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SUPER (a, d)-CYCLES-ANTIMAGIC LABELING OF SUBDIVISION OF A FAN GRAPH

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Abstract

We consider a simple, connected and undirected graph $G(V, E)$ with vertex set $V(G)$ and edge set $E(G)$. There is a super (a, d)-$\mathcal{H}$-*antimagic* total labeling on the graph $G(V, E)$ if there exists a bijection $f : V \cup E \rightarrow \{1, 2, ..., |V| + |E|\}$ such that for all subgraphs isomorphic to H, the total H-weights $W(H) = \sum_{v \in V(H)} f(v) + \sum_{e \in E(H)} f(e)$ form an arithmetic sequence

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$\{a, a+d, a+2d, ..., a+(m-1)d\}$, where $a > 0$ is the smallest value, d is the feasible difference, and m is the number of all subgraphs isomorphic to H. In this paper, we investigate the existence of super (a, d)-$\mathcal{H}$-*antimagic* total labeling for subdivisions of a fan graph $S(F_m)$, when subgraphs H are cycles.

1. Introduction

Given that a graph $G = (V, E)$ is nontrivial, finite, simple, undirected and connected graph of vertex set V and edge set E. For more details on graph, see [10, 3, 4]. A covering of G is a family of subgraphs $H_1, H_2, ..., H_n$ such that all vertices $V(G)$ and edges $E(G)$ belong to at least one of the subgraphs H_i, $i = 1, 2, ..., n$ taken into account as a cover. In this case, we say that G admits $(H_1, H_2, ..., H_n)$-*covering* if every subgraph H_i is isomorphic to a given graph H admits a special property to be an H-labeling.

A graph G is said to be an (a, d)-H-*antimagic total graph* if there exists a bijective function $f: V(G) \cup E(G) \to \{1, 2, ..., |V(G)| + |E(G)|\}$ such that for all subgraphs isomorphic to H, the total H-weights

$$w(H) = \sum_{v \in V(H)} f(v) + \sum_{e \in E(H)} f(e)$$

form an arithmetic sequence $\{a, a+d, a+2d, ..., a+(m-1)d\}$, where a and d are positive integers and n is the number of all subgraphs isomorphic to H. If such a function exists, then f is called an (a, d)-H-*antimagic total labeling* of G, see [11]. The total H-weight is the sum of both vertex and edge labels belonging to a subgraph H, under a given labeling f. The H-weight under a labeling f is denoted by $w(H)$. Such a labeling is called *super* if the smallest possible labels appear on the vertices. If G admits a super (a, d)-H-antimagic total labeling, then we say that G is a *super* (a, d)-H-*antimagic graph*. For $d = 0$, it is called H-*magic* or H-*supermagic*.

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Some relevant results have been published in many journals, some of them can be found in [1, 2, 8, 9]. Furthermore, Lladó and Moragas [12] proved that wheels, windmills, books and prisms are C_t^k-magic for some t. Inayah et al. in [11] proved that for any H and any integer $k \geq 2$, shack(H, v, k) which contains exactly k subgraphs isomorphic to H admits H-super antimagic. Dafik et al. in [5, 6] also obtained a cycle-super antimagicness of connected and cisconnected tensor product of graphs, and constructed H-antimagic graphs by using smaller edge-antimagic graphs. Furthermore, Dafik et al. in [7] also determined the super H-antimagicness of an edge comb product of graphs with subgraph as a terminal of its amalgamation.

2. The Results

We study the subdivision of graph G. By subdivision of graph, denoted by $S(G)$, we mean a graph obtained from G by replacing each edge uv of G by a new vertex y and the two new edges uy and vy. For details on the subdivision of graph G, see [4]. The vertex y is called a *subdivision vertex* on uv.

We deal with the super cycle-antimagic total labelings of subdivision of a fan graph, denoted by $S(F_m)$.

Observation 1. Let $S(F_m)$ be a subdivision of a fan graph. The order and size of graph $S(F_m)$ are, respectively, $|V(S(F_m))| = 3m$ and $|E(S(F_m))| = 4m - 2$.

Proof. The graph $S(F_m)$ is a connected graph with vertex set $V(S(F_m)) = \{x\} \cup \{x_i; 1 \leq i \leq m\} \cup \{y_i; 1 \leq i \leq m\} \cup \{z_i; 1 \leq i \leq m-1\}$ and edge set

$$E(S(F_m)) = \{y_i z_i; 1 \leq i \leq m-1\} \cup \{xx_i; 1 \leq i \leq m\}$$
$$\cup \{z_i y_{i+1}; 1 \leq i \leq m-1\} \cup \{y_i x_i; 1 \leq i \leq m\}.$$

Thus, the order of the graph $S(F_m)$ is $|V(S(F_m))| = 3m$ and the size of the graph $S(F_m) |E(S(F_m))| = 4m - 2$. □

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For illustration, we give an example of subdivision of a fan graph $S(F_m)$ depicted in Figure 1.

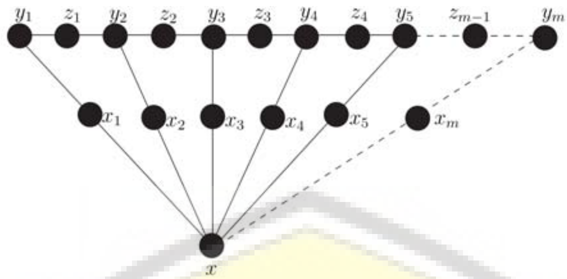

Figure 1. Example of subdivision of a fan graph $S(F_m)$.

Observation 2. Let C_t^k be a cycle of t vertices of subdivision of a fan graph $S(F_m)$, where $t = 6, 8, \ldots, 2m - 2$. The number of cycles of order t which is a cover $H \cong C_t^k$ of $S(F_m)$ is given by $|H| = m - \frac{t-4}{2}$.

Proof. Let C_t^k be a cycle of t vertices of subdivision of a fan graph $S(F_m)$, where $t = 6, 8, \ldots, 2m - 2$ for $3 \le m \le 4$ and $t = 6, 8, \ldots, 2m - 2$ for $m \ge 5$. The tth cycle of C_t^k can be formed by the following set of vertices

$$C_t^k = \{x, x_k, y_k, z_k, y_{k+1}, z_{k+1}, y_{k+2}, z_{k+2}, \ldots,$$
$$z_{k+\frac{t-6}{2}}, y_{k+\frac{t-6}{2}}, y_{k+\frac{t-4}{2}}, x_{k+\frac{t-4}{2}}, x\}.$$

It is easy to see that $k = 1, 2, \ldots, m - \left(\frac{t-4}{2}\right)$. Thus $|C_t^k| = m - \frac{t-4}{2}$. It concludes the proof. □

Furthermore, we can determine the C_k^t-weight of the cycle C_k^t, $k = 1, 2, \ldots, m - \left(\frac{t-4}{2}\right)$ under a total labeling g:

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$$w_g(C_t^k) = \sum_{v \in V(C_t^k)} f(v) + \sum_{e \in E(C_t^k)} f(e)$$

$$= \sum_{s=0}^{\frac{t-6}{2}} [g(y_{k+s}) + g(z_{k+s}) + g(y_{k+s}z_{k+s}) + g(z_{k+s}y_{k+s+1})]$$

$$+ g(y_{k+\left(\frac{t-4}{2}\right)}) + g(x_k) + g(x_{k+\left(\frac{t-4}{2}\right)}) + g(x)$$

$$+ g(y_{k+\left(\frac{t-4}{2}\right)}x_{k+\left(\frac{t-4}{2}\right)}) + g(y_k x_k) + g(x_k x) + g(x_{k+\left(\frac{t-4}{2}\right)}x). \quad (1)$$

From now on, we show our main results. We have found that the graph $S(F_m)$ admits super (a, d)-C_t^k antimagic labeling for differences $d = \{0, 1, 2, 4\}$.

Theorem 1. *Let $t = 6, 8, \ldots, 2m - 2$ for $3 \leq m \leq 4$ and $t = 6, 8, \ldots, 2m - 2$ for $m \geq 5$. Let $k = 1, 2, \ldots, m - \left(\frac{t-4}{2}\right)$. The subdivision of fan $S(F_m)$ admits a super (a, d)-C_t^k-antimagic labeling for $d = 0$.*

Proof. We define the labeling

$$g_1, g_1 : V(S(F_m)) \cup E(S(F_m)) \to \{1, 2, \ldots, p_{S(F_m)} + q_{S(F_m)}\}$$

in the following way:

$g_1(y_i) = 2i - 1; 1 \leq i \leq m$ $\qquad$ $g_1(z_i) = 2i; 1 \leq i \leq m - 1$

$g_1(x_i) = 3m + i - 1; 1 \leq i \leq m$ $\qquad$ $g_1(x) = 2m$

$g_1(y_i z_i) = 5m - 2i; 1 \leq i \leq m - 1$ $\qquad$ $g_1(y_i x_i) = 5m + 2i - 3; 1 \leq i \leq m$

$g_1(xx_i) = 7m - 2i; 1 \leq i \leq m$ $\qquad$ $g_1(z_i y_{i+1}) = 5m - 2i - 1; 1 \leq i \leq m - 1.$

Evidently, it is easy to see that g_1 is a bijective function, as it is a map $g_1 : V(S(F_m)) \cup E(S(F_m)) \to \{1, 2, \ldots, 3m, \ldots, 5m - 1, 5m, \ldots, 7m - 2\}$. The total weight of $V(S(F_m)) \cup E(S(F_m)) = \{y_i; 1 \leq i \leq m\} \cup \{y_i z_i; 1 \leq i \leq m - 1\}$

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under the labeling g_1, is given by

$$g_1(y_i) + g_1(y_i z_i) = g_1(z_i) + g_1(z_i y_{i+1}) = [2i - 1] + [5m - 2i] = 5m - 1. \quad (2)$$

The total edge-weight of

$$E(S(F_m)) = \{xx_i; 1 \leq i \leq m\} \cup \{y_i x_i; 1 \leq i \leq m\}$$

is

$$g_1(xx_i) + g_1(y_i x_i) = [5m + 2i - 3] + [7m - 2i] = 12m - 3. \quad (3)$$

From equations (1), (2) and (3), we obtain the total C_t^k-weight as follows:

$$\begin{aligned} w_{g_1}(C_t^k) &= \sum_{v \in V(C_t^k)} f(v) + \sum_{e \in E(C_t^k)} f(e) \\ &= [(t-4)(5m-1)] + 2 \times [12m - 3] + \left[3m + k + \frac{t-4}{2}\right] \\ &\quad + [3m - k + 1] + 2m \\ &= 5m(t-4) - t + 4 + 24m - 6 + 3m + k + \frac{t}{2} - 2 \\ &\quad + 3m - k + 1 + 2m \\ &= m(5t + 12) - \frac{t}{2} - 3. \end{aligned}$$

It is easy to see that the total C_t-weights of $S(F_m)$, under the labeling g_1, when $t = 6, 8, \ldots, 2m$ for $3 \leq m \leq 4$ and when $t = 6, 8, \ldots, 2m - 2$ for $m \geq 5$, and for $k = 1, 2, \ldots, m - \left(\frac{t-4}{2}\right)$, constitute the following sets:

$$C_t^k = \left\{m(5t + 12) - \frac{t}{2} - 3, m(5t + 12) - \frac{t}{2} - 3, \ldots, m(5t + 12) - \frac{t}{2} - 3\right\}.$$

It concludes that the subdivision of fan $S(F_m)$ admits a super (a, d)-C_t^k-antimagic total labeling with feasible $d = 0$. $\square$

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Theorem 2. *Let* $t = 6, 8, ..., 2m-2$ *for* $3 \le m \le 4$ *and* $t = 6, 8, ..., 2m-2$ *for* $m \ge 5$. *Let* $k = 1, 2, ..., m - \left(\frac{t-4}{2}\right)$. *Then subdivision of fan* $S(F_m)$ *admits a super* (a, d)-C_t^k *-antimagic labeling for* $d = 1$.

Proof. We define the labeling

$$g_2,\ g_2 : V(S(F_m)) \cup E(S(F_m)) \to \{1, 2, ..., p_{S(F_m)} + q_{S(F_m)}\}$$

in the following way:

$g_2(y_i) = i; 1 \le i \le m$ $\quad$ $g_2(x_i) = 2m - i + 1; 1 \le i \le m$

$g_2(z_i) = 3m - i; 1 \le i \le m - 1$ $\quad$ $g_2(x) = 3m$

$g_2(y_i z_i) = 3m + i; 1 \le i \le m - 1$ $\quad$ $g_2(y_i x_i) = 5m + 2i - 3; 1 \le i \le m$

$g_2(xx_i) = 7m - 2i; 1 \le i \le m$ $\quad$ $g_2(z_i y_{i+1}) = 5m - i - 1; 1 \le i \le m - 1.$

Evidently, it is easy to see that g_2 is a bijection, as it is a map $g_2 : V(S(F_m)) \cup E(S(F_m)) \to \{1, 2, ..., 3m, ..., 5m - 1, 5m, ..., 7m - 2\}$. The total weight of

$$\begin{aligned} & V(S(F_m)) \cup E(S(F_m)) \\ &= \{y_i; 1 \le i \le m\} \cup \{z_i; 1 \le i \le m-1\} \\ & \cup \{y_i z_i; 1 \le i \le m-1\} \cup \{z_i y_{i+1}; 1 \le i \le m-1\} \end{aligned}$$

under the labeling g_2, is

$$\begin{aligned} & g_2(y_i) + g_2(y_i z_i) + g_2(z_i) + g_2(z_i y_{i+1}) \\ &= i + 3m + i + 3m - i + 5m - i - 1 = 11m - 1. \end{aligned} \tag{4}$$

The total edge-weight of $E(S(F_m)) = \{xx_i; 1 \le i \le m\} \cup \{y_i x_i; 1 \le i \le m\}$ is as follows:

$$g_2(xx_i) + g_2(y_i x_i) = [5m + 2i - 3] + [7m - 2i] = 12m - 3. \tag{5}$$

From equations (1), (4) and (5), we obtain the total C_t^k -weight as follows:

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$$w_{g_2}(C_t^k) = \sum_{v \in V(C_t^k)} f(v) + \sum_{e \in E(C_t^k)} f(e)$$

$$= \left[\left(\frac{t-4}{2}\right)(11m-1)\right] + 2 \times [12m-3] + [2m+1]$$

$$+ [2m-k+1] + 3m$$

$$= 11m\left(\frac{t-4}{2}\right) - \frac{t-4}{2} + 24m - 6 + 2m + 1 + 2m - k + 1 + 3m$$

$$= 11m\frac{t}{2} + 9m - \frac{t}{2} - 3 - k.$$

It is easy to see that the total C_t-weights of $S(F_m)$, under the labeling g_2, $t = 6, 8, \ldots, 2m$ for $3 \le m \le 4$ and $t = 6, 8, \ldots, 2m-2$ for $m \ge 5$ and $k = 1, 2, \ldots, m - \left(\frac{t-4}{2}\right)$, constitute the following sets:

$$C_t^k = \left\{11m\frac{t}{2} + 9m - \frac{t}{2} - 3 - k, \ldots, 11m\frac{t}{2} + 9m - \frac{t}{2} - 3 - 2,\right.$$

$$\left. 11m\frac{t}{2} + 9m - \frac{t}{2} - 3 - 1\right\}.$$

It concludes that the subdivision of fan $S(F_m)$ admits a super (a, d)-C_t^k-antimagic total labeling with feasible $d = 1$. □

Theorem 3. *Let* $t = 6, 8, \ldots, 2m-2$ *for* $3 \le m \le 4$ *and* $t = 6, 8, \ldots, 2m-2$ *for* $m \ge 5$. *Let* $k = 1, 2, \ldots, m - \left(\frac{t-4}{2}\right)$. *Then subdivision of fan* $S(F_m)$ *admits a super* (a, d)-C_t^k *-antimagic labeling for* $d = 4$.

Proof. We define the labeling

$$g_3,\ g_3 : V(S(F_m)) \cup E(S(F_m)) \to \{1, 2, \ldots, p_{S(F_m)} + q_{S(F_m)}\}$$

in the following way:

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$g_3(y_i) = 2i - 1;\ 1 \le i \le m$ $\qquad g_3(z_i) = 2i;\ 1 \le i \le m - 1$

$g_3(x_i) = 2m + i - 1;\ 1 \le i \le m$ $\qquad g_3(x) = 3m$

$g_3(y_i z_i) = 7m - 2i;\ 1 \le i \le m - 1$ $\qquad g_3(y_i x_i) = 5m - 2i + 1;\ 1 \le i \le m$

$g_3(xx_i) = 3m + 2i;\ 1 \le i \le m$ $\qquad g_3(z_i y_{i+1}) = 7m - 2i - 1;\ 1 \le i \le m - 1.$

Evidently, it is easy to see that g_3 is a bijection as it is a map $g_3 : V(S(F_m)) \cup E(S(F_m)) \to \{1, 2, \ldots, 3m, \ldots, 5m - 1, 5m, \ldots, 7m - 2\}$. The total weight of $V(S(F_m)) \cup E(S(F_m)) = \{y_i;\ 1 \le i \le m\} \cup \{y_i z_i;\ 1 \le i \le m - 1\}$ under the labeling g_3, is given by

$$g_3(y_i) + g_3(y_i z_i) = g_3(z_i) + g_3(z_i y_{i+1}) = [2i - 1] + [7m - 2i] = 7m - 1. \quad (6)$$

The total edge-weight of $E(S(F_m)) = \{xx_i;\ 1 \le i \le m\} \cup \{y_i x_i;\ 1 \le i \le m\}$ is as follows:

$$g_3(xx_i) + g_3(y_i x_i) = [5m - 2i + 1] + [3m + 2i] = 8m + 1. \quad (7)$$

From equations (1), (6) and (7), we obtain the total C_t^k-weight as follows:

$$w_{g_3}(C_t^k) = \sum_{v \in V(C_t^k)} f(v) + \sum_{e \in E(C_t^k)} f(e)$$

$$= [(t - 4)(7m - 1)] + 2 \times [8m + 1] + \left[2(m + 2) + 3\left(k + \frac{t}{2} - 4\right)\right]$$

$$+ [2m + k - 1] + 3m$$

$$= (t - 4)7m - t + 4 + 16m + 2 + 2m + 4 + 3\left(k + \frac{t}{2}\right)$$

$$- 12 + 2m + k - 1 + 3m$$

$$= m(7t - 5) - 3 + \frac{t}{2} + 4k.$$

It is easy to see that the total C_t-weights of $S(F_m)$, under the labeling g_3, $t = 6, 8, \ldots, 2m$ for $3 \le m \le 4$ and $t = 6, 8, \ldots, 2m - 2$ for $m \ge 5$ and $k = 1, 2, \ldots, m - \left(\frac{t - 4}{2}\right)$, constitute the following sets:

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$$C_t^k = \left\{m(7t-5)-3+\frac{t}{2}+4,\ m(7t-5)-3+\frac{t}{2}+8,\ \ldots,\ m(7t-5)-3+\frac{t}{2}+4k\right\}.$$

It concludes that the subdivision of fan $S(F_m)$ admits a super (a, d)-C_t^k-antimagic total labeling with feasible $d = 4$. □

Theorem 4. *Let* $t = 6, 8, \ldots, 2m-2$ *for* $3 \le m \le 4$ *and* $t = 6, 8, \ldots, 2m-2$ *for* $m \ge 5$. *Let* $k = 1, 2, \ldots, m - \left(\frac{t-4}{2}\right)$. *Then subdivision of fan* $S(F_m)$ *admits a super* (a, d)-C_t^k*-antimagic labeling for* $d = 2$.

Proof. We define the labeling g_3,

$$g_3 : V(S(F_m)) \cup E(S(F_m)) \to \{1, 2, \ldots, p_{S(F_m)} + q_{S(F_m)}\}$$

in the following way:

$g_4(y_i) = 2i-1;\ 1 \le i \le m$ $\quad$ $g_4(z_i) = 2i;\ 1 \le i \le m-1$

$g_4(x_i) = 3m-i+1;\ 1 \le i \le m$ $\quad$ $g_4(x) = 2m$

$g_4(y_i z_i) = 5m-i-1;\ 1 \le i \le m-1$ $\quad$ $g_4(y_i x_i) = 5m+2i-3;\ 1 \le i \le m$

$g_4(xx_i) = 7m-2i;\ 1 \le i \le m$ $\quad$ $g_4(z_i y_{i+1}) = 4m-i;\ 1 \le i \le m-1.$

Evidently, it is easy to see that g_4 is a bijection as it is a map $g_3 : V(S(F_m)) \cup E(S(F_m)) \to \{1, 2, \ldots, 3m, \ldots, 5m-1, 5m, \ldots, 7m-2\}$. The total weight of

$$V(S(F_m)) \cup E(S(F_m)) = \{z_i; 1 \le i \le m-1\} \cup \{y_i z_i; 1 \le i \le m-1\} \cup \{z_i y_{i+1}; 1 \le i \le m-1\}$$

under the labeling g_3, is as follows:

$$g_4(z_i) + g_4(y_i z_i) + g_4(z_i y_{i+1}) = (5m-i-1) + 2i + 4m - i = 9m-1. \quad (8)$$

The total edge-weights of $E(S(F_m)) = \{xx_i; 1 \le i \le m\} \cup \{y_i x_i; 1 \le i \le m\}$ are as follows:

$$g_4(xx_i) + g_4(y_i x_i) = [5m+2i-3] + [7m-2i] = 12m-3. \quad (9)$$

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From equations (1), (8) and (9), we obtain the total C_t^k-weight in the following way:

$$w_{g_1}(C_t^k) = \sum_{v \in V(C_t^k)} f(v) + \sum_{e \in E(C_t^k)} f(e)$$

$$= \left[\left(\frac{t-4}{2}\right)(9m-1)\right] + 2 \times [12m-3] + [2m] + [2k-1+2k+t-5]$$

$$+ [3m-k+1] + 3m - \frac{t}{2} + 2 - k$$

$$= 9m\left(\frac{t-4}{2}\right) + 32m + 2k - 7.$$

It is easy to see that the total C_t-weights of $S(F_m)$, under the labeling g_4, $t = 6, 8, ..., 2m$ for $3 \leq m \leq 4$ and $t = 6, 8, ..., 2m-2$ for $m \geq 5$ and $k = 1, 2, ..., m - \left(\frac{t-4}{2}\right)$, constitute the following sets:

$$C_t^k = \left\{9m\left(\frac{t-4}{2}\right) + 32m + 2 - 7,\ 9m\left(\frac{t-4}{2}\right) + 32m + 4\right.$$

$$\left. - 7, ..., 9m\left(\frac{t-4}{2}\right) + 32m + 2\left(m - \left(\frac{t-4}{2}\right)\right) - 7\right\}.$$

It concludes that the subdivision of fan $S(F_m)$ admits a super (a, d)-C_t^k-antimagic total labeling with feasible $d = 2$. □

3. Concluding Remarks

We have shown the existence of super (a, d)-H-antimagicness of subdivision of fan graphs $S(F_m)$, when H is a cycle. We can prove that $d = \{0, 1, 2, 4\}$. As we have not found the result for another difference, we propose the following:

Problem. Find a super (a, d)-H-antimagic labeling of the subdivision of a fan graph for $d \neq \{0, 1, 2, 4\}$.

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