Preface: International Conference on Science and Applied Science (ICSAS) 2018

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Preface: International Conference on Science and Applied Science (ICSAS) 2018

International Conference on Science and Applied Science (ICSAS) 2018 was held at the Solo Paragon Hotel, Surakarta, Indonesia on 12 May 2018. The ICSAS 2018 conference is aimed to bring together scholars, leading researchers and experts from diverse backgrounds and applications areas in Science. Special emphasis is placed on promoting interaction between the science theoretical, experimental, and education sciences, engineering so that a high level exchange in new and emerging areas within Mathematics, Chemistry, Physics and Biology, all areas of sciences and applied mathematics and sciences is achieved.

In ICSAS 2018, there are eight parallel sessions and four keynote speakers. It is an honour to present this volume of AIP Conference Proceedings and we deeply thank the authors for their enthusiastic and high-grade contribution. From the review results, there are 166 papers which will be published in AIP Conference Proceedings We would like to express our sincere gratitude to all in the Programming Committee who have reviewed the papers and developed a very interesting Conference Program, as well as thanking the invited and plenary speakers. Finally, we would like to thank the conference chairman, the members of the steering committee, the organizing committee, the organizing secretariat and the financial support from the Sebelas Maret University that allowed ICSAS 2018 to be a success.

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On super local antimagic total edge coloring of some wheel related graphs

Ika Hesti Agustin, Ridho Alfarisi, Dafik, A. I. Kristiana, R. M. Prihandini, and E. Y. Kurniawati

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On Super Local Antimagic Total Edge Coloring of Some Wheel Related Graphs

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Abstract. Let G be a connected graph, let $V(G)$ be the vertex set of graph G, and let $E(G)$ be the edge set of graph G. Thus, the bijective function $f : V(G) \cup E(G) \longrightarrow \{1, 2, 3, ..., |V(G)| + |E(G)|\}$ is called a local antimagic total edge labeling if for two adjacent edges e_1 and e_2, $w_t(e_1) \neq w_t(e_2)$, where for $e = uv \in G$, $w_t(e) = f(u) + f(v) + f(uv)$. Thus, the local antimagic total edge labeling by induces a proper edge coloring of a graph G if each edge e is assigned the color $w_t(e)$. The local antimagic total edge coloring, denoted by $\gamma_{leat}(G)$ is the minimum number of colors taken over all colorings induced by local antimagic total edge labelings of a graph G. In this research, we determine the local super antimagic total edge coloring of some wheel related graph including fan, wheel, gear and friendship graph.

INTRODUCTION

For the definition about basic graphs, we can see in [1, 2]. A graph vertex labeling is a bijective function that mapping natural number to all vertices of a graph. A graph vertex labeling consider with all the sum of vertex label on each edge have different value is called antimagic vertex labeling. The antimagic labeling of graph concept introduces by Hartsfield and Ringel [3].

In recent year, there are a few results that connected with antimagic labeling such as Super antimagicness total edge labelings of $mK_{n,n}$ and Super edge-antimagicness for families graph of disconnected graphs by Dafik, *et al.* [5, 6]. Beside that, On r-dynamic coloring of graphs resulting some operations is related with coloring of a graph who has researched by ika, *et al.* [7]. The local antimagic labeling with $\gamma_{lae}(G) = 2$is called bimagic labeling. For the this concept introduces by Marr *et al.* [8].

The local antimagic edge labeling by induces a proper edge coloring of G where the edge e is assigned the color $w(e)$. Thus, we have the following concept.

Definition 0.1 *[9] Let $G = (V(G), E(G)$ be a connected graph, on the vertex set $V(G)$ and the edge set $E(G)$. Define a bijection $f : V(G) \longrightarrow \{1, 2, 3, ..., |V(G)|\}$ is called a local antimagic edge labeling if for two adjacent edges e_1 and e_2, $w(e_1) \neq w(e_2)$, where for $e = uv \in G$, $w(e) = f(u) + f(v)$.*

Hence, the local antimagic edge labeling induces a proper edge coloring of a graph G if each edge e is assigned the color $w(e)$. The local antimagic edge chromatic number $\gamma_{lae}(G)$ is the minimum number of colors taken over all colorings induced by local antimagic edge labelings of G. Furthermore, we will introduce the concept of the local antimagic total edge coloring of graphs. Define a bijection $f : V(G) \cup E(G) \longrightarrow \{1, 2, 3, ..., |V(G)| + |E(G)|\}$ is called a local antimagic total edge labeling if for two adjacent edges e_1 and e_2, $w_t(e_1) \neq w_t(e_2)$, where for $e = uv \in E(G)$, $w_t(e) = f(u) + f(v) + f(uv)$. Hence, the local antimagic total edge labeling induces a proper edge coloring of G if each edge e is assigned the color $w_t(e)$. The local antimagic total edge coloring $\gamma_{leat}(G)$ is the minimum number of colors taken over all colorings induced by local antimagic total edge labelings of G.

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The first person who introduced local antimagic coloring is Arumugam *et al.* [4]. They have interval value of local antimagic vertex coloring of graph resulting of joint product graph and exact value of local antimagic vertex coloring of some special graphs such as path, cycle, complete graphs, friendship, wheel, bipartite and complete bipartite graph. Furthemore, it is develops into local antimagic edge coloring who introduced by Agustin, *et al.* [9]. Their analysis is about the bounds of its local antimagic edge coloring, $\gamma_{lae}(G) \geq \Delta(G)$. Recently, there some result about local antimagic coloring of graphs such as [9, 10, 11, 12, 13].

Observation 0.1 *[4] For a graph G, $\chi_{la}(G) \geq \chi(G)$, where $\chi(G)$ is called a coloring of vertex coloring of a graph G.*

Theorem 0.1 *[9] For any graph G with $\Delta(G)$ is a maximum degrees of G, we have $\gamma_{lea}(G) \geq \Delta(G)$.*

MAIN RESULTS

In this section, we investigated the local antimagic total edge coloring of some wheel related graphs including fan, wheel, gear and helm graph. We will present several basic results in the following theorems.

Lemma 0.1 *Let G be a wheel related graphs with the maximum degree is Δ, the local antimagic total edge coloring of G is $\gamma_{leat}(G) \geq \Delta + 1$.*

Proof: Wheel related graphs is defined a graph with characterized of wheel. Wheel has vertices in outer cycle which adjacent with one central vertex which has the maximum degree Δ. Wheel has some properties namely rims and spokes. Rims are the edges in outer cycle of wheel and spokes are the edges incident with central vertex. If we assign all edges in spokes, then the edges in spokes have Δ distinct colors. We need at least two colors to assign edges in the rims. If we choose two colors in spokes to assign the rims, then there will be at least two incident edges which have same colors. Hence, we must take at least one different color from spokes to assign edges in outer cycle. Thus at least we have $\Delta + 1$ distinct colors to assign wheel related graphs.

Theorem 0.2 *Let F_n be a fan graph with $n \geq 3$, the local antimagic total edge coloring of F_n is $\gamma_{leat}(F_n) = n + 1$.*

Proof. The graph F_n is a related wheel graph with vertex set $V(F_n) = \{x, x_i : 1 \leq i \leq n\}$ and edge set $E(F_n) = \{xx_i : 1 \leq i \leq n\} \cup \{x_i x_{i+1} : 1 \leq i \leq n-1\}$. The cardinality of vertices $|V(F_n)| = n + 1$, the cardinality of edges $|E(F_n)| = 2n - 1$, and $\Delta(F_n) = n$. Based on Lemma 0.1 that $\gamma_{leat}(F_n) \geq n + 1$. Furthermore, we prove that the upper bound of the local antimagic total edge coloring of F_n is $\gamma_{leat}(F_n) \leq n + 1$. Define a bijection function $f : V(F_n) \cup E(F_n) \longrightarrow \{1, 2, 3, \ldots, |V(F_n)| + |E(F_n)|\}$ as follows
When n odd,

$$f(v) = \begin{cases} n+1, & \text{if } v = x \\ \frac{i+1}{2}, & \text{if } v = x_i,\ 1 \leq i \leq n,\ i \text{ is odd} \\ \frac{n+i+1}{2}, & \text{if } v = x_i,\ 2 \leq i \leq n-1,\ i \text{ is even} \end{cases}$$

$$f(e) = \begin{cases} 2n+2+\frac{i-1}{2}, & \text{if } e = xx_i, \text{ when } i \text{ odd and } i = 3, 4, \ldots, n-2 \\ 2n+2+\frac{n+i-3}{2}, & \text{if } e = xx_i,\ 2 \leq i \leq n-1,\ i \text{ is even} \\ 2n+3-i, & \text{if } e = x_i x_{i+1},\ 1 \leq i \leq n-2,\ i \text{ is odd} \\ 2n+1-i, & \text{if } e = x_i x_{i+1},\ 2 \leq i \leq n-1,\ i \text{ is even} \\ 2n+1, & \text{if } e = xx_1 \\ n+3, & \text{if } e = xx_n \end{cases}$$

We can see that f is a local antimagic labeling of F_n and the weights of edge labeling are as follows:

$$w(e) = \begin{cases} \frac{5n+9}{2}, & \text{if } e = xx_n, e = x_i x_{i+1},\ 1 \leq i \leq n-2,\ i \text{ is odd} \\ \frac{5n+5}{2}, & \text{if } e = x_i x_{i+1},\ 2 \leq i \leq n-1,\ i \text{ is even} \\ 3n+3, & \text{if } e = xx_1 \\ 3n+i+3, & \text{if } e = xx_i,\ 2 \leq i \leq n-1,\ i \text{ is odd} \\ 4n+i+2, & \text{if } e = xx_i,\ 2 \leq i \leq n-1,\ i \text{ is even} \end{cases}$$

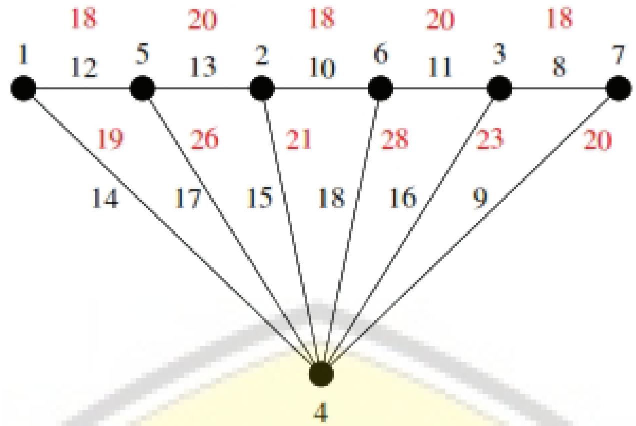

FIGURE 1. The local antimagic total edge coloring of F_6

For n even,

$$f(v) = \begin{cases} \frac{n}{2}+1, & \text{if } v = x \\ \frac{i+1}{2}, & \text{if } v = x_i,\ 1 \le i \le n-1,\ i \text{ is odd} \\ \frac{n+i+2}{2}, & \text{if } v = x_i,\ 2 \le i \le n,\ i \text{ is even} \end{cases}$$

$$f(e) = \begin{cases} 2n+2+\frac{i-1}{2}, & \text{if } e = xx_i,\ 3 \le i \le n-1,\ i \text{ is odd} \\ 2n+2+\frac{n+i-2}{2}, & \text{if } e = xx_i,\ 2 \le i \le n-2,\ i \text{ is even} \\ 2n+3-i, & \text{if } e = x_ix_{i+1},\ 2 \le i \le n-2,\ i \text{ is even} \\ 2n+1-i, & \text{if } e = x_ix_{i+1},\ 1 \le i \le n-1,\ i \text{ is odd} \\ 2n+2, & \text{if } e = xx_1 \\ n+3, & \text{if } e = xx_n \end{cases}$$

We can see that f is a local antimagic labeling of F_n and the weights of edge labeling are as follows

$$w(e) = \begin{cases} \frac{5n+6}{2}, & \text{if } e = x_ix_{i+1},\ 1 \le i \le n-1,\ i \text{ is odd} \\ \frac{5n+10}{2}, & \text{if } e = xx_n,\ e = x_ix_{i+1}, \text{when } i \text{ even and } i = 2,3,\dots,n-2 \\ \frac{5n+8}{2}, & \text{if } e = xx_1 \\ \frac{5}{2}n+i+3, & \text{if } e = xx_i,\ 2 \le i \le n-1,\ i \text{ is odd} \\ \frac{7}{2}n+i+3, & \text{if } e = xx_i,\ 2 \le i \le n-1,\ i \text{ is even} \end{cases}$$

Hence, it is easy to see that f induces a proper edge coloring of F_n and it gives $\gamma_{leat}(F_n) \le n+1$. We obtain that the lower bound and upper bound of the local antimagic total edge coloring of F_n is $n+1$. It concludes that $\gamma_{leat}(F_n) = n+1$.

We can see the illustration the local antimagic total edge coloring of F_6 in figure 1.

Theorem 0.3 *Let W_n be a wheel graph with $n \ge 3$, the local antimagic total edge coloring of W_n is $\gamma_{leat}(W_n) = n+1$.*

Proof. The graph W_n is a connected graph. The vertex set of graph W_n is $V(W_n) = \{x, x_i; \text{for } i = 1,\dots,n\}$ and edge set $E(W_n) = \{xx_i : \text{for } i = 1,\dots,n\} \cup \{x_1x_n, x_ix_{i+1} : \text{for } i = 1,\dots,n-1\}$. The cardinality of vertices $|V(W_n)| = n+1$, the cardinality of edges $|E(W_n)| = 2n$, and $\Delta(W_n) = n$. Based on Lemma 0.1 that $\gamma_{leat}(W_n) \ge n+1$. Furthermore, we prove that the upper bound of the local antimagic total edge coloring of W_n is $\gamma_{leat}(W_n) \le n+1$. We divide into two cases to show the upper bound.

Case 1: For n is even, define a bijection function $f : V(W_n) \cup E(W_n) \longrightarrow \{1,2,3,\dots,|V(W_n)|+|E(W_n)|\}$ by the following

$$f(v) = \begin{cases} \frac{n}{2}+1, & \text{if } v = x \\ \frac{i+1}{2}, & \text{if } v = x_i,\ 1 \le i \le n-1,\ i \text{ is odd} \\ \frac{n+i+2}{2}, & \text{if } v = x_i,\ 2 \le i \le n,\ i \text{ is even} \end{cases}$$

$$f(e)=\begin{cases} 2n+3-i, & \text{if } e=x_ix_{i+1}, \text{ when } i \text{ even and } i=2,3,\dots,n-2\\ 2n+1-i, & \text{if } e=x_ix_{i+1},\ 1\le i\le n-1,\ i \text{ is odd}\\ 2n+2, & \text{if } e=xx_1\\ n+3, & \text{if } e=xx_n \end{cases}$$

We can see that f is a local antimagic labeling of W_n and the weights of edge labeling are as follows

$$w(e)=\begin{cases} \frac{5n+6}{2}, & \text{if } e=x_ix_{i+1},\ 1\le i\le n-1,\ i \text{ is odd}\\ \frac{5n+10}{2}, & \text{if } e=xx_n, e=x_ix_{i+1}, \text{ when } i \text{ even and } i=2,3,\dots,n-2\\ \frac{5n+8}{2}, & \text{if } e=xx_1 \end{cases}$$

Case 2: For n is odd, we define a bijection function $f : V(W_n)\cup E(W_n) \longrightarrow \{1,2,3,\dots,|V(W_n)|+|E(W_n)|\}$ by the following

$$f(v)=\begin{cases} n+1, & \text{if } v=x\\ \frac{i+1}{2}, & \text{if } v=x_i,\ 1\le i\le n,\ i \text{ is odd}\\ \frac{n+i+1}{2}, & \text{if } v=x_i,\ 2\le i\le n-1,\ i \text{ is even} \end{cases}$$

$$f(e)=\begin{cases} 2n+3-i, & \text{if } e=x_ix_{i+1},\ 1\le i\le n-2,\ i \text{ is odd}\\ 2n+1-i, & \text{if } e=x_ix_{i+1},\ 2\le i\le n-1,\ i \text{ is even}\\ 2n+1, & \text{if } e=xx_1\\ n+3, & \text{if } e=xx_n \end{cases}$$

We can see that f is a local antimagic labeling of W_n and the weights of edge labeling are as follows

$$w(e)=\begin{cases} \frac{5n+9}{2}, & \text{if } e=xx_n, e=x_ix_{i+1},\ 1\le i\le n-2,\ i \text{ is odd}\\ \frac{5n+5}{2}, & \text{if } e=x_ix_{i+1},\ 2\le i\le n-1,\ i \text{ is even}\\ 3n+3, & \text{if } e=xx_1 \end{cases}$$

For other edges labeling, we use construction to give labels on $e=xx_i,\ 2\le i\le n-1$ and $e=x_1x_n$. We define an algorithm of construction of edges labeling by the following.

1. Determined the weights of edge, $e=x_1x_i, 2\le i\le n-1$ and $e=x_1x_n$, through the labels of vertex.
2. Arranged the edge weights by ascending order.
3. Arranged the labels of edges $e = xx_i$ and x_1x_n by descending order such that the total edge weights are same.
4. $e=x_1x_n$ and one of $e=xx_i$ must have the same weights. Since they are not adjacent.
5. Put the other labels of edges $e = xx_i$ randomly such that they have the distinct total edge weights.

Hence, it is easy to see that the function f and algorithm induces a proper edge coloring of W_n and it gives $\gamma_{leat}(W_n)\le 3+(n-2)=n+1$. We obtain that the lower bound and the upper bound of the local antimagic total edge coloring of W_n is $n+1$ It concludes that $\gamma_{leat}(W_n)=n+1$ for $n\ge 3$.

We can see the illustration the local antimagic total edge coloring of W_7 in figure 2.

Theorem 0.4 *Let $J_{n,1}$ be a gear graph with $n\ge 6$, the local antimagic total edge coloring of $J_{n,1}$ is* $\gamma_{leat}(J_{n,1}) = n+1$.

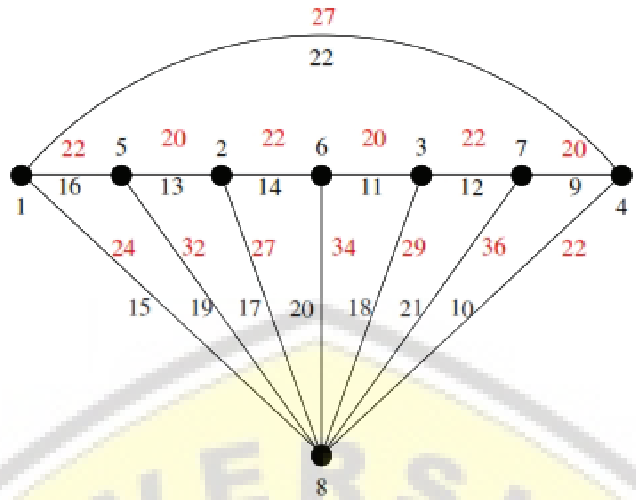

FIGURE 2. The local antimagic total edge coloring of W_7

Proof. The graph $J_{n,1}$ is a related wheel graph with vertex set $V(J_{n,1}) = \{x, x_i, y_i : 1 \leq i \leq n$ and edge set $x_1x_n; xx_i, 1 \leq i \leq n; x - iy_i, 1 \leq i \leq n; y_ix_{i+1}, 1 \leq i \leq n-1$, such that $|V(J_{n,1})| = 2n + 1$ and $|E(J_{n,1})| = 3n$, respectively. It has $\Delta(J_{n,1}) = n$, based on Lemma 0.1 that $\gamma_{leat}(J_{n,1}) \geq n + 1$. Furthermore, we prove that the upper bound of the local antimagic total edge coloring of $J_{n,1}$ is $\gamma_{leat}(J_{n,1}) \leq n + 1$.

We define a bijection function $f : V(J_{n,1}) \cup E(J_{n,1}) \longrightarrow \{1, 2, 3, ..., |V(J_{n,1})| + |E(J_{n,1})|\}$ by the following

$$f(v) = \begin{cases} 2n, & \text{if } v = x \\ 2n + 1, & \text{if } v = y_n \\ i, & \text{if } v = x_i,\ 1 \leq i \leq n \\ n + i, & \text{if } v = y_i,\ 1 \leq i \leq n-1 \end{cases}$$

$$f(e) = \begin{cases} 4n + 3 - 2i, & \text{if } e = x_iy_i,\ 1 \leq i \leq n-1 \\ 4n - 2i, & \text{if } e = y_ix_{i+1},\ 1 \leq i \leq n-1 \\ 4n, & \text{if } e = xx_1 \\ 2n + 3, & \text{if } e = xx_n \end{cases}$$

We can see that f is a local antimagic labeling of $J_{n,1}$ and the weights of edge labeling are as follows

$$w(e) = \begin{cases} 5n + 3, & \text{if } e = xx_n, e = x_iy_i,\ 1 \leq i \leq n-1 \\ 5n + 1, & \text{if } e = y_ix_{i+1},\ 1 \leq i \leq n-1 \\ 6n + 1, & \text{if } e = xx_1 \end{cases}$$

For other edges labeling, we use construction to give labels on $e = xx_i,\ 2 \leq i \leq n-1, x_1y_n$ and $e = x_ny_n$. We define an algorithm of construction of edges labeling by the following.

1. Determined the weights of edge, $e = x_1x_i, 2 \leq i \leq n-1$ and $e = x_1x_n$, through the labels of vertex.

2. Arranged the edge weights by ascending order.

3. Arranged the labels of edges $e = xx_i$ and x_1x_n by descending order such that the total edge weights are same.

4. $e = x_1y_n$ and $e = x_ny_n$ must have the same weights with one of $e = xx_i$. Since they are not adjacent.

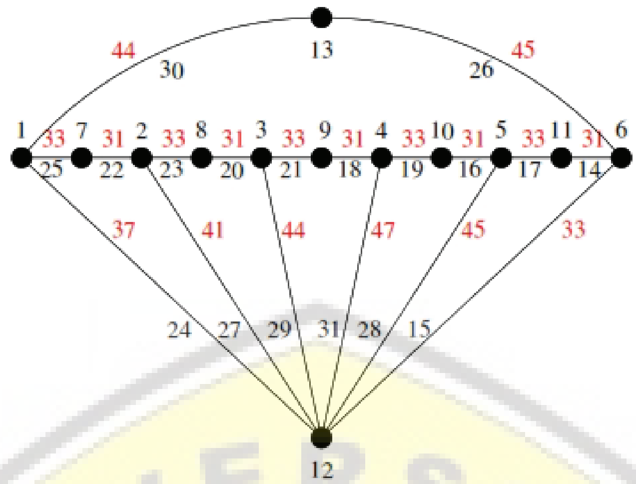

FIGURE 3. Illustration of the local antimagic total edge coloring of $J_{6,1}$

5. Put the other labels of edges $e = xx_i$ randomly such that they have the distinct total edge weights.

Hence, it is easy to see that the function f and algorithm induces a proper edge coloring of $J_{n,1}$ and it gives $\gamma_{leat}(J_{n,1}) \leq 3 + (n-2) = n + 1$. We obtain that the lower bound and the upper bound of the local antimagic total edge coloring of $J_{n,1}$ is $n+1$ It concludes that $\gamma_{leat}(J_{n,1}) = n + 1$ for $n \geq 6$.

We can see the illustration the local antimagic total edge coloring of $J_{6,1}$ in figure 3.

Theorem 0.5 *Let $\mathcal{F}_n$ be a friendship graph with $n \geq 3$, the local antimagic total edge coloring of $\mathcal{F}_n$ is* $\gamma_{leat}(\mathcal{F}_n) = 2n + 1$.

Proof. The friendship graph $\mathcal{F}_n$ is a related wheel. The vertex set of the friendship graph can be define as follow: $V(\mathcal{F}_n) = \{x, x_i, y_i : 1 \leq i \leq n\}$ and edge set $E(\mathcal{F}_n) = \{xx_i, xy_i, x_iy_i : 1 \leq i \leq n\}$. The cardinality of vertices $|V(\mathcal{F}_n)| = 2n+1$, $|E(\mathcal{F}_n)| = 3n$, and $\Delta(\mathcal{F}_n) = 2n$. Based on Lemma 0.1 that $\gamma_{leat}(\mathcal{F}_n) \geq 2n + 1$. Furthermore, we prove that the upper bound of the local antimagic total edge coloring of $\mathcal{F}_n$ is $\gamma_{leat}(\mathcal{F}_n) \leq 2n+1$. We define a bijection function $f : V(\mathcal{F}_n) \cup E(\mathcal{F}_n) \longrightarrow \{1, 2, 3, \ldots, |V(\mathcal{F}_n)| + |E(\mathcal{F}_n)|\}$ by the following

$$f(v) = \begin{cases} 1, & \text{if } v = x \\ i+1, & \text{if } v = x_i,\ 1 \leq i \leq n \\ n+i+1, & \text{if } v = y_i,\ 1 \leq i \leq n \end{cases}$$

$$f(e) = \begin{cases} 4n-2i+3, & \text{if } e = xx_i,\ 1 \leq i \leq n \\ 4n+1+i, & \text{if } e = xy_i,\ 1 \leq i \leq n \\ 4n-2i+2, & \text{if } e = x_iy_i,\ 1 \leq i \leq n \end{cases}$$

We can see that f is a local antimagic labeling of $\mathcal{F}_n$ and the weights of edge labeling are as follows

$$w_t(e) = \begin{cases} 4n-i+5, & \text{untuk } e = xx_i,\ 1 \leq i \leq n \\ 5n+2i+3, & \text{untuk } e = xy_i,\ 1 \leq i \leq n \\ 5n+4, & \text{untuk } e = x_iy_i,\ 1 \leq i \leq n \end{cases}$$

Hence, it is easy to see that f induces a proper edge coloring of $\mathcal{F}_n$ and it gives $\gamma_{leat}(\mathcal{F}_n) \leq 2n+1$. Thus, we obtain that the upper bound and lower bound of the local antimagic total edge coloring of $\mathcal{F}_n$ is $2n+1$. It concludes that $\gamma_{leat}(\mathcal{F}_n) = 2n + 1$.

We can see the illustration the local antimagic total edge coloring of $\mathcal{F}_4$ in figure 4.

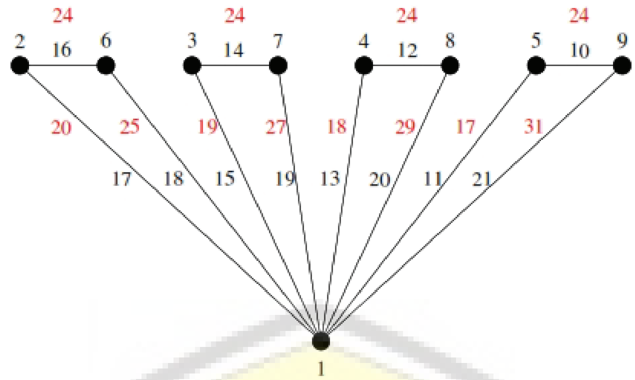

FIGURE 4. Illustration of the local antimagic total edge coloring of $\mathcal{F}_4$

CONCLUSION

In this paper we have given an asymptotically tight result on local antimagic total edge coloring of some wheel related graphs including wheel, fan, gear and frienship graph. Hence the following problem aries naturally.

Open Problem 0.1 *Determine exact value local antimagic total edge coloring of Gear graph $J_{n,k}$ for $k \geq 2$ or other some wheel related graphs?*

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