



## Research Article

### Impact of Infusing Truth-Seeking and Open-Minded Behaviors on Mathematical Problem-Solving

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#### Abstract

The aims to investigate the impact of infusing truth-seeking and open-mindedness on mathematical problem-solving toward students' dispositions and achievement. It was qualitative in nature, with an exploratory descriptive methodology, and a total of 20 in-service mathematics teachers from Indonesia were involved in the study. These participants were taught using two different treatments during a one-semester mathematical problem-solving course. In the first treatment, students were encouraged to assume that all provided information in the problems is correct; meanwhile, in the second treatment, students were encouraged to seek the truth and analyze the possible different point of view. Both treatments were employed alternately every week in one semester (14 weeks). The result of the study indicated that after the infusion period that students tend to show critical thinking dispositions when they have to solve a mathematical problem, and their mathematics achievement also outperforms students who never informed truth-seeking and open-minded behaviors.

#### Keywords:

critical thinking disposition; infusing; mathematical problem-solving; truth-seeking; open-minded.

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## Introduction

Helping students to become critical thinkers has become the focus of current educational objectives (As'ari, Mahmudi, & Nuerlaelah, 2017; Di Giacomo, Fishbein, Monthey, & Pack, 2012; National Education Association, 2014). The industry agenda of World Economic Forum (World Economic Forum, 2015) even explicitly set critical thinking as one among the essential competencies needed by anyone to approach the complex problems (Sudirman, Irwanto, & Basuki, 2017), and they put it in a paper entitled "New Vision for Education." Scholars believe that critical thinking skills help students develop their learning skills (Bell & Loon, 2015), which enable them to produce better decision-making (Wongsila & Yuenyong, 2019; Cottrell, 2005), better problem-solving products (Walid, Sajidan, Ramli, & Kusumah, 2019; Tumkaya, Aybek, & Aldag, 2009), and having a better life (Starkey, 2004). Therefore, educators should facilitate their students to become critical thinkers. Students should be trained to consistently show their critical thinking dispositions, whether in the classroom or daily life.

Discussion with mathematics teachers indicated that their teaching focus is still on mathematical content acquisition. Very rare of them who try to develop students critical thinking skills during their lesson (Edwards, 2017). Even, current researches indicated that prospective mathematics teachers are not critical thinkers yet (As'ari et al., 2017; Yorgancı, 2016).

There is no such significant change up to now. When the author currently conducted a small survey through Facebook, in which the author asked the facebookers to comment on an elementary mathematics problem, most of them, including them who are mathematics teachers, did not think critically. They did not do truth-seeking, analyze the adequateness, the completeness, and the reasonableness of the information. Students' critical thinking dispositions did not appear. It is in line with the results of research which states that the pre-service mathematics teachers had not yet attained critical thinking dispositions pertinent to truth-seeking (Kurniati, Purwanto, As'ari, & Dwiyanu, 2019).

This humiliating condition challenged the authors to find a way on how to help students to become critical thinkers. The results of searching research articles from eric.ed.gov website, a few scholars have studied it (Gotoh, 2016; Darby & Rashid, 2017; Semerci & Elaldi, 2014; Weatherspoon, 2013). They used different teaching methods to develop their students' critical thinking dispositions, such as metacognition (Gotoh, 2016; Semerci & Elaldi, 2014), electronic simulation (Weatherspoon, 2013), RED model (Bloch & Spataro, 2014), and infusion critical thinking disposition (Darby & Rashid, 2017).

Considering the characteristics of Indonesian citizens who tend to imitate their idol behavior, and the teacher could be the idol for their students. Based on that result, the authors are interested in using the infusion approach to help students to become critical thinkers. However, in infusing critical thinking dispositions, the

teacher should model all of its components, namely: truth-seeking, open-mindedness, systematicity, analyticity, self-confident, inquisitiveness, and cognitive maturity (Facione, 2000). This infusion, of course, is not an easy task to implement it. Modeling all of these components demand a significant effort, and it will take a very long period. Therefore, there is a need to find the most straightforward way in the infusion model, which has the same power for developing critical thinking disposition. The authors, therefore, are interested in finding which one among all of the critical thinking dispositions above that has the best representation of critical thinking disposition. Unfortunately, the existing theory and results of the previous studies do not provide the prescription for it.

The authors then decided to investigate the truth-seeking and open-mindedness. Based on the definition of critical thinking (Ennis, 2011), where students should use reasonable and reflective thinking for deciding something, truth-seeking should be the first and most crucial activity in practicing critical thinking. The result of the truth-seeking can be used as the foundation for other critical thinking skills activities, such as inference, interpretation, evaluation, and explanation. All of these activities will be meaningless if the data are wrong. Meanwhile, the open-mindedness has the potentials to lead the students to consider different points of view. This open-mindedness will lead the students to become objective and come up with a more mature decision. Furthermore, truth-seeking and open-minded behaviors are predictors of students or teachers having a critical thinking disposition (Kurniati, Purwanto, As'ari, Dwiyan, Subanji, & Susanto, 2019; Kurniati, Purwanto, As'ari, & Dwiyan, 2018).

Truth-seeking and open-minded of in-service mathematics teacher have not become a habit that is done when solving the mathematical problems (Kurniati et al., 2019). Therefore an attempt should be made to accustom truth-seeking and open-minded. However, not many studies have examined the appropriate learning to accustom in-service mathematics teachers to have truth-seeking and open-minded. Therefore, the main focus of this research is to investigate the impact of infusing truth-seeking and open-minded behaviors on mathematical problem-solving.

### **Problem of Research**

Truth-seeking and open-minded behaviors are predictors of students or teachers having a critical thinking disposition, but not many studies have examined the appropriate learning to accustom in-service mathematics teachers to have truth-seeking and open-minded. Therefore, the research problem is “how the impact of infusing truth-seeking and open-minded behaviors on mathematical problem-solving toward students’ dispositions and achievement?”.

## Method

### Research Design

This research was qualitative in nature and employed an exploratory descriptive method. It was projected to investigate the impact of infusing truth-seeking and open-mindedness on mathematical problem-solving toward students' dispositions and achievement. The design was operationalized because qualitative descriptive research constituted a design that was aimed at describing, combining, analysing, and interpreting the situation of an individual (Kurniati et al., 2019).

### Participants

The study was implemented at a post-graduate program at a state university of Malang in East Java, Indonesia. A total of 20 in-service mathematics teachers from various districts in East Java who are taking a master degree program in Mathematics Education study program were involved voluntarily. Demographic Structures of Participants were presented in Table 1.

**Table 1.**

*Demographic Structures of Participants*

	Aspects	Total
Gender	Male	12
	Female	8
Age	25-30 years	13
	31-35 years	7
Education Background	Mathematics Education	20
Culture	Javanese	17
	Madurese	3

### Data Collection Methods

Research data were collected through giving the mathematical problems, direct observation, and interviewing. The types of mathematical problems presented in two approaches (regular and infusion approach) were also different. In the regular approach, the type of its mathematical problems is a complete self-explained problem. All of the information in the problems are clear and has only one meaning for every student. Meanwhile, the type of problems in the infusion approach was different. Two types of problems were given to the students, namely: (a) problems with contradictory information, and (b) problems with no specified universal set given. Problem with contradictory information is a problem which contains, directly or indirectly, contradictive statements. Problem with no specified universal set given is a problem where there is no specific information about the universal set of the

values of its variables. The table 2 can briefly describe the differences between the two approaches.

**Table 2.**

*Difference Focus of Regular Approach and Infusion Approach*

<b>Instructional Aspects</b>	<b>Regular Approach</b>	<b>Infusion Approach</b>
Types of Mathematical Problems presented to ISMT students	Regular Mathematics Problems	Problem with contradictory information or algebraic problems with no specified universal set of its variable was given
The focus of problem-solving modeling activities done before routine problem-solving activities	Helping the ISMT students to see ways to solve the problem entirely and accurately	Helping the ISMT students to become critical thinkers before solving the problems
Leading or guiding Questions asked to the students during problem-solving activities	Helping students better understand the problem, select and apply the appropriate problem-solving strategy	Helping students to check whether the problems are worth doing firstly and to open up their mind to see other alternative solutions

The first mathematical problem was as follows:

If two machines can produce two cakes for 2 seconds, how long it takes to produce

- 100 cakes with 100 machines
- 100 cakes with 10 machines
- 50 cakes with 50 machines.

This problem was intended to measure the impact of the instruction toward their immediate critical thinking disposition, i.e., critical thinking disposition that they show right after the end of the semester.

Another problem that was given after a semester independent learning period or at the end of the second semester is as follows.

Given 8 points as the following.

- |   |   |   |   |
|---|---|---|---|
| A | B | C | D |
| • | • | • | • |
| • | • | • | • |
| E | F | G | H |

If you connect three non-collinear points, you will have a triangle. Use the limitation that two triangles are considered different if they are not congruent to answer the following questions.

1. What is the possible number of different right-angled triangles can be identified?
2. What is the possible number of different acute triangles can be identified?
3. What is the possible number of different obtuse triangles can be identified?

This problem was intended to see the long term impact of the instruction.

During the giving the mathematical problems, the researchers also observed and recorded the participants' behaviour especially truth-seeking and open-mindedness. The last data collection method was the in-depth interview. Each interview was recorded using a video, and the results of mathematical problem-solving were collected.

### **Data Analysis**

The data were analysed through the application of data reduction and data presentation, leading to the study's findings and conclusion. In the first stage, the data reduction dealt with including pertinent variables which were in line with the research objectives and excluding unnecessary variables. The second stage, data presentation, was related to classifying the reduced data into two groups, namely, (1) the impact of infusing truth-seeking and open-mindedness on mathematical problem-solving toward students' dispositions and (2) the impact of infusing truth-seeking and open-mindedness on mathematical problem-solving toward students' achievement. The final stage entailed drawing conclusions based on the findings and the data presentation

### **Procedure**

Using the Selected Topics of School Mathematics as the course, these 20 mathematics in-service mathematics teacher (ISMT) students were taught using two different approaches alternately. Each week these ISMT students were taught for 2 × 50 minutes. In the odd weeks (1, 3, 5, 7, 9, 11, 13), these in-service mathematics students were guided and trained to solve mathematical problems using the conventional approach. In the even weeks (2, 4, 6, 8, 10, 12, 14), they were guided and trained to solve mathematical problems using critical thinking disposition approach. In the week 15<sup>th</sup>, these ISMT students were encouraged to work independently and do a self-reflection. In the week 16<sup>th</sup>, they were given a final test problem where they have to solve the problem independently.

The difference treatment between both methods is mostly in understanding the problem stage, especially the type of questions that arose in this stage. If in the odd week, the focus of the understanding the problem stage was to identify what is given and what is asked from the problem. In the even weeks, the focus of the understanding stage was to check the truth of all information provided in the problem (truth-seeking behavior) and to explore several possible conditions behind the information provided in the problem (open-mindedness).

Type of questions in the understanding the problem stage for conventional approach as mostly: (a) what is given in the problem, (b) what is asked in the problem. Other questions that can be raised to guide ISMT students to understand the problem could be: (a) what mathematical concepts involved in the problem, (b) what do you know about theorems related to that concept?, (c) have you ever solved similar problems, and (d) what ideas that can be used to solve this problem.

Meanwhile, the guided questions in the truth-seeking and open-mindedness approach were mostly: (a) is there any information in the problem that could have different meanings?, (b) are there any contradictory information in the problem?, (c) is there variable in the problem which produce different answer if the universe is changed? (d) is there a possible different point of views related to the information provided in the problem?, and (e) what other people would interpret the information provided in the problem?.

### **Reliability and Validity**

To ensure the validity and reliability of this research, data analysis was performed by triangulation. The process of triangulation in this research was the data from the participants' problem solving was displayed and compared with the direct observations and the interviews. This was carried out to determine the impact of infusing truth-seeking and open-mindedness on mathematical problem-solving toward students' dispositions and achievement. The data resulting from the triangulation were analysed to find the tendencies of the impact of infusing truth-seeking and open-mindedness toward students' dispositions and achievement.

## **Results**

The final test problem is a regular problem. Similar problems can be found in many mathematics textbooks, or even in the collection of national examination problems. Usually, students directly use proportional reasoning strategy to solve it since everything is clear and no need to use reflective thinking in solving it.

In the beginning period of the final test, they used proportional reasoning in solving the problem. However, having received a small reminder from the authors that the problem was not as a routine, as usual, they showed a refinement answer. This reminder influenced their thinking habits, and the refinement results indicated they are capable of thinking critically. Their refinement indicated that they could perform critical thinking dispositions. So, they are actually at the developing critical thinkers level.

**First**, in the refinement product, they showed that they could become critical thinkers by analyzing the problem deeply. They explored several possible meanings of the machines potencies, its strengths, and also its weaknesses. They did not assume directly anymore that all of the machines have the same conditions. Followings are some examples of students responses regarded to this truth-seeking activity, as seen in Figure 1.

Translate Version

Dari yang diketahui bisa memberikan penjelasan yg berbeda, misalkan kedua mesin tsb dinamakan mesin 1 dan mesin 2, bisa diartikan bahwa mesin 1 dalam 2 detik menghasilkan 1 kue dan mesin 2 dalam 2 detik menghasilkan 1 kue, sehingga kalau bekerja bersama berarti mesin 1 dan mesin 2 dalam 2 detik menghasilkan 2 kue. atau bisa juga diartikan mesin 1 dalam 1 detik menghasilkan 1 kue dan mesin 2 dalam 1 detik menghasilkan 1 kue sehingga jika digabungkan dalam 2 detik kedua mesin menghasilkan 2 kue.

Based on information, we can give different interpretations, for example the two machines are called machine 1 and machine 2, then we can interpret that machine 1 in 2 seconds produces 1 cake and machine 2 in 2 seconds produces 1 cake so that if working together then machine 1 and machine 2 in 2 seconds produce 2 cakes. In addition to this interpretation, machine 1 in 1 second produces 1 cake and machine 2 in 1 second produces 1 cake so that in 2 seconds both machines produce 2 cakes.

Figure 1.

First ISMT Student Response

This ISMT student stated that the information in the problem could lead to different interpretation. The first interpretation is that each machine produces one cake in two seconds. The second interpretation is that machine 1 produces one cake in 1 second, and the next second (or 2 second), machine 2 produces another cake. He analyzed the problem a little bit deeper.

Second ISMT student response is presented in Figure 2. This ISMT student even questioned whether both machines are healthy, have the same working power, produced the same cake, the same size, and whether each machine work for 2 seconds or not.

Translate

Dari soal diketahui ada 2 mesin dalam waktu 2 detik menghasilkan 2 kue. Dari ~~pernyataan~~ kalimat tersebut tidak ada penjelasan secara rinci bagaimana proses untuk menghasilkan kue :

- Apakah mesin bekerja normal / Tidak
- Apakah 2 mesin tersebut sama kerjanya
- Apakah kue yang dihasilkan jenisnya sama
- Apakah kue yang dihasilkan besarnya sama
- Apakah mesin bekerja penuh selama 2 detik

From the problem, it is known that there are 2 machines in 2 seconds producing 2 cakes. Based on this sentence there is no detailed explanation of how the process of producing cakes

1. Does the machine produce normally/ not?
2. Do 2 machines simultaneously produce?
3. Are the same type of cakes that produced?
4. Is the cake produced the same size?
5. Does the machine work fully for 2 seconds?

Figure 2.

Second ISMT Student Response

Second, through their refinement work too, these ISMT students tried to show that they are capable of becoming open-minded students. They stated clearly their assumption in solving this problem. They wanted to show that the truth of their answer was based on the stated assumption, and other solution could be correct if they have a different reasonable assumption.

Following are some examples of their stated assumption in answering the problem, as seen in Figure 3 and Figure 4.



Saya asumsikan semua mesin sama & dapat digunakan  
 semua serta kue yang dihasilkan ukurannya sama  
 (dengan kata lain semua berada pada kondisi ideal)

Translate Version

I assumed that all of the machines are similar and functional. I also assumed that all of the produced cakes are having the same size. I assumed that all are in their ideal condition.

Figure 3.

Third ISMT Student Response

Apabila diasumsikan mesin berjalan normal, identik satu sama lain  
 dan kue yang dihasilkan tidak ada perbedaan, maka permasalahan  
 dapat dibuat bentuk tabel berikut

Translate Version

I assumed that the machines are working regularly, having identical potencies, and produce similar cakes.

Figure 4.

Forth ISMT Student Response

**Third**, although they realize the existence of some limitations on the problems, these ISMT students still tried to make their lecturer happy by adhering to their lecturer request. They still answered the problem since they know that if they did not submit their answer, they would have a bad consequence from their lecturers. However, they did not just solve the problem procedurally. They incorporated their analysis, the results of their analysis, and their preferred solution using a specific assumption.

**Forth**, they tried to solve the problem systematically. They used a table and defined a variable to make it become a more apparent problem which enables these ISMT students to process and solve the problem.

**Finally**, they consistently used logical inferences solving the problem, although they did not have long enough time for doing it, as seen Figure 5.

mesin	lama waktu	produk
2 mesin	2 detik	2 kue
100 mesin	$x_1$	100 kue
10 mesin	$x_2$	10 kue
50 mesin	$x_3$	50 kue

misalkan:  $x_i =$  lama waktu untuk busus- $i$

Translate Version

Machine	Time (seconds)	Product (cake)
2	2 seconds	2
100	$X_1$	100
10	$X_2$	10
50	$X_3$	50

Figure 5.

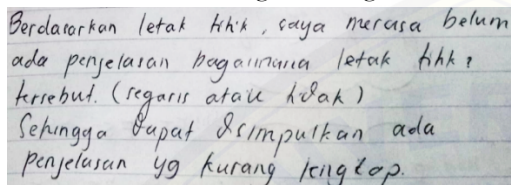
Fifth ISMT Student Response

Based on the above results, several or even most of the critical thinking dispositions indicators are shown by these ISMT students. Based on As'ari et al. (2017), at the end of this critical thinking disposition instructional infusion approach, these ISMT students are actually at the developing level of critical thinkers. They still did not reach the highest level of critical thinkers, which is the mastery level.

A better and promising result seems to emerge from this long term assessment. They directly showed a tendency to think and behave critically when they were asked

to solve a mathematical problem. They did not assume that the problem is formulated correctly or accurately, and therefore, they did not directly use mathematical procedures to solve it. They spent plenty enough time analyzing the problem. They used several perspectives and many other critical thinking dispositions indicators in solving this problem. They kept questioning everything and tried to get the best and accurate possible information and present their best possible solutions.

Most of these ISMT, except one, students realized that additional information is needed to make this problem is worth doing. Following are comments from several students, as seen in Figure 6, Figure 7, and Figure 8.



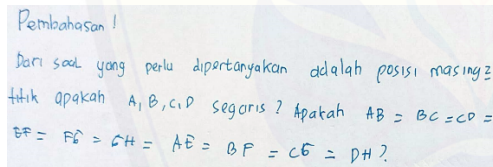
Translate Version

Based on the location of the point, I feel there is no explanation of how the location of the point (whether collinear or not). So that I can conclude that there is an incomplete explanation.

**Figure 6.**

*The First Type of ISMT Student Response*

She realized that the positions of points are unclear. She could not be able to determine whether the points are collinear or not. Another ISMT student responded as the following.



Translate Version

He said that he needs additional information whether  $A, B, C,$  and  $D$  are collinear and whether  $AB = BC = CD = EF = FG = GH = AE = BF = CG = DH?$

**Figure 7.**

*The Second Type of ISMT Student Response*

Another response from an ISMT student was as follows:

- \* Harus kita asumsikan dahulu :
- 1. titik-titik  $A, B, C, D$  berjarak sama satu dengan lainnya dan sejajar (segaris)
- 2. titik-titik  $B, F, G, H$  berjarak sama satu dengan lainnya dan sejajar (segaris)
- 3. jarak  $A$  dan  $E$ ,  $B$  dan  $F$ ,  $C$  dan  $G$ ,  $D$  dan  $H$  adalah sama dan sejajar

Translate Version

He said that he need to assume that  $A, B, C,$  and  $D$  are having equal distance, collinear, parallel to  $E, F, G, G$  and distance from point  $A$  to point  $E$ , from point  $B$  to point  $F$ , from point  $C$  to point  $G$  and from point  $D$  to point  $H$  are equal and collinear.

**Figure 8.**

*The Third Type of ISMT Student Response*

These responses mean that they first analyzed the problem and realized that the information in the problem is inadequate. These ISMT students seem to show additional information to ensure that they can answer it accurately. They showed several indicators of being critical thinkers.

However, unfortunately, did not show their need explicitly. Most of these ISMT students did not raise any clarification questions to the lecturers. They did not criticize the limitation of the problem. They still answered the problem, although they need additional information, just to please their lecturers. They try to become obedient students. So, they can think critically, but not at the mastery level. They just at the *emergent critical thinkers* level (As'ari et al., 2017).

There was only one among the 20 ISMT students, asked the lecturer to provide additional information about the conditions of the points. He asked the following as seen in Figure 9.

Maaf pak, ada beberapa hal yang perlu diperjelas sebelum menjawab, hal ini terkait konfigurasi titik A,B,C,D,E,F,G,H Pada gambar yang belum dijelaskan.

1. Apakah titik A,B,C,D terletak pada satu garis?
2. Apakah titik D,E,F,G terletak pada satu garis?
3. Apakah garis pd poin 1 dan poin 2 sejajar?
4. Apakah ruas garis AD Tegak lurus dengan garis pada poin 1 dan garis pada poin 2
5. Apakah jarak  $AB=BC=CD=AD=DE=EF=FG?$

Translate Version

I am sorry, there are some corrections before we solve the problem.

1. Are points A, B, C, D collinear?
2. Are points D, E, F, G collinear?
3. Are the lines at point 1 and point 2 parallel?
4. Is the line segment AD perpendicular to the line at point 1 and the line at point 2?
5. Is the distance  $AB = BC = CD = AD = DE = EF = FG?$

Figure 9.

*The Forth Type of ISMT Student Response*

He did not directly accept the picture as a picture with a perfect dimension. He asked the lecturers whether A, B, C, D are collinear. He asked whether E, F, G, H (although he mistyped it into D, E, F, G) are collinear. He also asked whether line containing A, B, C, D and line containing E, F, G, H are parallel. Even, he also asked whether  $AB = BC = CD = AE = EF = FG = GH$  or not? He showed curiosity and tried to seek the truth. He showed an indicator of critical thinking disposition.

However, when there was no answer provided by the lecturers, he finally answered the problem using the following assumption, as seen in Figure 10.

ASumsi

- 1) titik A, B, C, D terletak pd satu garis
- 2) titik E, F, G, H terletak pd satu garis
- 3) AD sejajar EF
- 4) jarak  $AB = BC = CD = AE = EF = FG = GH$

Translate Version

Assumption:

1. A, B, C, D are collinear
2. E, F, G, H are collinear
3. line AD is parallel to line EF,
4. the distances among the closest points are equidistance

Figure 10.

*The Fifth Type of ISMT Student Response*

He assumed that points A, B, C, D are collinear, points E, F, G, H are also collinear, line AD is parallel to line EF, and the distances among the closest points are equidistance.

One missing thing from his answer was that he was unable to present all possible alternatives related to the position of the points, including its distances. He did not differentiate at least three different possible vertical distances compare to horizontal distances, which influence the type of the formed triangles (acute, or obtuse). He just focused on one alternative above, namely equidistance vertically and horizontally. So, although he questioned the information in the problem to the teacher, he cannot be considered as at the level of *mastery of critical thinker*. He is at *emergent critical thinker level* still.

These immediate and long-term results indicated that infusing critical thinking dispositions (especially truth-seeking and open-mindedness) have the powers in developing ISMT critical thinking dispositions. The critical thinking disposition infusion approach has made students reach at most at the *developing* (but not mastery) critical thinking disposition level. Additional efforts are still needed to help them to become master critical thinkers.

### Discussion and Conclusion

The results of this study provide a positive message, i. e. there is a chance to improve Mathematics ISMT students' critical thinking dispositions level. The infusion approach has the potency to improve ISMT critical thinking dispositions. The infusion has the potential to improve students' critical thinking disposition. Infusing critical thinking disposition is useful for developing students' critical thinking disposition (Abrami, Bernard, Borokhovski, Wade, Surkes, Tamim, & Zhang, 2008; Aarsal, 2015; Bandura, 1971).

Several things need to address in discussing the result of this study. These include: (a) activities of modeling the truth-seeking, (b) activities of modeling the open-mindedness, (c) questioning activities, (d) monitoring and feedback activities, and (e) types of mathematical problems presented to the students.

Truth-seeking and Open-mindedness modeling that was practiced by the authors during the infusion model of teaching, according to the social learning theory (Bandura, 1971), provides examples of behavior that be imitated by students. Truth-seeking and open-mindedness questions modeled by authors lead to positive classroom interaction and at the same time, also provide examples of how to think critically while solving the mathematical problems. The interaction that the teacher made with their students can be a tool for students to learn and imitate the behavior (Burdick, 2014).

However, students would imitate if they pay attention to the intended behaviors, they have much memory on that behavior, and they get a reward by imitating the behavior (Bandura, 1971). In this study, those three requirements were stimulated by authors. In each infusion approach time, authors always apply truth-seeking and open-mindedness behaviors and keep remind ISMT students to apply truth-seeking and open-mindedness while solving mathematical problems. Guided questions for

being critical thinkers, which always referred during the infusion approach has made the development of dynamic interaction. The questioning activity is in line with the previous research that questioning (especially using several types of questions) could promote interaction (Al-Zahrani & Al-Bargi, 2017). Also, by giving good reward for these ISMT students when they perform truth-seeking and open-mindedness, these ISMT students have a good memory about implementing truth-seeking and being open-mindedness.

Another thing that makes the students have a better memory on this truth-seeking and open-mindedness is types of problems presented to them. Problems presented to them during the infusion approach were two types, namely: (a) problems with contradictory information (Kurniati et al., 2019) and (b) problems with no specified universal set. If usually those ISMT students just solve any mathematical problems procedurally, through solving problems with contradictory information, problems with no specified universal set, they will experience conflict cognitive. They will see something that contradicts to what they have already in their schemata. This cognitive conflict will usually lead to conceptual change (Kang, Scgharmann, & Noh, 2004), which will be remembered for an extended period.

According to the Planned Behavior Theory (Ajzen, 2012; Munro, Lewin, Swart, & Volmink, 2007), there are three significant aspects to consider if we want successfully change one behavior into an expected behavior. The three aspects are: (1) attitude toward the behavior, (2) supporting subjective norms, and (3) perceived control behavior.

Authors' observation on ISMT students' behaviors during the *Kapita Selektta Matematika Sekolah* (Selected Topics of School Mathematics) course and other interactions outside the classroom indicated that they have big respect to anyone who shows their critical thinking disposition consistently. Anyone who practicing critical thinking dispositions is considered as having some advantages from the others. They valued critical thinking dispositions as good practices that everybody should perform routinely. So, based on five stages of the affective domain (Krathwohl, Bloom, & Masia, 1973), these ISMT students' affective aspects toward critical thinking dispositions are at least at valuing level already. According to the authors, their active involvement in the full semester implementation of this infusion approach seems to enhance students' attitude and perception toward critical thinking disposition. The freely and conducive environments have provided very supportive norms for ISMT students to get used to practicing and developing critical thinking dispositions. They also feel that they have the full autonomous control of their critical thinking behaviors. Therefore, this is in line what the Planned Behavior Theory has mentioned (Ajzen, 2012; Munro et al., 2007). The three aspects that are needed to change behaviors, i.e.: (1) attitude toward the behavior, (2) supporting subjective norms, and (3) perceived control behavior were fulfilled in the implementation of critical thinking dispositions infusion approach.

However, since this study was unable to develop students' critical thinking disposition into the highest possible critical thinkers level (mastery level), several recommendations are provided here. First, a replication study with several refinements of its infusion approach is needed. Second, since thinking disposition is closely related to thinking style (Emir, 2013), and this study did not address the thinking style issue yet, the authors also recommend other researchers to investigate the impact of teachers' thinking style on the development of critical thinking disposition. Whenever possible, another research on how to treat and facilitate ISMT students with specific thinking style to become better critical thinkers is needed.

Third, based on a glance observation, students' content mastery seems to influence their students' critical thinking dispositions level. Students with better content mastery tend to have better critical thinking disposition than those from the lower mathematical content acquisition. The authors are happy to support everybody investigating the relationship between content mastery and critical thinking disposition level, especially in the mathematics domain. Finally, since valid and reliable standardized instruments to measure the critical thinking dispositions and skills in mathematics are not developed yet, another recommendation is to conduct research and development (R & D) approach to construct and develop standardized tests for critical thinking skills and disposition in the mathematical domain.

It can be concluded that infusing critical thinking dispositions (especially truth-seeking and open-mindedness) in mathematical problem-solving instruction is useful for developing ISMT students' critical thinking disposition. The impact is also long-lasting and growing. However, refinement of the instructional implementation is still needed to enable students to reach the highest level of critical thinkers, that is the mastery level. Further studies which consider additional aspects such as students' thinking styles and their mathematical content mastery level are also strongly recommended. Also, to ensure better critical thinking assessment in mathematics, there is a need also to construct and develop standardized critical thinking skills and dispositions inventory tests in mathematics using research and development (R & D) methodological approach.

### **Disclosure and Conflicts of Interest**

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article. This research is original work and does not contain any libelous or unlawful statements or infringe on the rights or privacy of others or contain material or instructions that might cause harm or injury.

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