Preface: International Conference on Science and Applied Science (ICSAS) 2018

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International Conference on Science and Applied Science (ICSAS) 2018 was held at the Solo Paragon Hotel, Surakarta, Indonesia on 12 May 2018. The ICSAS 2018 conference is aimed to bring together scholars, leading researchers and experts from diverse backgrounds and applications areas in Science. Special emphasis is placed on promoting interaction between the science theoretical, experimental, and education sciences, engineering so that a high level exchange in new and emerging areas within Mathematics, Chemistry, Physics and Biology, all areas of sciences and applied mathematics and sciences is achieved.

In ICSAS 2018, there are eight parallel sessions and four keynote speakers. It is an honour to present this volume of AIP Conference Proceedings and we deeply thank the authors for their enthusiastic and high-grade contribution. From the review results, there are 166 papers which will be published in AIP Conference Proceedings We would like to express our sincere gratitude to all in the Programming Committee who have reviewed the papers and developed a very interesting Conference Program, as well as thanking the invited and plenary speakers. Finally, we would like to thank the conference chairman, the members of the steering committee, the organizing committee, the organizing secretariat and the financial support from the Sebelas Maret University that allowed ICSAS 2018 to be a success.

The Editors

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Dewanta Arya Nugraha, S.Pd., M.Pd., M.Si.

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The antimagicness of super (a, d) - $P_2 \trianglerighteq H$ total covering on total comb graphs

Rafiantika M. Prihandini, Dafik, Slamin, and I. H. Agustin

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The Antimagicness of Super $(a, d) - P_2 \dot{\trianglerighteq} H$ Total Covering on Total Comb Graphs

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Abstract. A graph can be constructed in several ways. One of them is by operating two or more graphs. The resulting graphs will be a new graph which has certain characteristics. One of the latest graph operations is total comb of two graphs. Let L, H be a finite collection of nontrivial, simple and undirected graphs. The total comb product is a graph obtained by taking one copy of L and $|V(L)| + |E(L)|$ copies of H and grafting the i-th copy of H at the vertex o and edge uv to the i-th vertex and edge of L. The graph G is said to be an (a, d)-$P_2 \dot{\trianglerighteq} H$-antimagic total graph if there exists a bijective function $f : V(G) \cup E(G) \rightarrow \{1, 2, \dots, |V(G)| + |E(G)|\}$ such that for all subgraphs isomorphic to $P_2 \dot{\trianglerighteq} H$, the total $P_2 \dot{\trianglerighteq} H$-weights $W(P_2 \dot{\trianglerighteq} H) = \sum_{v \in V(P_2 \dot{\trianglerighteq} H)} f(v) + \sum_{e \in E(P_2 \dot{\trianglerighteq} H)} f(e)$ form an arithmetic sequence. An (a, d)-$P_2 \dot{\trianglerighteq} H$-antimagic total covering f is called super when the smallest labels appear in the vertices. By using partition technique has been proven that the graph $G = L \dot{\trianglerighteq} H$ admits a super (a, d)-$P_2 \dot{\trianglerighteq} H$ antimagic total labeling with different value $d = d^* + d^*(d_{v_1} + d_{e_1}) + d_{v_2} + d_{e_2} + 1$.

INTRODUCTION

The graph theory was introduced by Leonhard Euler in 1736 while trying to prove the possibility of passing through four regions which was connected with seven bridges over the Pregel river in Konigsberg, Russia at a time. The problem can be expressed by the term graph in determining the four regions as the vertex and the seven bridges as the edges, which connect the corresponding vertex pairs.

One of the topic in graph theory is labeling. The labeling concept has evolved from the total super (a, d) edge antimagic total labeling (*SEATL*) into the super labeling $(a, d) - \mathcal{H}$ - *Antimagic total covering* ($S\mathcal{H}$ATC). Inayah et al in [7] develop the concept of labeling ($S\mathcal{H}ATC$), with the explanation that an (a, d)-$\mathcal{H}$ $-$ *antimagic* on the graph G if there exists a bijective function $f : V(G) \cup E(G) \rightarrow \{1, 2, \dots, |V(G)| + |E(G)|\}$ such that for all subgraphs isomorphic to H, the total H-weights $W(H) = \sum_{v \in V(H)} f(v) + \sum_{e \in E(H)} f(e)$ form an arithmetic sequence $\{a, a+d, a+2d, ..., a+(n-1)d\}$, where a and d are positive integers and n is the number of all subgraphs isomorphic to H.

Some of the labeling related studies that have been developed include [2, 3, 8, 9] and [10, 11, 12, 13, 15]. One of article by Inayah *et al* in [8] explain about connected graph. They prove that, graph $G = shack(H, v, k)$ with $k \geq 2$ and k is the number of graph which isomorphic to H has a super $(a, d) - \mathcal{H}$ antimagic total covering. Dafik *et al* in [5] have proved a super $(a, d) - H$ antimagic total covering of graph $G = c(K \trianglerighteq Amal(H, L \subset H, n))$ with feasible difference d, namely $d = d_v + d_e$. d_v is dependent on the number of vertex and d_e is dependent on the number of edge. Agustin *et al* in [6] have investigated that graph $G = L \triangleright H$ when $L = C_n$ has an (a, d)- super antimagic total covering. This paper is concerned with antimagic total covering of graph $G = L \dot{\trianglerighteq} H$ for connected graph when L, H is any graph as well as selected some case examples for $L = P_n$ and $L = C_n$. The total comb product is a graph obtained by taking one copy of L and $|V(L)| + |E(L)|$ copies of H and grafting the i-th copy of H at the vertex o and edge uv to the i-th vertex and edge of L. We also investigate the connection between edge antimagic vertex labeling (EAVL) and $P_2 \dot{\trianglerighteq} H$-antimagic total covering.

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The labeling results in graph $G = L \trianglerighteq H$ can be used to develop key in ciphertext (secret message). The key is got from edge label that will be used to build the key stream. The key stream is a flowing and unlimited key. In addition it is also sensitive to initial values, so the secret key of the initial value when replaced will result in a different key stream. Complicated key combinations will make the system harder to be hacked by hackers.

To simplify the labeling process will be used partition technique first introduced by [1, 3] is operative. This partition is used to build super $(a, d) - H$ antimagic total labeling on connected graphs $G = L \trianglerighteq H$. The partition technique is the set of integers placed on the table so that each row and column have different numbers. Partition technique can make it easier for us to get all feasible differences d.

A USEFUL LEMMA AND COROLLARY

Graph G is simple and nontrivial. The order of graph L is $|V(L)|$ and the order of graph H is $|V(H)|$. The size of graph L is $|E(L)|$ and the size of graph H is $|E(H)|$. Now we will be determined the vertex and edge set of the graph $G = L \trianglerighteq H$. The vertex set of graph G is $V(G) = \{x_{i,k}; 1 \leq i \leq p_H; 1 \leq k \leq p_L\} \cup \{y_{i,k}; 1 \leq i \leq p_H - 2; 1 \leq k \leq q_L\}$ and the edge set of graph G is $E(G) = \{e_k; 1 \leq k \leq q_L\} \cup \{e_{l,k}; 1 \leq l \leq q_H; 1 \leq k \leq p_L\} \cup \{e'_{l,k}; 1 \leq l \leq q_H - 1; 1 \leq k \leq q_L\}$. Thus we obtain $|V(G)| = |V(L)||V(H)| + |E(L)|(|V(H)| - 2)$ and $|E(G)| = |E(L)| + |E(L)|(|E(H)| - 1) + |V(L)||E(H)|$. Thus $|V(G)| = p_L p_H + q_L(p_H - 2)$ and $|E(G)| = q_L + p_L q_H + q_L(q_H - 1)$. Lemma 1 is the lemma which determine the upper bound of feasible difference d.

Lemma 1 [2] *Let G be a simple graph of order p and size q. If G is super (a, d)-H-antimagic total labeling then $d \leq \frac{(p_G - p_H)p_H + (q_G - q_H)q_H}{n-1}$, for $p_G = |V(G)|$, $q_G = |E(G)|$, $p_H = |V(H)|$, $q_H = |E(H)|$, and $n = |H|$.*

Corollary 1 *If the graph $G = L \trianglerighteq H$ admits super (a, d)-$P_2 \trianglerighteq H$-antimagic total covering for integer $n \geq 2$, then $d \leq \frac{(3p_L + 3q_L - 9)(p_H^2 + q_H^2) + 4(q_L - 1) + p_H(12 - 2p_L - 8q_L)}{q_L - 1}$*

Lemma 2 [4] *Let n and c be positive integers. The sum of $\mathcal{P}^n_{c,d_1}(i, k) = \{(i - 1)n + k,\ \ 1 \leq i \leq c\}$ and $\mathcal{P}^n_{c,d_2}(i, k) = \{(k - 1)c + i;\ \ 1 \leq i \leq c\}$ form an aritmatic sequence of difference $d_1 = c, d_2 = c^2$, respectively.*

Lemma 3 [6] *Let n, c be positive integers. For $1 \leq k \leq n$, the sum of*

$$\mathcal{P}^n_{c,d_3}(i, k) \quad = \begin{cases} (\frac{k+1}{2}) + (i - 1)n; 1 \leq i \leq c; j \text{ odd} \\ \lceil \frac{n}{2} \rceil + \frac{k}{2} + (i - 1)n; 1 \leq i \leq c; j \text{ even} \end{cases}$$

form an arithmetic sequence of difference $d_3 = c$.

Lemma 4 *Let n, c be positive integers. For $1 \leq k \leq n$, the sum of*

$$\mathcal{P}^n_{c,d_4}(i, k) \quad = \begin{cases} (\frac{k-1}{2})c + i; 1 \leq i \leq c; k \text{ odd} \\ c\lceil \frac{n}{2} \rceil + i + (\frac{k-2}{2})c; 1 \leq i \leq c; k \text{ even} \end{cases}$$

form an arithmetic sequence of difference $d_4 = c^2$.

Lemma 5 [14] *Given that $G = L \triangleright H$. If L admits an edge antimagic vertex labeling (EAVL), then the sum of the corresponding partition label graph of H form an arithmetic sequence.*

MAIN RESULTS

Theorem 1 *Given the any graph H. If L admits $(b, d^*) - EAVL$, then the total comb product of the connected graph $G = L \trianglerighteq H$ admits super $(a, d) - P_2 \trianglerighteq H$ antimagic total covering with $d = d^* + d^*(d_{v_1} + d_{e_1}) + d_{v_2} + d_{e_2} + 1$.*

Proof. Let graph L contains edge antimagic vertex labeling (EAVL) with a bijective function $l : \{w^l(e) : e \in E(L)\} = \{b, b + d^*, b + 2d^*, \ldots, b + (q_L - 1)d^*\}$. Denote the edges of graph L by the symbols $e_1, e_2, \ldots, e_{q_L}$ such that: $\{w^l(e) = b + (j - 1)d^*$ with $1 \leq j \leq q_L\}$. Let H be a connected graph, and $G = L \trianglerighteq H$ contains $p_L + q_L$ subgraph isomorphic to H, say $H_1, H_2, \ldots, H_{p_L + q_L}$ where the subgraph H_i replaces the vertex v_i, $i = 1, 2, \ldots, p_L$ and edge e_i, $i = 1, 2, \ldots, q_L$ in graph L. Vertex and edge labeling on the graph H which is grafted at the vertex in graph L follow the edge antimagic vertex labeling, while vertex and edge labeling on the graph H which is grafted at the edge in graph L follow the smallest edge weight. Construct a total labeling $f^*, f^* : V(L \trianglerighteq H) \cup E(L \trianglerighteq H) \rightarrow \{1, 2, \ldots, p_L(p_H+q_H)+q_L(p_H+q_H)-2q_L\}$ constitutes the following set:

$$\begin{aligned}
f_1^*(V_{p_H}) &= \{\mathcal{P}^{p_L}_{p_H-1,d_{v_1}}(i,j) \oplus p_L\} \\
f_2^*(V_{p_H}) &= \{\mathcal{P}^{q_L}_{p_H-2,d_{v_2}}(i,j) \oplus p_H p_L\} \\
f^*(E_{q_L}) &= p_H p_L + (p_H-2)q_L + j; 1 \le j \le q_L \\
f_1^*(E_{q_H}) &= \{\mathcal{P}^{p_L}_{q_H,d_{e_1}}(i,j) \oplus [p_L p_H + (p_H-2)q_L + q_L]\} \\
f_2^*(E_{q_H}) &= \{\mathcal{P}^{q_L}_{q_H-1,d_{e_2}}(i,j) \oplus [p_L p_H + (p_H-2)q_L + q_L + p_L q_H]\}
\end{aligned}$$

where d_{v_1} and d_{v_2} respectively depend on p_H-1 and p_H-2 also d_{e_1} and d_{e_2} respectively depend on q_H and q_H-1. Furthermore the weight of the subgraph H_i, $i = 1, 2, \ldots, p_L + q_L$ in the following way:

$$\begin{aligned}
W &= \sum_{v \in V(H_i)} f(v) + \sum_{e \in E(H_i)} f(e) \\
&= (b+(j-1)d^*) + (\sum_{i=1}^{p_H-1} (\mathcal{P}^{p_L}_{p_H-1,d_{v_1}}(j) \oplus p_L)) + (\sum_{i=1}^{p_H-2} (\mathcal{P}^{q_L}_{p_H-2,d_{v_2}}(j) \oplus p_H p_L) \\
&\quad +(p_L p_H + (p_H-2)q_L + q_L) + (\sum_{i=1}^{q_H} (\mathcal{P}^{p_L}_{q_H,d_{e_1}}(j) \oplus p_H p_L + (p_H-2)q_L + q_L)) \\
&\quad +(\sum_{i=1}^{q_H-1} (\mathcal{P}^{q_L}_{q_H-1,d_{e_2}}(j) \oplus p_H p_L + (p_H-2)q_L + q_L + p_L q_H))
\end{aligned}$$

Using lemma 5 generated the following results:

$$\begin{aligned}
&= [b+(j-1)d^*] + [2C^{p_L}_{p_H-1,d_{v_1}}(j) + d_{v_1}(b+(j-1)d^*)] + [C^{q_L}_{p_H-2,d_{v_2}}(j) + d_{v_2} j] \\
&\quad +[p_H p_L + (p_H-2)q_L + j] + [2C^{p_L}_{q_H,d_{e_1}}(j) + d_{e_1}(b+(j-1)d^*)] + \\
&\quad [C^{q_L}_{q_H-1,d_{e_2}}(j) + d_{e_2} j] \\
&= b - d^* + 2C^{p_L}_{p_H-1,d_{v_1}}(j) + bd_{v_1} - d_{v_1}d^* + C^{q_L}_{p_H-2,d_{v_2}}(j) + p_H p_L + (p_H-2)q_L + \\
&\quad 2C^{p_L}_{q_H,d_{e_1}}(j) + bd_{e_1} - d_{e_1}d^* + C^{q_L}_{q_H-1,d_{e_2}}(j) + [d^* + d^*(d_{v_1} + d_{e_1}) + d_{v_2} + d_{e_2} + 1]j
\end{aligned}$$

Hence reffering to the above mentioned evidences, we prove this theorem. □

Theorem 2 *Suppose $G = P_n \dot{\trianglerighteq} H$ with $n \ge 2$. Let $K = P_2 \trianglerighteq H$ and let $p_H - 1 = c_1$, $q_H = t_1$ be the number of vertices and edges on the graph H which is grafted at the vertex in graph L respectively, also $p_H - 2 = c_2$, $q_H - 1 = t_2$ be the number of vertices and edges on the graph H which is grafted at the edge in graph L, respectively. If we assign the linear combination of $\mathcal{P}^n_{c_1,c_1}$, $\mathcal{P}^n_{c_2,c_2{}^2}$, $\mathcal{P}^n_{t_1,t_1}$, and $\mathcal{P}^n_{t_2,t_2{}^2}$ as a label of all elements in G, then G admits super (a,d)-$P_2 \dot{\trianglerighteq} \mathcal{H}$-antimagic total covering with the smallest $K-$weight $a = 4 + n(c_1^2 - c_1) + c_1 + 2nc_1 + \frac{(c_2 - c_2{}^2)}{2} + c_2 n(c_1+1) + n(c_1+1) + (n-1)c_2 + n(t_1{}^2 - t_1) + t_1 + 2t_1(n(2+c_1+c_2) - c_2 - 1)) + \frac{t_2 - t_2^2}{2} + t_2(n(2+c_1+c_2+t_1) - c_2 - 1) + 2(c_1+t_1) + c_2{}^2 + t_2{}^2$ and difference $d = [2 + 2(c_1+t_1) + c_2{}^2 + t_2{}^2 + 1]$.*

Proof. The graph $G = P_n \dot{\trianglerighteq} H$ is a connected graph with vertex and edge set of the graph $G = P_n \dot{\trianglerighteq} H$ which can be split in the following property: $V(G) = \{x_{1,k}; 1 \le k \le n\} \cup \{x_{i,k}; 1 \le i \le p_H - 1; 1 \le k \le n\} \cup \{y_{i,k}; 1 \le i \le p_H - 2, 1 \le k \le n-1\}$ and $E(G) = \{x_{1,k}x_{1,k+1}; 1 \le k \le n-1\} \cup \{e_{lk}; 1 \le l \le q_H, 1 \le k \le n\} \cup \{e'_{lk}; 1 \le l \le q_H - 1, 1 \le k \le n-1\}$. Thus $p_G = |V(G)| = np_H + (n-1)(p_H-2)$ and $q_G = |E(G)| = nq_H + (n-1) + (n-1)(q_H-1)$. Since the cover is $K = P_2 \trianglerighteq H$, by using the linear combination of $\mathcal{P}^n_{c,c}$ and $\mathcal{P}^n_{c,c^2}$, we can determine the labeling under the function $f_1 : V(G) \cup E(G) \rightarrow$

$\{1, 2, \ldots, p_G + q_G\}$. Define $f_1 : V(G) \cup E(G) \rightarrow \{1, 2, \ldots, (np_H + (n-1)(p_H - 2)) + (nq_H + (n-1) + (n-1)(q_H - 1))\}$ in such a way that:

$$
\begin{aligned}
f_1(x_{1,k}) &= k; 1 \leq k \leq n \\
f_1(x_{i,k}) &= \{\mathcal{P}^n_{c_1,c_1}(i,k) \oplus n\} \\
f_1(y_{i,k}) &= \{\mathcal{P}^n_{c_2,{c_2}^2}(i,k) \oplus n(c_1 + 1)\} \\
f_1(x_{1,k}x_{1,k+1}) &= n(c_1 + 1) + (n-1)c_2 + k; 1 \leq k \leq n-1 \\
f_1(e_{l,k}) &= \{\mathcal{P}^n_{t_1,t_1}(i,k) \oplus [n(c_1 + 1) + (n-1)(c_2 + 1)]\} \\
f_1(e'_{l,k}) &= \{\mathcal{P}^n_{t_2,{t_2}^2}(i,k) \oplus [n(2 + c_1 + c_2 + t_1) - c_2 - 1]\}
\end{aligned}
$$

It has been verified that f_1 is a bijective function. For $1 \leq k \leq n-1$, the total edge weights of graph $G = P_n \dot{\trianglerighteq} H$ have the following property:

$$
\begin{aligned}
w_{f_1(x_{1,k})} &= k + k + 1 = 2k + 1 \\
w_{f_1(x_{i,k})} &= [\sum_{i=1}^{c_1} \mathcal{P}^n_{c_1,c_1}(i,k) + c_1 n + \sum_{i=1}^{c_1} \mathcal{P}^n_{c_1,c_1}(i,k+1) + c_1 n] \\
&= [\mathcal{P}^n_{c_1,c_1}(k) + nc_1] + [\mathcal{P}^n_{c_1,c_1}(k+1) + nc_1] \\
&= \{[\frac{n}{2}(c_1^2 - c_1) + c_1 k + nc_1] + [\frac{n}{2}(c_1^2 - c_1) + c_1 k + c_1 + nc_1]\} \\
&= \{n(c_1^2 - c_1) + 2c_1 k + c_1 + 2nc_1\} \\
w_{f_1(y_{i,k})} &= \sum_{i=1}^{c_2} \mathcal{P}^n_{c_2,c_2^2}(i,k) + nc_2(c_1 + 1) = \mathcal{P}^n_{c_2,c_2^2}(k) + nc_2(c_1 + 1) \\
&= \{\frac{(c_2 - {c_2}^2)}{2} + {c_2}^2 k + c_2 n(c_1 + 1)\} \\
w_{f_1(e_{l,k})} &= [\sum_{i=1}^{t_1} \mathcal{P}^n_{t_1,t_1}(i,k) + t_1(n(2 + c_1 + c_2) - c_2 - 1)] + \\
&\quad [\sum_{i=1}^{t_1} \mathcal{P}^n_{t_1,t_1}(i,k+1) + t_1(n(c_1 + 1) + (n-1)(c_2 + 1))] \\
&= [\mathcal{P}^n_{t_1,t_1}(j) + [t_1(n(c_1 + 1) + (n-1)(c_2 + 1))]] + [\mathcal{P}^n_{t_1,t_1}(k+1) \\
&\quad +[t_1(n(2 + c_1 + c_2) - c_2 - 1)]] \\
&= \{[\frac{n}{2}(t_1^2 - t_1) + t_1 k + t_1(n(2 + c_1 + c_2) - c_2 - 1)] + \\
&\quad [\frac{n}{2}(t_1^2 - t_1) + t_1 k + t_1 + t_1(n(2 + c_1 + c_2) - c_2 - 1)]\} \\
&= \{ n({t_1}^2 - t_1) + 2t_1 k + t_1 + 2t_1(n(2 + c_1 + c_2) - c_2 - 1)\} \\
w_{f_1(e'_{l,k})} &= [\sum_{i=1}^{t_2} \mathcal{P}^n_{t_2,t_2^2}(i,k) + t_2(n(c_1 + 1) + (n-1)(c_2 + 1) + nt_1) \\
&= \mathcal{P}^n_{t_2,t_2^2}(k) + t_2(n(2 + c_1 + c_2 + t_1) - c_2 - 1) \\
&= \frac{t_2 - t_2^2}{2} + t_2^2 k + t_2(n(2 + c_1 + c_2 + t_1) - c_2 - 1)\}
\end{aligned}
$$

For $k = 1, 2, \ldots, n$, the sum of the weights of graph $G = P_n \dot{\trianglerighteq} H$ under the labeling f_1 constitute the following sets:

$$
\begin{aligned}
W^1_{f_1} &= w_{f_1(x_{1,k})} + w_{f_1(x_{i,k})} + w_{f_1(y_{i,k})} + f_1(e_K) + w_{f_1(e_{i,k})} + w_{f_1(e'_{i,k})} \\
&= 2k + 1 + w_{f_1(x_{i,j})} + w_{f_1(y_{i,j})} + n(c_1 + 1) + (n-1)c_2 + k + w_{f_1(e_{l,k})} + w_{f_1(e'_{l,k})} \\
&= C + [2 + 2(c_1 + t_1) + {c_2}^2 + {t_2}^2 + 1]k; 1 \leq k \leq n-1
\end{aligned}
$$

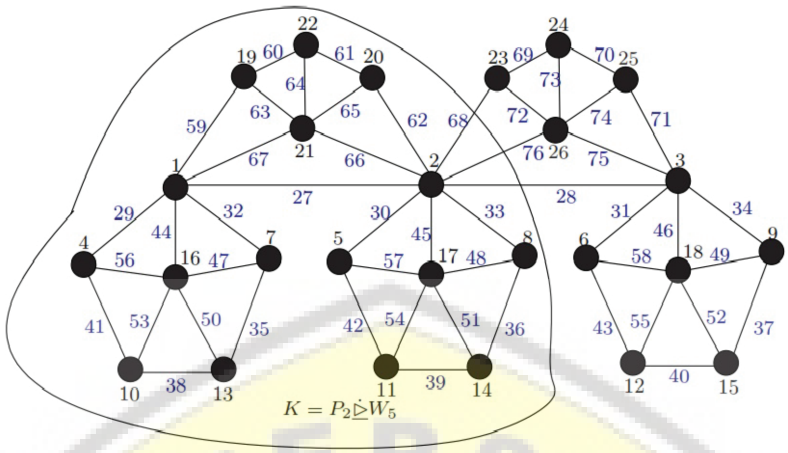

FIGURE 1. Super $(1644, 130)$-$P_2 \dot{\trianglerighteq} W_5$-*antimagic* total *covering* of graph $G = P_3 \dot{\trianglerighteq} W_5$

with $C = 1 + n(c_1^2 - c_1) + c_1 + 2nc_1 + \frac{(c_2 - {c_2}^2)}{2} + c_2 n(c_1 + 1) + n(c_1 + 1) + (n-1)c_2 + n({t_1}^2 - t_1) + t_1 + 2t_1(n(2 + c_1 + c_2) - c_2 - 1) + \frac{t_2 - t_2^2}{2} + t_2(n(2 + c_1 + c_2 + t_1) - c_2 - 1)$. It is simple to verify that the set of total edge-weights $W_{f_1}^1$ consists of an arithmetic sequence with the smallest value $a = 4 + n(c_1^2 - c_1) + c_1 + 2nc_1 + \frac{(c_2 - {c_2}^2)}{2} + c_2 n(c_1 + 1) + n(c_1 + 1) + (n-1)c_2 + n({t_1}^2 - t_1) + t_1 + 2t_1(n(2 + c_1 + c_2) - c_2 - 1) + \frac{t_2 - t_2^2}{2} + t_2(n(2 + c_1 + c_2 + t_1) - c_2 - 1) + 2(c_1 + t_1) + {c_2}^2 + {t_2}^2$ when the total edge weight lies at $k = 1$ and the difference $d = [2 + 2(c_1 + t_1) + {c_2}^2 + {t_2}^2 + 1]$. □

The Super (a, d)-*antimagic total covering* of graph $G = P_3 \dot{\trianglerighteq} W_5$ use the linear combination of $\mathcal{P}^n_{c,c}(i, k)$ and $\mathcal{P}^n_{c,c^2}(i, k)$ displayed in **FIGURE 1**. We use the linear combination of $\mathcal{P}^3_{6,6}(i, k)$ and $\mathcal{P}^2_{4,4^2}(i, k)$ for vertex labeling and the linear combination of $\mathcal{P}^3_{10,10}(i, k)$ and $\mathcal{P}^2_{9,9^2}(i, j)$ for edge labeling. Thus the value of $d = 2 + 2(5 + 10) + 4^2 + 9^2 + 1 = 130$ and the smallest value is $a = 1644$.

Theorem 3 *Suppose $G = C_n \dot{\trianglerighteq} H$, with $n \geq 3$ and n odd. Let $K = P_2 \dot{\trianglerighteq} H$, and let $p_H - 1 = c_1$, $q_H = t_1$ be the number of vertices and edges on the graph H which is grafted at the vertex in graph L respectively, also $p_H - 2 = c_2$, $q_H - 1 = t_2$ be the number of vertices and edges on the graph H which grafted at the edge in graph L, respectively. If we assign the linear combination of $\mathcal{P}^n_{c_1,c_1}$, $\mathcal{P}^n_{c_2,{c_2}^2}$, $\mathcal{P}^n_{t_1,t_1}$, and $\mathcal{P}^n_{t_2,{t_2}^2}$ as a label of all elements in G, then G admits super (a,d)-$P_2 \dot{\trianglerighteq} \mathcal{H}$-antimagic total labeling with the smallest $K-$weight $a = 1 + (\frac{n+1}{2}) + c_1\lceil \frac{n}{2} \rceil + c_1 + {c_1}^2 n + c_1 n + \frac{(c_2 - {c_2}^2)}{2} + c_2 n(c_1 + 1) + {c_2}^2 + n(1 + c_1 + c_2) + 1 + t_1\lceil \frac{n}{2} \rceil + t_1 + {t_1}^2 n - nt_1 + 2t_1(n(2 + c_1 + c_2)) + \frac{t_2 - t_2^2}{2} + t_2(n(2 + c_1 + c_2 + t_1) + t_2)$ and difference $d = [1 + (c_1 + t_1) + {c_2}^2 + {t_2}^2 + 1]$.*

Proof. The vertex and edge set of the graph $G = C_n \dot{\trianglerighteq} H$ can be split in the following sets: $V(G) = \{x_{1,k}; 1 \leq k \leq n\} \cup \{x_{i,k}; 1 \leq i \leq p_H - 1; 1 \leq k \leq n\} \cup \{y_{i,k}; 1 \leq i \leq p_H - 2, 1 \leq k \leq n - 1\}$ and $E(G) = \{x_{1,1}x_{1,n}\} \cup \{x_{1,k}x_{1,k+1}; 1 \leq k \leq n - 1\} \cup \{e_{lk}; 1 \leq l \leq q_H, 1 \leq k \leq n\} \cup \{e'_{lk}; 1 \leq l \leq q_H - 1, 1 \leq k \leq n - 1\}$. Thus $p_G = |V(G)| = np_H + (n-1)(p_H - 2)$ and $q_G = |E(G)| = nq_H + n + (n-1)(q_H - 1)$. Since the cover is $K = P_2 \dot{\trianglerighteq} H$. We can define the vertex labeling $f_1 : V(G) \cup E(G) \to \{1, 2, \ldots, p_G + q_G\}$ by using the linear combination of $\mathcal{P}^n_{c,c}$ and $\mathcal{P}^n_{c,c^2}$. By Lemma 3 and Lemma 4, we use c_1 and t_1 for the partition $\mathcal{P}^n_{c,c}(i, k)$ and we use c_2 and t_2 for the partition $\mathcal{P}^n_{c,c^2}(i, k)$. For $i = 1, 2, \ldots, c$, $l = 1, 2, \ldots, t$ and $k = 1, 2, \ldots, n$, the total labels can be expressed as follows:

$$
\begin{aligned}
f_2(x_{1,k}) &= \frac{k+1}{2}; k \text{ odd} \\
f_2(x_{1,k}) &= \frac{n+1}{2} + \frac{k}{2}; k \text{ even} \\
f_2(x_{i,k}) &= \{\mathcal{P}^n_{c_1,c_1}(i, k) \oplus n\} \\
f_2(y_{i,k}) &= \{\mathcal{P}^n_{c_2,{c_2}^2}(i, k) \oplus n(c_1 + 1)\}
\end{aligned}
$$

$$\begin{aligned}
f_2(x_{1,1}x_{1,n}) &= n(c_1+1)+nc_2+1 \\
f_2(x_{1,k}x_{1,k+1}) &= n(c_1+c_2+1)+1+k; 1\le k\le n-1 \\
f_2(e_{l,k}) &= \{\mathcal{P}^n_{t_1,t_1}(i,k)\oplus[n(c_1+c_2+2)]\} \\
f_2(e'_{l,k}) &= \{\mathcal{P}^n_{t_2,t_2{}^2}(i,k)\oplus[n(c_1+c_2+t_1+2)]
\end{aligned}$$

After obtaining the vertex label of graph G, now we construct the total edge-weights of $G = C_n \dot{\trianglerighteq} H$ under the labeling f_2, for $1\le k\le n-1$, which constitute the following sets:

$$\begin{aligned}
w_{f_2(x_{1,k})} &= k+1+(\frac{n+1}{2}) \\
w_{f_2(x_{i,k})} &= [\sum_{i=1}^{c_1}\mathcal{P}^n_{c_1,c_1}(i,k)+c_1n+\sum_{i=1}^{c_1}\mathcal{P}^n_{c_1,c_1}(i,k+1)+c_1n] \\
&= [\mathcal{P}^n_{c_1,c_1}(k)+nc_1]+[\mathcal{P}^n_{c_1,c_1}(k+1)+nc_1] \\
&= \{[c_1(\frac{k+1}{2})+(\frac{c_1+c_1^2}{2})n-c_1n+nc_1]+[c_1(\lceil\frac{n}{2}\rceil+\frac{k+1}{2})+(\frac{c_1{}^2+c_1}{2})n]\} \\
&= c_1\lceil\frac{n}{2}\rceil+c_1k+c_1+c_1{}^2n+c_1n \\
w_{f_2(y_{i,k})} &= \sum_{i=1}^{c_2}\mathcal{P}^n_{c_2,c_2^2}(i,k)+nc_2(c_1+1)=\mathcal{P}^n_{c_2,c_2^2}(k)+nc_2(c_1+1) \\
&= \frac{(c_2-c_2{}^2)}{2}+c_2{}^2k+c_2n(c_1+1)+c_2{}^2 \\
w_{f_2(e_{l,k})} &= [\sum_{i=1}^{t_1}\mathcal{P}^n_{t_1,t_1}(i,k)+t_1(n(c_1+1)+n(c_2+1))]+ \\
&\quad [\sum_{i=1}^{t_1}\mathcal{P}^n_{t_1,t_1}(i,k+1)+t_1(n(c_1+1)+n(c_2+1))] \\
&= [\mathcal{P}^n_{t_1,t_1}(k)+[t_1n(c_1+c_2+2)]]+[\mathcal{P}^n_{t_1,t_1}(k+1)+[t_1n(c_1+c_2+2)]] \\
&= \{[t_1(\frac{k+1}{2})+(\frac{t_1+t_1^2}{2})n-nt_1+t_1(n(2+c_1+c_2))]+ \\
&\quad [t_1(\lceil\frac{n}{2}\rceil+\frac{k+1}{2})+(\frac{t_1{}^2-t_1}{2})n+t_1(n(2+c_1+c_2))]\} \\
&= t_1\lceil\frac{n}{2}\rceil+t_1k+t_1+t_1{}^2n-nt_1+2t_1(n(2+c_1+c_2)) \\
w_{f_2(e'_{l,k})} &= [\sum_{i=1}^{t_2}\mathcal{P}^n_{t_2,t_2^2}(i,k)+t_2(n(2+c_1+c_2+t_1)) \\
&= \mathcal{P}^n_{t_2,t_2^2}(k)+t_2(n(2+c_1+c_2+t_1)) \\
&= \frac{t_2-t_2^2}{2}+t_2^2k+t_2(n(2+c_1+c_2+t_1)+t_2) \\
W^2_{f_2} &= w_{f_2(x_{1,k})}+w_{f_2(x_{i,k})}+w_{f_2(y_{i,k})}+f_2(x_{1,k}x_{1,k+1})+w_{f_2(e_{l,k})}+w_{f_2(e'_{l,k})} \\
&= 2(k+1)+(\frac{n+1}{2})+w_{f_2(x_{i,k})}+w_{f_2(y_{i,k})}+n(c_1+c_2+1)+w_{f_2(e_{l,k})}+w_{f_2(e'_{l,k})} \\
&= C_1+[1+(c_1+t_1)+c_2{}^2+t_2{}^2+1]k; 1\le k\le n-1
\end{aligned}$$

with $C_1 = 1+(\frac{n+1}{2})+c_1\lceil\frac{n}{2}\rceil+c_1+c_1{}^2n+c_1n+\frac{(c_2-c_2{}^2)}{2}+c_2n(c_1+1)+c_2{}^2+n(1+c_1+c_2)+1+t_1\lceil\frac{n}{2}\rceil+t_1+t_1{}^2n-nt_1+2t_1(n(2+c_1+c_2))+\frac{t_2-t_2^2}{2}+t_2(n(2+c_1+c_2+t_1)+t_2)$. While the total K-weight for $k=1$ and $k=n$ can be expressed in the following sets:

$$w_{f_2(x_{1,k})} = 1+(\frac{n+1}{2})$$

$$
\begin{aligned}
w_{f_2(x_{i,k})} &= [\sum_{i=1}^{c_1} \mathcal{P}^n_{c_1,c_1}(i,1) + c_1 n + \sum_{i=1}^{c_1} \mathcal{P}^n_{c_1,c_1}(i,n) + c_1 n] \\
&= [\mathcal{P}^n_{c_1,c_1}(1) + nc_1] + [\mathcal{P}^n_{c_1,c_1}(n) + nc_1] \\
&= \{[c_1 + (\frac{c_1 + c_1^2}{2})n - c_1 n + nc_1] + [c_1(\frac{n+1}{2}) + (\frac{c_1 + {c_1}^2}{2})n - c_1 n + nc_1]\} \\
&= \frac{3c_1}{2} + \frac{c_1 n}{2} + (c_1 + {c_1}^2)n \\
w_{f_2(y_{i,k})} &= \sum_{i=1}^{c_2} \mathcal{P}^n_{c_2,c_2^2}(i,n) + nc_2(c_1+1) = \mathcal{P}^n_{c_2,c_2^2}(n) + nc_2(c_1+1) \\
&= \frac{(c_2 - {c_2}^2)}{2} + {c_2}^2 + c_2 n(c_1+1) \\
w_{f_2(e_{l,k})} &= [\sum_{i=1}^{t_1} \mathcal{P}^n_{t_1,t_1}(i,1) + t_1(n(c_1+1) + n(c_2+1))] + \\
& \quad [\sum_{i=1}^{t_1} \mathcal{P}^n_{t_1,t_1}(i,n) \oplus t_1(n(c_1+1) + n(c_2+1))] \\
&= [\mathcal{P}^n_{t_1,t_1}(1) + [t_1 n(c_1 + c_2 + 2)]] + [\mathcal{P}^n_{t_1,t_1}(n) + [t_1 n(c_1 + c_2 + 2)]] \\
&= \{[t_1(\frac{1+1}{2}) + (\frac{t_1 + t_1^2}{2})n - nt_1 + t_1(n(2 + c_1 + c_2))] + \\
& \quad [t_1(\frac{n+1}{2}) + (\frac{{t_1}^2 + t_1}{2})n - nt_1 + t_1(n(2 + c_1 + c_2))]\} \\
&= \frac{3t_1}{2} + \frac{t_1 n}{2} + (t_1 + {t_1}^2)n - 2nt_1 + 2t_1(n(2 + c_1 + c_2)) \\
w_{f_2(e'_{l,k})} &= [\sum_{i=1}^{t_2} \mathcal{P}^n_{t_2,t_2^2}(i,n) + t_2(n(2 + c_1 + c_2 + t_1)) \\
&= \mathcal{P}^n_{t_2,t_2^2}(n) + t_2(n(2 + c_1 + c_2 + t_1)) \\
&= \frac{t_2 - t_2^2}{2} + t_2^2 + t_2(n(2 + c_1 + c_2 + t_1)) \\
W^2_{f_2} &= w_{f_2(x_{1,k})} + w_{f_2(x_{i,k})} + w_{f_2(y_{i,k})} + f_2(x_{1,1}x_{1,n}) + w_{f_2(e_{l,k})} + w_{f_2(e'_{l,k})} \\
&= k + 2 + (\frac{n+1}{2}) + w_{f_2(x_{i,k})} + w_{f_2(y_{i,k})} + n(c_1 + c_2 + 1) + w_{f_2(e_{l,k})} + w_{f_2(e'_{l,k})} \\
&= C_1, \text{ is the smallest value.}
\end{aligned}
$$

From the two K-weights, we have the following

$$
\begin{aligned}
\bigcup_{m=1}^{2} W^m_{f_2} &= \{C_1, C_1 + [1 + (c_1 + t_1) + {c_2}^2 + {t_2}^2 + 1], C_1 + 2[1 + (c_1 + t_1) + {c_2}^2 + {t_2}^2 + 1], \\
& \quad \ldots, C_1 + (n-1)[1 + (c_1 + t_1) + {c_2}^2 + {t_2}^2 + 1]\}
\end{aligned}
$$

It concludes the proof. □

Conjecture 1 *if the graph $G = L \trianglerighteq H$ admits a super $(a,d) - P_2 \trianglerighteq H$ antimagic total covering with the feasible difference $d = d^* + d^*(d_{v_1} + d_{e_1}) + d_{v_2} + d_{e_2} + 1$ where d^* is the feasible value of difference in edge antimagic vertex labeling of graph L and d_{v_1}, d_{v_2} respectively depends on $p_H - 1$ and $p_H - 2$ also d_{e_1}, d_{e_2} respectively depends on q_H and $q_H - 1$, then the disjoint union of graph G admits a super $(a,d) - P_2 \trianglerighteq H$ antimagic total covering with the feasible difference $d = d^* + d^*(d_{v_1} + d_{e_1}) + d_{v_2} + d_{e_2} + 1$.*

CONCLUDING REMARKS

Use partition technique, we have shown that the graph $G = L\dot{\trianglerighteq}H$ admits a super $(a,d)-P_2\dot{\trianglerighteq}H$ antimagic total covering with the feasible difference $d = d^* + d^*(d_{v_1} + d_{e_1}) + d_{v_2} + d_{e_2} + 1$ and there is a connection between a super $(a,d)-P_2\dot{\trianglerighteq}H$ and edge antimagic vertex labeling (EAVL).

Direction for further research, the author write some open problem as follows:

Open Problem 1 *Does the disjoint union of graph $G = s(L\dot{\trianglerighteq}H)$ admit a super $(a,d) - P_2\dot{\trianglerighteq}H$ antimagic total covering?*

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