| = 1$ and $w_i \in S$ such that there are 1 vertices in C_n as the element of local edge metric generator. If we take 1 vertex as local edge metric generator in any vertices of C_n suppose we take x_i, there will be an adjacent edges such as x_ix_{i+1} and x_ix_{i-1} which has the same representation respect to S. The representation of x_ix_{i+1} and x_ix_{i-1} respect to S is (0). Thus, its contradiction with our definition of local edge metric dimension, where $r(vc|S) \neq r(cu|S)$ for every pair vc, cu of adjacent edges of G. It can be concluded that $diml_E(C_n) \geq 2$. Based on that observation, it can be concluded that $diml_E(C_n) = 2$ for n is even.

Theorem 2.4 *Let S_n be a star graph with $n \geq 4$. The local edge metric dimension of S_n is $n-1$.*

Proof. The star graph consist of $n+1$ vertices. It is a tree graph with the set of vertices $\{b\} \cup \{x_i; 1 \leq i \leq n\}$ and the set of edges $E(S_n) = \{bx_i; 1 \leq i \leq n\}$.

In order to prove this theorem, we should analyze the bound of local edge metric dimension of S_n, which respectively are $diml_E(S_n) \leq n-1$ and $diml_E(S_n) \geq n-1$.

First, it will be shown that $diml_E(S_n) \leq n-1$. By choosing the edge metric generator $S = \{x_i, 1 \leq i \leq n-1\}$, we will find the representation of all edges $e \in E(S_n)$ respect to S in 4. Based on Table 4, it can be seen that all adjacent edges representation in star graph order $n+1$ with respect to S are distinct. Hence, S is the local edge metric generator of S_n with $|S| = n-1$. It can be concluded that the upper bound of the local edge metric dimension of star graph is $dim_E(S_n) \leq n-1$.

The next step, we should analyze the lower bound of of local edge metric dimension of star graph order $n+1$ S_n. We will prove $diml_E(S_n) \geq n-1$ by assuming the $diml_E(S_n) < n-1$. Let we take $|S| = n-2$ where $k_i \in S$ such that there are $n-2$ vertices in pendant as the element of edge metric generator in S_n. Since we have $n-2$ vertices in pendant as the element of edge metric generator in S_n, we still have two pendants rest in S_n which is not belong to edge metric generator S. Suppose that the pendant which is not belong to S is $x_nb, x_{n-1}b$ or $x_nb, x_{n-1}b \notin S$. Let we consider the distance of $x_{n-1}b$ and x_nb to k_i. By the definition of distace of edge and vertex, we have $d(x_{n-1}b, k_1) = min\{d(x_{n-1}, k_1), d(b, k_1)\} = min\{d(x_{n-1}, b) + d(b, k_1), d(b, k_1)\} = d(b, k_1)$, $d(x_{n-1}b, k_2) = min\{d(x_{n-1}, k_2), d(c, k_2)\} = min\{d(x_{n-1}, b) + d(b, k_2), d(b, k_2)\} = d(b, k_2)$, ... , $d(x_{n-1}b, k_i) = min\{d(x_{n-1}, k_i), d(b, k_i)\} = min\{d(x_{n-1}, b) + d(b, k_i), d(b, k_i)\} = d(b, k_i)$, with $1 \leq i \leq |S|$ and $d(x_nb, k_1) = min\{d(x_n, k_1), d(b, k_1)\} = min\{d(x_n, b) + d(x_n, k_1), d(x_n, k_1)\} = d(x_n, k_1)$, $d(x_nb, k_2) = min\{d(x_n, k_2), d(b, k_2)\} = min\{d(x_n, b) + d(b, k_2), d(b, k_2)\} = d(b, k_2)$, ... , $d(x_nb, k_i) = min\{d(x_n, k_i), d(b, k_i)\} = min\{d(x_n, b) + d(b, k_i), d(b, w_i)\} = d(b, k_i)$, with $1 \leq i \leq |S|$.

Then, the representation of two pendant vertex $x_{n-1}c$ and x_nc respect to S can be written as $r(x_{n-1}b|S) = (d(x_{n-1}c, k_1), d(x_{n-1}c, k_2), ..., d(x_{n-1}b, k_{n-2})) = (d(b, k_1), d(b, k_2), ..., d(b, k_{n-2}))$ and $r(x_nb|S) = (d(x_nb, k_1), d(x_nb, k_2), ..., d(x_nb, k_{n-2})) = (d(b, k_1), d(b, k_2), ..., d(b, k_{n-2}))$. We can see that $r(x_{n-1}b|S) = r(x_nb|S)$. Since every edge in star graph is adjacent, thus $x_{n-1}b$ and x_nb are adjacent. It is a contradiction. Furthermore, we should have $n-1$ vertex as local edge metric generator in star's pendant. Since it has been proven that $diml_E(S_n) \leq n-1$ and $diml_E(S_n) \geq n-1$, It can be conclude that $dim_E(S_n) = n-1$. □

Theorem 2.5 *Let W_n be a wheel graph with $n \geq 4$. The local edge metric dimension of W_n is $n-1$.*

Proof. The wheel graph is a connected graph with the cardinality of vertex is $n+1$. The vertex set $\{z\} \cup \{x_i; 1 \leq i \leq n\}$ and edge set $E(W_n) = \{zx_i; 1 \leq i \leq n\} \cup \{x_ix_{i+}; 1 \leq i \leq n-1\}$. Vertex c is a central vertex and x_i are pendant vertex. Wheel graph is the union of cycle graph and star graph. In order to prove the local edge metric dimension of W_n is $n-1$ or $diml_E(W_n) = n-1$, we will show the lower bound and upper bound of local edge metric dimension of wheel graph respectively are $diml_E(W_n) \leq n-1$ and $diml_E(W_n) \geq n-1$.

Table 4. The representation of edges in star graph S_n respect to local edge metric generator S

e	$r(e\|S)$	condition
cx_1	$(0,\underbrace{1,\ldots,1}_{n-2})$	
cx_i	$(\underbrace{1,\ldots,1}_{i-1},0,\underbrace{1,\ldots,1}_{n-i-1})$	$2 \le i \le n-1$
cx_n	$(\underbrace{1,\ldots,1}_{n-1})$	$n \ge 2$

Table 5. The representation of edges in wheel graph W_n respect to local edge metric generator S

e	$r(e\|S)$	condition
cx_1	$(0,\underbrace{1,\ldots,1}_{n-2})$	
cx_i	$(\underbrace{1,\ldots,1}_{i-1},0,\underbrace{1,\ldots,1}_{n-i-1})$	$2 \le i \le n-1$
cx_n	$(\underbrace{1,\ldots,1}_{n-1})$	$n \ge 3$
x_1x_2	$(0,0,1,\underbrace{2,\ldots,2}_{n-4})$	$n \ge 3$
x_nx_1	$(0,1,\underbrace{2,\ldots,2}_{n-4},1)$	$n \ge 3$
x_ix_{i+1}	$(\underbrace{2,\ldots,2}_{i-2},1,0,0,1,\underbrace{2,\ldots,2}_{n-i-3})$	$2 \le i \le n-2$

First, we should investigate the upper bound of local edge metric dimension of W_n. It will be shown that $diml_E(S_n) \le n-1$. By choosing the edge metric generator $S = \{x_i, 1 \le i \le n-1\}$, we will have the edge representation in W_n, $e \in E(W_n)$ respect to S in 5. By analyzing 5, it can be seen that all adjacent edges representation in W_n with respect to S are distinct. Hence, S is the local edge metric generator of W_n with $|W| = n-1$. It can be concluded that the upper bound of the local edge metric dimension of wheel graph is $n-2$, $dim_E(S_n) \le n-1$

The next step, we should analyze the lower bound of of local edge metric dimension of wheel graph W_n. We will prove $diml_E(W_n) \ge n-1$ by assuming the $diml_E(W_n) < n-1$. We know that wheel graph is a graph consist of the union of star graph and cycle graph. Let we consider the star graph as the component of wheel graph to prove the lower bound. Let we take $|S| = n-2$ where $k_i \in S$. there will be $n-2$ vertices in pendant as the element of edge metric generator in W_n. Since we have $n-2$ vertices in pendant as the element of edge metric generator in W_n, we still have two pendants left that connect central vertex with another vertex in spoke in W_n which is not belong to edge metric generator S. Assume that the pendant left uc and vc. By the definition of distance in edge and vertex, especially the distance of uc and vc to k_i, we will have the following term : $d(uz,k_1) = min\{d(u,k_1), d(z,k_1)\} = min\{d(u,z) + d(z,k_1), d(z,k_1)\} = d(z,k_1)$, $d(uz,k_2) = min\{d(u,k_2), d(z,k_2)\} = min\{d(u,z) + d(z,k_2), d(z,k_2)\} = d(z,k_2)$, ... , $d(uz,k_i) = min\{d(u,k_i), d(z,k_i)\} = min\{d(u,z) + d(z,k_i), d(z,k_i)\} = d(z,k_i)$, with $1 \le i \le |S|$ and $d(vz,k_1) = min\{d(v,k_1), d(z,k_1)\} = min\{d(v,z) + d(z,k_1), d(z,k_1)\} = d(z,k_1)$, $d(vz,k_2) = min\{d(v,k_2), d(z,k_2)\} = min\{d(v,z) + d(z,k_2), d(z,k_2)\} = d(z,k_2)$, ... , $d(vz,k_i) =$

$min\{d(v,k_i),d(z,k_i)\} = min\{d(v,z)+d(z,k_i),d(z,k_i)\} = d(z,k_i)$, with $1 \le i \le |S|$.
Then, the metric representation of two pendant vertex uz and vz respect to S can be written as $r(uz|S) = (d(uz,k_1),d(uz,k_2),...,d(uz,k_{n-2})) = (d(z,k_1),d(z,k_2),...,d(z,k_{n-2}))$ and $r(vz|S) = (d(vz,k_1),d(vz,k_2),...,d(vz,k_{n-2})) = (d(z,k_1),d(z,k_2),...,d(z,k_{n-2}))$. We can see that $r(uz|S) = r(vz|S)$. Since uz and vz in wheel graph are adjacent, thus it is a contradiction. Furthermore, we should have $n-1$ local edge metric generator. Since it has been proven that $diml_E(W_n) \le n-1$ and $diml_E(W_n) \ge n-1$, it can be concluded that $dim_E(W_n) = n-1$. □

3. Concluding Remarks
We have found the exact values of local edge metric dimension on some graphs namely path graph, cycle graph, ladder graph, star graph, and wheel graph. Since we just initiate this study, there still have many open problems related to the local edge metric dimension topic. Thus, we propose the following open problems:

Open Problem 1 *Characterize the local edge metric dimension on special families of graph especially for any reguler graphs, families of tree, planar graphs, graph operation of any simple graphs.*

Open Problem 2 *Find the sharpest upper bound of local edge metric dimension of any graph.*

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