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SOME FAMILIES OF TREE ARE ELEGANT

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ABSTRACT. An elegant labeling on the graph G with n vertex and m edge is a one-to-one mapping (injection function) of the vertex set $V(G)$ to the set of non negative integers $\{0, 1, 2, 3, \dots, m\}$ such that each edge gets the label of the sum from the adjacent vertex label in modulo number $(m+1)$ all different and nonzero, that is: $g(e) = g(uv) = [g(u)+g(v)] \mod (m+1)$ and $g(e) \neq 0$, where u and v are adjacent vertices. In this research, we investigate the elegant labelling of selected graph from some families of tree. A tree graph is a graph that does not contain a circle. The selected some families of tree are generalized of star and amalgamation of star. The results of this research have shown that some families of tree graph are elegant.

1. INTRODUCTION

This research discusses about Elegant Labeling with the domain is the set of vertices. Elegant Labeling of graph G with n vertex and m edge used one-to-one mapping (injection function) of the vertex set $V(G)$ to the set of non negative integers $\{0, 1, 2, 3, \dots, m\}$ such that each edge gets the label of the sum from the adjacent vertex label in modulo number $(m + 1)$ all different and nonzero, that is: $g(e) = g(uv) = [g(u)+g(v)] \mod (m+1)$ and $g(e) \neq 0$, where u and v are adjacent vertices.

Elumalai in [1] has been proved that : (i) graph P_n^2 is elegant for all $n \geq 1$; (ii) the graph $P_m^2 + \bar{K}_n$, $S_n + S_m$, and $S_m + \bar{K}_m$ are elegant for all $m, n \geq 1$; (iii) every

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even cycle $C_{2n} :< a_0, a_2, \ldots, a_{2n-1}, a_0 >$ with $2n-3$ chords $a_0, a_2, a_0a_3, a_0a_{2n-2}$ is elegant, for all $n \geq 2$ and (iv) The graph $C_3 \times P_m$ is elegant, for all $m \geq 1$. Some of related studies about the labeling have been developed include [2], [4], [5] and [3].

In this paper, there are certain families of trees as follow: the generalized star and the amalgamation of star. A tree graph is a graph that does not contain a circle.

2. Main Results

Theorem 2.1. *The generalized star $S_{n,m}$ for $m \geq 2$ even is elegant.*

Proof. The generalized star is a graph obtained by connecting one pendant vertex of every m copy of a path graph P_n to a new vertex called a root. Let The vertex and edge set of graph $G = S_{n,m}$ be defined as follows:$p = V(S_{n,m}) = \{x\} \cup \{x_i^k; 1 \leq k \leq m; 1 \leq i \leq n-1\}$ and $q = E(S_{n,m}) = \{xx_1^k; 1 \leq k \leq m\} \cup \{x_i^k x_{i+1}^k; 1 \leq k \leq m; 1 \leq i \leq n-2\}$. Here $q = m(n-1)$. An elegant labeling of $S_{n,m}$ is exhibited below:

$$g(x) = 0,$$

$$g(x_i^k) = \begin{cases} k + \frac{m}{2}(i-1); & 1 \leq k \leq \frac{m}{2}; 1 \leq i \leq n-1 \\ nm + k - \frac{m}{2}(i+1); & \frac{m+2}{2} \leq k \leq m; 1 \leq i \leq n-1 \end{cases}.$$

We can see clearly that g is injective function. The edge label of the graph $S_{n,m}$ is exhibited below:

$$g(xx_1^k) = \begin{cases} k; & 1 \leq k \leq \frac{m}{2} \\ m(n-1)+k; & \frac{m+2}{2} \leq k \leq m \end{cases},$$

$$g(x_i^k x_{i+1}^k) = (2k + m(\frac{2i-1}{2})) \mod m(n-1)+1$$

if $1 \leq k \leq \frac{m}{2}$, $1 \leq i \leq n-2$; and

$$g(x_i^k x_{i+1}^k) = (m(n - \frac{3}{2} - i) + 2k - 1) \mod m(n-1)+1$$

if $\frac{m+2}{2} \leq k \leq m$, $1 \leq i \leq n-2$.

Based on the edge label, the edge sequence is as follows: $S = \{xx_k; 1 \leq k \leq \frac{m}{2}\} \bigcup_{i=1}^{n-1} \{x_{n-i}^{\frac{m}{2}+k} x_{n-i+1}^{\frac{m}{2}+k}, x_i^k x_{i+1}^k; 1 \leq k \leq \frac{m}{2}\} \cup \{xx_{k+1}; 1 \leq k \leq \frac{m}{2}\}$ Hence g is an Elegant labeling of G. □

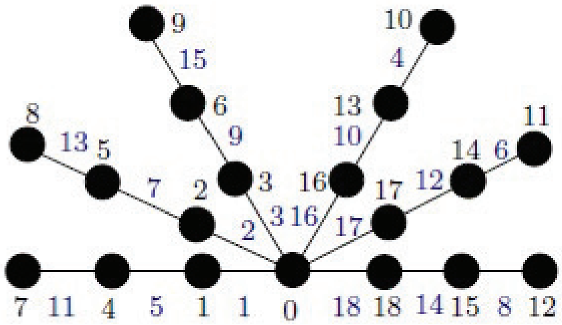

FIGURE 1. The Elegant labeling of $S_{3,6}$

Figure 1 shows an illustration of the Elegant labeling of $S_{3,6}$. Hence $q = 18$ the edge sequence as follows:

$g(xx_1) = 0 + 1 = 1$	$g(xx_2) = 0 + 2 = 2$
$g(xx_3) = 0 + 3 = 3$	$g(x_3^4 x_4^4) = [13 + 10] \mod 19 = 4$
$g(x_1^1 x_2^1) = 1 + 4 = 5$	$g(x_3^5 x_4^5) = [14 + 11] \mod 19 = 6$
$g(x_1^2 x_2^2) = 2 + 5 = 7$	$g(x_3^6 x_4^6) = [15 + 12] \mod 19 = 8$
$g(x_1^3 x_2^3) = 3 + 6 = 9$	$g(x_2^4 x_3^4) = [16 + 13] \mod 19 = 10$
$g(x_3^1 x_4^1) = 4 + 7 = 11$	$g(x_2^5 x_3^5) = [17 + 14] \mod 19 = 12$
$g(x_3^2 x_4^2) = 5 + 8 = 13$	$g(x_2^6 x_3^6) = [18 + 15] \mod 19 = 14$
$g(x_3^3 x_4^3) = 6 + 9 = 15$	$g(xx_4) = 0 + 16 = 16$
$g(xx_5) = 0 + 17 = 17$	$g(xx_6) = 0 + 18 = 18$

Theorem 2.2. *The Amalgamation of star* $Amal(S_n, v, m)$ *for* $m \geq 2$ *is elegant.*

Proof. Let G_i be a finite collection of graphs and let each G_i have a fixed vertex v_{oi} called the terminal. The amalgamation $Amal(G_i, v_{oi})$ is formed by taking all the $G_i s$ and identifying their terminals. The terminal of the Amalgamation of star is a pendant vertex of S_m. Let The vertex and edge set of graph $Amal(S_n, v, m)$ are defined as follows: $p = V(Amal(S_n, v, m)) = \{x\} \cup \{x_k; 1 \leq k \leq m\} \cup \{x_i^k; 1 \leq k \leq$

$m; 1 \leq i \leq n-1\}$ and $q = E(S_{n,m}) = \{xx^k; 1 \leq k \leq m\} \cup \{x^k x_i^k; 1 \leq k \leq m; 1 \leq i \leq n-1\}$, where $q = mn$. An Elegant labeling of $S_{n,m}$ is exhibited below:

$$g(x) = 0,$$

$$g(x^k) = \begin{cases} k; & 1 \leq k \leq \frac{m}{2} \\ m(n-1)+k; & \frac{m+2}{2} \leq k \leq m \end{cases},$$

$$g(x_i^k) = \begin{cases} m(\frac{2i-1}{2})+k; & 1 \leq k \leq \frac{m}{2}; 1 \leq i \leq n-1 \\ mi+k-\frac{m}{2}; & \frac{m+2}{2} \leq k \leq m; 1 \leq i \leq n-1 \end{cases}.$$

Clearly the g is injective function. The edge label of the graph $Amal(S_n, v, m)$ is exhibited below:

$$g(xx^k) = \begin{cases} k; & 1 \leq k \leq \frac{m}{2} \\ m(n-1)+k; & \frac{m+2}{2} \leq k \leq m \end{cases},$$

$$g(x^k x_i^k) = (2k + m(\frac{2i-1}{2})) \mod nm+1$$

if $1 \leq k \leq \frac{m}{2}$, $1 \leq i \leq n-1$; and

$$g(x^k x_i^k) = (m(n-\frac{3}{2}) + 2k - mi - 1) \mod nm+1$$

if $\frac{m+2}{2} \leq k \leq m$, $1 \leq i \leq n-1$.

Based on the edge label, the edge sequence is as follows: $S = \{xx_k; 1 \leq k \leq \frac{m}{2}\} \bigcup_{i=1}^{n-1}\{x_i^{\frac{m}{2}+k} x_{k+\frac{m}{2}}, x_k x_i^k; 1 \leq k \leq \frac{m}{2}\} \cup \{xx_{k+\frac{m}{2}}; 1 \leq k \leq \frac{m}{2}\}$. Hence g is an Elegant labeling of G. $\square$

Figure 2 shows an illustration of the Elegant labeling of $Amal(S_4, v, 6)$. Hence $q = 24$ the edge sequence is as follows:

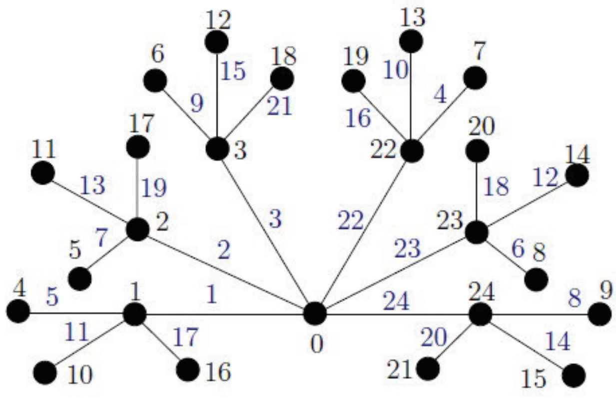

FIGURE 2. The Elegant labeling of $Amal(S_4, v, 6)$

$$g(xx_1) = 0 + 1 = 1 \qquad g(xx_2) = 0 + 2 = 2$$
$$g(xx_3) = 0 + 3 = 3 \qquad g(x_1^4x_4) = [7 + 22] \mod 25 = 4$$
$$g(x_1^1x_1) = 1 + 4 = 5 \qquad g(x_1^5x_5) = [8 + 23] \mod 25 = 6$$
$$g(x_1^2x_2) = 2 + 5 = 7 \qquad g(x_1^6x_6) = [9 + 24] \mod 25 = 8$$
$$g(x_1^3x_3) = 3 + 6 = 9 \qquad g(x_2^4x_4) = [13 + 22] \mod 25 = 10$$
$$g(x_2^1x_1) = 1 + 10 = 11 \qquad g(x_2^5x_5) = [14 + 23] \mod 25 = 12$$
$$g(x_2^2x_2) = 2 + 11 = 13 \qquad g(x_2^6x_6) = [15 + 24] \mod 25 = 14$$
$$g(x_2^3x_3) = 3 + 12 = 15 \qquad g(x_3^4x_4) = [19 + 22] \mod 25 = 16$$
$$g(x_3^1x_1) = 1 + 16 = 17 \qquad g(x_3^5x_5) = [20 + 23] \mod 25 = 18$$
$$g(x_3^2x_2) = 2 + 17 = 19 \qquad g(x_3^6x_6) = [21 + 24] \mod 25 = 20$$
$$g(x_3^3x_3) = 3 + 18 = 21 \qquad g(xx_4) = [0 + 22] = 22$$
$$g(xx_5) = 0 + 23 = 23 \qquad g(xx_6) = [0 + 24] = 24$$

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