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Local antimagic r-dynamic coloring of graphs

A I Kristiana[1,2,3], M I Utoyo[3], Dafik[1,2], I H Agustin[1,2], R Alfarisi[1,5]

[1]CGANT-University of Jember, Jember, Indonesia
[2]University of Jember, Jember, Indonesia
[3]University of Airlangga, Surabaya, Indonesia

E-mail: arikakristiana@gmail.com

Abstract. Let $G = (V, E)$ be a connected graph. A bijection function $f : E(G) \to \{1, 2, 3, \cdots, E(G)|\}$ is called a local antimagic labeling if for all $uv \in E(G)$s, $w(u) \neq w(v)$, where $w(u) = \Sigma_{e \in E(u)} f(e)$. Such that, local antimagic labeling induces a proper vertex k-coloring of graph G that the neighbors of any vertex u receive at least $\min\{r, d(v)\}$ different colors. The local antimagic r-dynamic chromatic number, denoted by $\chi_r^{la}(G)$ is the minimum k such that graph G has the local antimagic r-dynamic vertex k-coloring. In this paper, we will present the basic results namely the upper bound of the local antimagic r-dynamic chromatic number of some classes graph.

1. Introduction

In this paper, all graph is simple, connected and undirected, $G = (V, E)$, on the vertex set $V(G)$ and the set $E(G)$. For vertex, v of G is denoted by $N(v)$ and the degree of v is denoted by $d(v)$. The maximum and minimum of graph G are denoted by $\Delta(G)$ and $\delta(G)$. Arumugam [4], introduced the concept of local antimagic chromatic number of graphs. Then it is followed by Albirri [2] which did some research of another graphs.

Definition 1.1 *[4] Let $G = (V(G), E(G)$ be a graph of order n and size m having no isolated vertices. A bijection $f : E(G) \to \{1, 2, 3, ..., m\}$ is called a local antimagic labeling if for all $uv \in E(G)$ we have $w(u) \neq w(v)$, where for $w(u) = \Sigma_{e \in E(u)} f(e)$. A graph G is local antimagic if G has a local antimagic labeling.*

Montgomery [8] introduced the concept of r-dynamic coloring, definition of r-dynamic coloring as follows,

Definition 1.2 *[8] An r-dynamic coloring of a graph G is defined to be a map c from V to the set of colors such that*

- *If $uv \in E(G)$, then $c(u) \neq c(v)$, and*
- *For each vertex $v \in V(G), |c(N(v))| \geq min\{r, d(v)\}$.*

The minimum k such that graph G with an r-dynamic k-coloring is called the r-dynamic chromatic number of graph G, $\chi_r(G)$. This concept was introduced by Montgomery [8]. He found lower bound of the r- dynamic chromatic number, $\chi_r(G) \geq \min\{\Delta(G), r\} + 1$.

In this paper, we introduce the new concept which the combination of local antimagic labeling [4] and r-dynamic chromatic number [3]. The local antimagic labeling induces a proper vertex k-coloring of graph G where the vertex v is assigned the color $w(v)$ such that the neighbors of any vertex v receive at least $\min\{r, d(v)\}$ different colors.

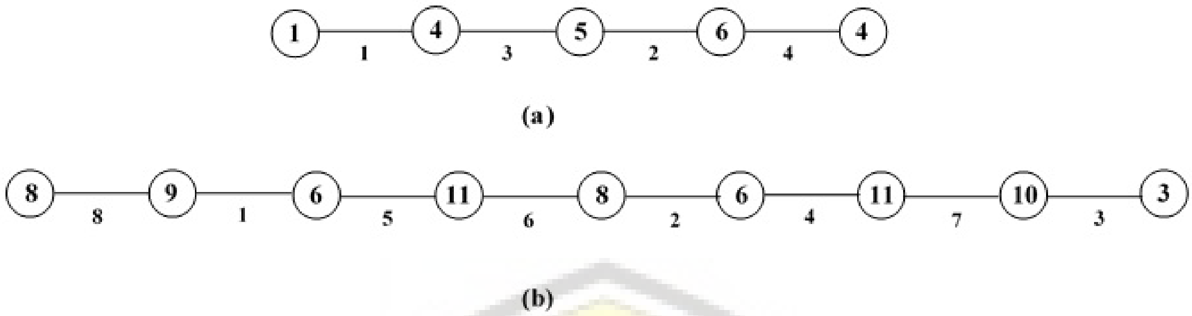

Figure 1. (a) $\chi_r^{la}(P_5) = 4$ (b) $\chi_r^{la}(P_9) = 6$

2. Main Result

In the paper, we have the new concept which combine of local antimagic labeling and r-dynamic chromatic number.

Definition 2.1 *Let $G = (V, E)$ be a graph of size m having no isolated vertices. A bijection $f : E(G) \to \{1, 2, 3, \cdots, m\}$ is called local antimagic r-dynamic coloring, such that:*

- *If $uv \in E(G)$, then $w(u) \neq w(v)$, where $w(u) = \Sigma_{e \in E(u)} f(e)$ and*
- *For each vertex $v \in V(G), |w(N(v))| \geq min\{r, d(v)\}$.*

Definition 2.2 *The local antimagic r-dynamic chromatic number of graph G, denoted by $\chi_r^{la}(G)$ is the minimum k such that graph G has an local antimagic r-dynamic vertex k-coloring induced by local antimagic labelings.*

We find the lower bound of local antimagic $r$dynamic chromatic number of G. We get the local antimagic r-dynamic chromatic number of some classes graphs namely path, cycle, path, star, and complete.

Lemma 2.1 *Let G be a connected graph with order at least 3, then local antimagic r-dynamis chromatic number is $\chi_r^{la}(G) \geq 2$.*

Theorem 2.1 *Let P_n be a path graph with order n, for $n \geq 2$ then local antimagic r-dynamic chromatic number is*

$$\chi_r^{la}(P_n) \leq \begin{cases} 3, & if\ r = 1 \\ 4, & if\ r \geq 2\ and\ n = 5 \\ n, & if\ r \geq 2\ and\ n = 3, 4 \\ \frac{n+4}{3}, & if\ r \geq 2\ and\ n \equiv 5\, mod\, 6,\ n \neq 5 \\ \frac{n+7}{3}, & if\ r \geq 2\ and\ n \equiv 2\, mod\, 6 \\ \frac{n+8}{3}, & if\ r \geq 2\ and\ n \equiv 1\, mod\, 3,\ n \neq 4 \\ \frac{n+9}{3}, & if\ r \geq 2\ and\ n \equiv 0\, mod\, 3,\ n \neq 3 \end{cases}$$

Proof 2.1 Case 1*: For $n \equiv 5$ mod 6, we define a bijection $f : E(P_n) \to \{1, 2, 3, ..., n\}$ as follows*

$$f(x_i x_{i+1}) = \begin{cases} \frac{i+1}{3}, & if\ i \equiv 2\, mod\, 3 \\ \frac{2n-i-1}{3}, & if\ i \equiv 0\, mod\, 3 \\ \frac{2n+i-5}{3}, & if\ i \equiv 1\, mod\, 3 \\ n-1, & if\ i = 1 \end{cases}$$

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From the labels f, we obtain the vertex weight $w(u) = \Sigma_{e\in E(u)} f(e)$ as follows.

$$w(x_i) = \begin{cases} \frac{2n-1}{3}, & \text{if } i \equiv 0 \, mod 3 \\ \frac{4n-5}{3}, & \text{if } i \equiv 1 \, mod 3 \\ \frac{2n+2i-5}{3}, & \text{if } i \equiv 2 \, mod 3 \\ n-1, & \text{if } i = 1 \\ n-2, & \text{if } i = n \\ n, & \text{if } i = 2 \end{cases}$$

From the weight of vertex x_i in path P_n, we can see that for every two adjacent vertices have distinct weight namely $w(v) = n-1, n, \frac{2n-1}{3}, \frac{4n-5}{3}, \frac{2n+5}{3}, \frac{2n-1}{3}, \frac{4n-5}{3}, \frac{2n+11}{3}, \frac{2n-1}{3}, \frac{4n-5}{3}, \frac{2n+17}{3}, \ldots, \frac{2n-1}{3}, \frac{4n-5}{3}, n-2$. Furthermore, it shows that every vertex has $|w(u)| \geq min\{r, d(u)\}$. We obtain that $\chi_r^{la}(P_n) \leq \frac{n+4}{3}$.

Case 2: *For $n \equiv 2$ mod 6, we define a bijection $f : E(P_n) \to \{1, 2, 3, ..., n\}$ as follows*

$$f(x_i x_{i+1}) = \begin{cases} \frac{i+1}{3}, & \text{if } i \equiv 2 \, mod 3 \\ \frac{2n-i-1}{3}, & \text{if } i \equiv 0 \, mod 3 \\ \frac{2n+i-5}{3}, & \text{if } i \equiv 1 \, mod 3 \\ n-1, & \text{if } i = 1 \end{cases}$$

From the labels f, we obtain the vertex weight $w(u) = \Sigma_{e\in E(u)} f(e)$ as follows.

$$w(x_i) = \begin{cases} \frac{2n-1}{3}, & \text{if } i \equiv 0 \, mod 3 \\ \frac{4n-5}{3}, & \text{if } i \equiv 1 \, mod 3 \\ \frac{2n+2i-5}{3}, & \text{if } i \equiv 2 \, mod 3 \\ n-1, & \text{if } i = 1 \\ n-2, & \text{if } i = n \\ n, & \text{if } i = 2 \end{cases}$$

From the weight of vertex x_i in path P_n, we can see that for every two adjacent vertices have distinct weight namely $w(v) = n-1, n, \frac{2n-1}{3}, \frac{4n-5}{3}, \frac{2n+5}{3}, \frac{2n-1}{3}, \frac{4n-5}{3}, \frac{2n+11}{3}, \frac{2n-1}{3}, \frac{4n-5}{3}, \frac{2n+17}{3}, \ldots, \frac{2n-1}{3}, \frac{4n-5}{3}, n-2$. Furthermore, it shows that every vertex has $|w(u)| \geq min\{r, d(u)\}$. We obtain that $\chi_r^{la}(P_n) \leq \frac{n+7}{3}$.

Case 3: *For $n \equiv 1$ mod 3, we define a bijection $f : E(P_n) \to \{1, 2, 3, ..., n\}$ as follows*

$$f(x_i x_{i+1}) = \begin{cases} \frac{i+1}{3}, & \text{if } i \equiv 2 \, mod 3 \\ \frac{2n-i+1}{3}, & \text{if } i \equiv 0 \, mod 3 \\ \frac{2n+i-3}{3}, & \text{if } i \equiv 1 \, mod 3 \\ n-1, & \text{if } i = 1 \end{cases}$$

From the labels f, we obtain the vertex weight $w(u) = \Sigma_{e\in E(u)} f(e)$ as follows.

$$w(x_i) = \begin{cases} \frac{2n+1}{3}, & \text{if } i \equiv 0 \, mod 3 \\ \frac{4n-1}{3}, & \text{if } i \equiv 1 \, mod 3 \\ \frac{2n+2i-3}{3}, & \text{if } i \equiv 2 \, mod 3 \\ n-1, & \text{if } i = 1 \\ \frac{n+2}{3}, & \text{if } i = n \\ n, & \text{if } i = 2 \end{cases}$$

From the weight of vertex x_i in path P_n, we can see that for every two adjacent vertices have distinct weight namely $w(v) = n-1, n, \frac{2n-1}{3}, \frac{4n-5}{3}, \frac{2n+5}{3}, \frac{2n-1}{3}, \frac{4n-5}{3}$,

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$\frac{2n+11}{3}, \frac{2n-1}{3}, \frac{4n-5}{3}, \frac{2n+17}{3}, \ldots, \frac{2n-1}{3}, \frac{4n-5}{3}, n-2$. *Furthermore, it shows that every vertex has* $|w(u)| \geq min\{r, d(u)\}$. *We obtain that* $\chi_r^{la}(P_n) \leq \frac{n+8}{3}$.

Case 4: *For* $n \equiv 0$ *mod* 3, *we define a bijection* $f : E(P_n) \rightarrow \{1, 2, 3, ..., n\}$ *as follows*

$$f(x_i x_{i+1}) = \begin{cases} \frac{i+1}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{2n-i}{3}, & if\ i \equiv 0\ mod 3 \\ \frac{2n+i-4}{3}, & if\ i \equiv 1\ mod 3 \\ n-1, & if\ i = 1 \end{cases}$$

From the labels f, *we obtain the vertex weight* $w(u) = \Sigma_{e \in E(u)} f(e)$ *as follows.*

$$w(x_i) = \begin{cases} \frac{2n}{3}, & if\ i \equiv 0\ mod 3 \\ \frac{4n-3}{3}, & if\ i \equiv 1\ mod 3 \\ \frac{2n+2i-4}{3}, & if\ i \equiv 2\ mod 3 \\ n-1, & if\ i = 1 \\ \frac{n}{3}, & if\ i = n \\ n, & if\ i = 2 \end{cases}$$

From the weight of vertex x_i *in path* P_n, *we can see that for every two adjacent vertices have distinct weight namely* $w(v) = n-1, n, \frac{2n-1}{3}, \frac{4n-5}{3}, \frac{2n+5}{3}, \frac{2n-1}{3}, \frac{4n-5}{3}, \frac{2n+11}{3}, \frac{2n-1}{3}, \frac{4n-5}{3}, \frac{2n+17}{3}, \ldots, \frac{2n-1}{3}, \frac{4n-5}{3}, n-2$. *Furthermore, it shows that every vertex has* $|w(u)| \geq min\{r, d(u)\}$. *We obtain that* $\chi_r^{la}(P_n) \leq \frac{n+9}{3}$.

Case 5: *For* $n = 5$, *we define a bijection* $f : E(P_5) \rightarrow \{1, 2, 3, ..., 4\}$. *We have edge label of path* P_5, $f(e) : 1, 3, 2, 4$ *and vertex weight* $w(v) : 1, 4, 5, 6, 4$. *Based on the vertex weight that for any two adjacent vertices have distict weight and satisfy* $|w(u)| \geq min\{r, d(u)\}$. *Such that, we obtain that* $\chi_2^{la}(P_5) \leq 4$.

Case 6: *For* $n = 3, 4$, *we define a bijection* $f : E(P_n) \rightarrow \{1, ..., n-1\}$. *We have edge label of path* P_3, $f(e) : 1, 2$ *and vertex weight* $w(v) : 1, 3, 2$. *We have edge label of path* P_4, $f(e) : 1, 3, 2$ *and vertex weight* $w(v) : 1, 4, 5, 2$. *Hence, we obtain that* $\chi_r^{la}(P_n) \leq n$.
The proof is complete.

Theorem 2.2 *Let* C_n *be a cycle graph with order* n, *for* $n \geq 3$ *then local antimagic* r*-dynamic chromatic number is*

$$\chi_r^{la}(C_n) \leq \begin{cases} 3, & if\ r = 1 \\ n, & if\ r \geq 2\ and\ n = 3, 4, 5 \\ \lceil \frac{n}{3} \rceil + 2, & if\ r \geq 2\ and\ n \equiv 1, 2, 3\ mod 6 \\ \lceil \frac{n}{3} \rceil + 1, & if\ r \geq 2\ and\ n \equiv 0, 4, 5\ mod 6 \end{cases}$$

Proof 2.2 *For* $r = 1$ *in [4],* $\chi_{la}(C_n) = \chi_1^{la}(C_n) = 3$. *For* $r \geq 2$, *we divide into some cases as follows.*

Case 1: *For* $n \equiv 1$ *mod* 6, *we define a bijection* $f : E(C_n) \longrightarrow \{1, 2, 3, ..., n\}$ *as follows*

$$f(x_i x_{i+1}) = \begin{cases} \frac{i+2}{3}, & if\ i \equiv 1\ mod 3 \\ \frac{2n-i}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{2n+i-2}{3}, & if\ i \equiv 0\ mod 3 \\ n, & if\ i = n \end{cases}$$

From the labels f, *we obtain the vertex weight* $w(u) = \Sigma_{e \in E(u)} f(e)$ *as follows.*

$$w(x_i) = \begin{cases} \frac{2n+1}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{4n-1}{3}, & if\ i \equiv 0\ mod 3 \\ \frac{2n+2i-5}{3}, & if\ i \equiv 1\ mod 3 \\ n+1, & if\ i = 1 \\ 2n-1, & if\ i = n \end{cases}$$

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From the weight of vertex x_i in cycle C_n, we can see that for every two adjacent vertices have distinct weight namely $w(v) = n+1, \frac{2n+1}{3}, \frac{4n-1}{3}, \frac{2n+3}{3}, \frac{2n+1}{3}, \frac{4n-1}{3}, \frac{2n+9}{3}, \frac{2n+1}{3}, \frac{4n-1}{3}, \frac{2n+15}{3}, \dots, \frac{2n+1}{3}, \frac{4n-1}{3}, 2n-1$. *Furthermore, it shows that every vertex has* $|w(u)| \geq min\{r, d(u)\}$. *We obtain that* $\chi_r^{la}(C_n) \leq \frac{n+8}{3}$.

Case 2*: For* $n \equiv 2 \bmod 6$*, we define a bijection* $f : E(C_n) \longrightarrow \{1, 2, 3, ..., n\}$ *as follows*

$$f(x_i x_{i+1}) = \begin{cases} \frac{i+2}{3}, & if\ i \equiv 1\ mod 3 \\ \frac{2n-i+1}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{2n+i-1}{3}, & if\ i \equiv 0\ mod 3 \\ n, & if\ i = n \end{cases}$$

From the labels f, we obtain the vertex weight $w(u) = \Sigma_{e \in E(u)} f(e)$ *as follows.*

$$w(x_i) = \begin{cases} \frac{2n+2}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{4n+1}{3}, & if\ i \equiv 0\ mod 3 \\ \frac{2n+2i}{3}, & if\ i \equiv 1\ mod 3 \\ n+1, & if\ i = 1 \end{cases}$$

From the weight of vertex x_i in cycle C_n, we can see that for every two adjacent vertices have distinct weight namely $w(v) = n+1, \frac{2n+2}{3}, \frac{4n+1}{3}, \frac{2n+8}{3}, \frac{2n+2}{3}, \frac{4n+1}{3}, \frac{2n+14}{3}, \frac{2n+2}{3}, \frac{4n+1}{3}, \frac{2n+20}{3}, \dots, \frac{2n+2}{3}, \frac{4n+1}{3}$. *Furthermore, it shows that every vertex has* $|w(u)| \geq min\{r, d(u)\}$. *We obtain that* $\chi_r^{la}(C_n) \leq \frac{n+7}{3}$.

Case 3*: For* $n \equiv 3 \bmod 6$*, we define a bijection* $f : E(C_n) \longrightarrow \{1, 2, 3, ..., n\}$ *as follows*

$$f(x_i x_{i+1}) = \begin{cases} \frac{i+2}{3}, & if\ i \equiv 1\ mod 3 \\ \frac{2n-i+2}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{2n+i}{3}, & if\ i \equiv 0\ mod 3 \\ n, & if\ i = n \end{cases}$$

From the labels f, we obtain the vertex weight $w(u) = \Sigma_{e \in E(u)} f(e)$ *as follows.*

$$w(x_i) = \begin{cases} \frac{2n+3}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{4n+3}{3}, & if\ i \equiv 0\ mod 3 \\ \frac{2n+2i+1}{3}, & if\ i \equiv 1\ mod 3 \\ n+1, & if\ i = 1 \end{cases}$$

From the weight of vertex x_i in cycle C_n, we can see that for every two adjacent vertices have distinct weight namely $w(v); n+1, \frac{2n+3}{3}, \frac{4n+3}{3}, \frac{2n+9}{3}, \frac{2n+3}{3}, \frac{4n+3}{3}, \frac{2n+15}{3}, \frac{2n+3}{3}, \frac{4n+3}{3}, \frac{2n+21}{3}, \dots, \frac{2n+3}{3}, \frac{4n+3}{3}$. *Furthermore, it shows that every vertex has* $|w(u)| \geq min\{r, d(u)\}$. *We obtain that* $\chi_r^{la}(C_n) \leq \frac{n+6}{3}$.

Case 4*: For* $n \equiv 4 \bmod 6$*, we define a bijection* $f : E(C_n) \longrightarrow \{1, 2, 3, ..., n\}$ *as follows*

$$f(x_i x_{i+1}) = \begin{cases} \frac{i+2}{3}, & if\ i \equiv 1\ mod 3 \\ \frac{2n-i}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{2n+i-2}{3}, & if\ i \equiv 0\ mod 3 \\ n, & if\ i = n \end{cases}$$

From the labels f, we obtain the vertex weight $w(u) = \Sigma_{e \in E(u)} f(e)$ *as follows.*

$$w(x_i) = \begin{cases} \frac{2n+1}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{4n-1}{3}, & if\ i \equiv 0\ mod 3 \\ \frac{2n+2i-1}{3}, & if\ i \equiv 1\ mod 3 \\ n+1, & if\ i = 1 \\ 2n-1, & if\ i = n \end{cases}$$

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From the weight of vertex x_i *in cycle* C_n*, we can see that for every two adjacent vertices have distinct weight namely* $w(v) = n+1, \frac{2n+1}{3}, \frac{4n-1}{3}, \frac{2n+7}{3}, \frac{2n+1}{3}, \frac{4n-1}{3}, \frac{2n+13}{3}, \frac{2n+1}{3}, \frac{4n-1}{3}, \frac{2n+19}{3}, \dots, \frac{2n+1}{3}, \frac{4n-1}{3}, 2n-1$. *Furthermore, it shows that every vertex has* $|w(u)| \geq min\{r, d(u)\}$. *We obtain that* $\chi_r^{la}(C_n) \leq \frac{n+5}{3}$.

Case 5*: For* $n \equiv 5 \bmod 6$*, we define a bijection* $f : E(C_n) \to \{1, 2, 3, ..., n\}$ *as follows*

$$f(x_i x_{i+1}) = \begin{cases} \frac{i+2}{3}, & if\ i \equiv 1\ mod 3 \\ \frac{2n-i+2}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{2n+i-1}{3}, & if\ i \equiv 0\ mod 3 \\ n, & if\ i = n \end{cases}$$

From the labels f*, we obtain the vertex weight* $w(u) = \Sigma_{e \in E(u)} f(e)$ *as follows.*

$$w(x_i) = \begin{cases} \frac{2n+2}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{4n+1}{3}, & if\ i \equiv 0\ mod 3 \\ \frac{2n+2i}{3}, & if\ i \equiv 1\ mod 3 \\ n+1, & if\ i = 1 \end{cases}$$

From the weight of vertex x_i *in cycle* C_n*, we can see that for every two adjacent vertices have distinct weight namely* $w(v) = n+1, \frac{2n+2}{3}, \frac{4n+1}{3}, \frac{2n+8}{3}, \frac{2n+2}{3}, \frac{4n+1}{3}, \frac{2n+14}{3}, \frac{2n+2}{3}, \frac{4n+1}{3}, \frac{2n+20}{3}, \dots, \frac{2n+2}{3}, \frac{4n+1}{3}$. *Furthermore, it shows that every vertex has* $|w(u)| \geq min\{r, d(u)\}$. *We obtain that* $\chi_r^{la}(C_n) \leq \frac{n+4}{3}$.

Case 6*: For* $n \equiv 0 \bmod 6$*, we define a bijection* $f : E(C_n) \to \{1, 2, 3, ..., n\}$ *as follows*

$$f(x_i x_{i+1}) = \begin{cases} \frac{i+2}{3}, & if\ i \equiv 1\ mod 3 \\ \frac{2n-i+2}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{2n+i}{3}, & if\ i \equiv 0\ mod 3 \end{cases}$$

From the labels f*, we obtain the vertex weight* $w(u) = \Sigma_{e \in E(u)} f(e)$ *as follows.*

$$w(x_i) = \begin{cases} \frac{2n+3}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{4n+3}{3}, & if\ i \equiv 0\ mod 3 \\ \frac{2n+2i+1}{3}, & if\ i \equiv 1\ mod 3 \\ n+1, & if\ i = 1 \end{cases}$$

From the weight of vertex x_i *in cycle* C_n*, we can see that for every two adjacent vertices have distinct weight namely* $w(v); n+1, \frac{2n+3}{3}, \frac{4n+3}{3}, \frac{2n+9}{3}, \frac{2n+3}{3}, \frac{4n+3}{3}, \frac{2n+15}{3}, \frac{2n+3}{3}, \frac{4n+3}{3}, \frac{2n+21}{3}, \dots, \frac{2n+3}{3}, \frac{4n+3}{3}$. *Furthermore, it shows that every vertex has* $|w(u)| \geq min\{r, d(u)\}$. *We obtain that* $\chi_r^{la}(C_n) \leq \frac{n+3}{3}$.

Case 7*: For* $n = 3, 4, 5$*, Define a bijection* $f : E(C_n) \to \{1, 2, 3, ..., n\}$*. We have edge label of cycle* C_n *as follows*

- *We have edge label of cycle* C_3*,* $f(e) : 1, 2, 3$ *and vertex weight* $w(v) : 4, 3, 5$
- *We have edge label of cycle* C_4*,* $f(e) : 1, 3, 4, 2$ *and vertex weight* $w(v) : 3, 4, 7, 6$
- *We have edge label of cycle* C_5*,* $f(e) : 1, 3, 5, 2, 4$ *and vertex weight* $w(v) : 5, 4, 8, 7, 6$

Hence, we obtain that $\chi_r^{la}(C_n) \leq n$.
From Case 1-7, we obtain that for $n \equiv 1, 2, 3$ *(mod 6),* $\chi_r^{la}(C_n) \leq \lceil \frac{n}{3} \rceil + 2$ *and for* $n \equiv 0, 4, 5$ *(mod 6),* $\chi_r^{la}(C_n) \leq \lceil \frac{n}{3} \rceil + 1$*. The proof is complete.*

Theorem 2.3 *Let* S_n *be a star graph with order* $n+1$*, for* $n \geq 3$ *then local antimagic* r*-dynamic chromatic number is* $\chi_r^{la}(S_n) = n + 1$.

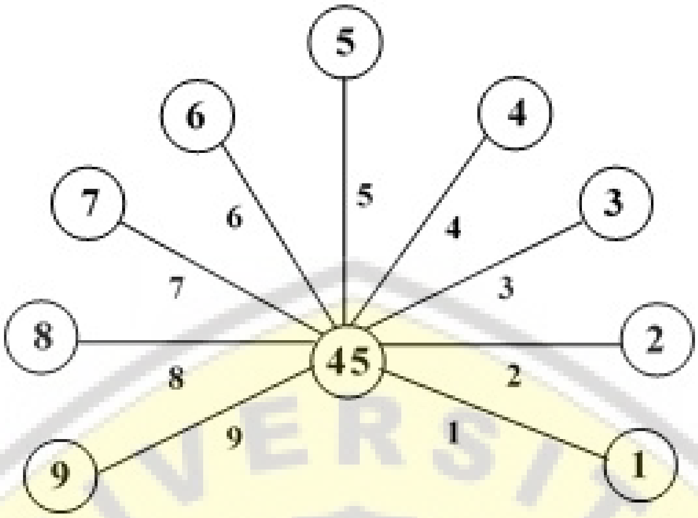

Figure 2. $\chi_r^{la}(S_9) = 10$

Proof 2.3 *Consider the star graph, S_n with central vertex v_0, $d(v_0) = n$ and vertices v_i, $d(v_i) = 1$, $1 \le i \le n$. The order of star graph is $n+1$ and the size of star graph is $|E(S_n)| = n$, namely $e_i = v_0v_i, 1 \le i \le n$. Define a bijection $f : E(S_n) \to \{1, 2, 3, ..., n\}$ as, $f(e_i) = i$, $1 \le i \le n$ such that $w(v_0) = \Sigma_{k=1}k$, $1 \le k \le n$ and $w(v_i) = i$, $1 \le i \le n$. Hence, it shows $|w(N(v_0))| = n \ge min\{r, d(v_0)\}$ and $|w(N(v_i))| = 1 \ge min\{r, d(v_i)\}$. We obtain that $\chi_r^{la}(S_n) = n + 1$.*

Theorem 2.4 *Let K_n be a complete graph with order n, for $n \ge 3$ then local antimagic r-dynamic chromatic number is $\chi_r^{la}(K_n) = n$.*

Proof 2.4 *Consider the complete graph, K_n vertices v_i, $d(v_i) = n - 1$, $1 \le i \le n$. The order of complete graph is n and the size of star graph is $|E(K_n)| = \frac{n(n-1)}{2}$, namely $e_j = v_iv_{i+k}, 1 \le i \le n, 1 \le k \le n - i$. Define a bijection $f : E(K_n) \to \{1, 2, 3, ..., \frac{n(n-1)}{2}\}$ as, $f(e_j) = j$, $1 \le j \le \frac{n(n-1)}{2}$ such that $w(v) \ne w(u)$ every $e = uv, e \in E(K_n)$. Hence, for every $v \in V(G)$, it shows $|w(N(v))| = n - 1 \ge min\{r, d(v)\}$. We obtain that $\chi_r^{la}(K_n) = n$.*

3. Conclusion

We have found the concept local antimagic r-dynamic coloring. We find the basic results namely the upper bound of the local antimagic r-dynamic chromatic number of some classes graph, namely path, cycle, star, and complete graph

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