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Non-Isolated Resolving Number of Graphs with Homogeneous Pendant Edges

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Abstract. All graphs in this paper are a simple, nontrivial and connected graph G. A set $W = \{w_1, w_2, w_3, \dots, w_k\}$ of vertex set of G, $r(v\,|W) = (d(v,w_1), d(v,w_2), \dots, d(v,w_k))$ is a representation of vertex v to W, the distance between the vertex v and the vertex w, denoted by $d(v,w)$. A set W is called a resolving set of G if every vertices of G have different representation. The minimum cardinality of resolving set W is metric dimension, denoted by $dim(G)$. Furthermore, the resolving set W of G, is called the non-isolated resolving set if there does not for all $v \in W$ induced by the non-isolated vertex. A non-isolated resolving number, denoted by $nr(G)$, is minimum cardinality of non-isolated resolving set in G. In this research, we obtain the lower bound of the non isolated resolving number of graphs with homogeneous pendant edges, $nr(G \odot mK_1) \geq |V(G)|m$ for $m \geq 2$ and these results of the non isolated resolving number of graphs with homogeneous pendant edges with G with a path P_n, complete graph K_n and cycle C_n.

INTRODUCTION

In this paper, given a graph $G = (V, E)$ is a connected graph in [1,2]. The concept of metric dimension was introduced by Slater [3] and Harrary and Melter [4]. This concept as a locating set in Slater. The distance is the length of a shortest path between two vertices u and v in G, denoted by $d(u,v)$. A set $W = \{w_1, w_2, w_3, \dots, w_k\}$ of vertices and a vertex $v \in G$, the ordered $r(v\,|W) = (d(v,w_1), d(v,w_2), \dots, d(v,w_k))$ of k-vector is a representation of vertex v to W**.** A set W is a resolving set of G if every vertices of G have different representation. The minimum cardinality of resolving set in G, is metric dimension of G, denoted by $dim(G)$.

A resolving set W of graph G without isolated vertex if each subgraph $\langle W \rangle$ induced by W does not isolated vertex in G. The minimum cardinality of a non isolated vertex set in G, is called non isolated resolving number, denoted by $nr(G)$ in [5].

In recent year, Chartrand *et.al.*[6] studied the bound of metric dimension of some families of graphs G. Saenpholphat *et.al.* in [7] investigated the concept of connected resolving partition in a graph G. Furthermore, Chitra and Arumugam [5] studied the lower bound of metric dimension without isolated vertex of some graph families and graph resulting of operation graphs namely cartesian product. There exists some results of other researcher in [7-19]. Thus, we will show some known results about the metric dimension without isolated vertex of graphs with homogeneous pendant edges.

Theorem 1.1. ([7]) Let G be a graph with $n \geq 3$. Then $cr(G) = n - 1$ if and only if $G = K_n$ or $K_{1,n-1}$.

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Theorem 1.2. ([5]) Let T be a tree graphs which does not path. Let s be the number of vertex v of T with $l_v > 1$, $nr(T) = dim(T) + s$.

The results of metric dimension without isolated vertex of standart graph are as follows.

Proposition 1.3 ([5]) Let G be a connected graph of order $n \geq 2$, then we have

For P_n, $n \geq 2, nr(P_n) = 2$.
For K_n, $n \geq 3, nr(K_n) = n - 1$.
For $K_{m,n}, m, n \geq 2$, $nr(K_{m,n}) = m + n - 2$.
For the friendship with k-triangles, $k \geq 2$, $nr(G) = k + 1$.
For $P_n + K_1, n \geq 2, nr(P_n + K_1) = \lfloor \frac{n}{2} \rfloor$.

In this paper, we use one of operation graphs namely corona product, a graph G corona product $H, G \odot H$, is defined as a graph obtained by taking one copy of G and $|V(G)|$ copies of graph $H_1, H_2, \ldots, H_n$ of H and connecting i-th vertex of a graph G to the vertices of $H_i, 1 \leq i \leq n$. The vertex and edge set, respectively are as follows

$$V(G \odot H) = V(G) \cup \bigcup_{i \in V(G)} V(H_i),$$

$$E(G \odot H) = E(G) \cup \bigcup_{i \in V(G)} E(H_i) \cup \{iu_i | u_i \in V(H_i)\},$$

where $H_i \cong H$, for all $i \in V(G)$. If $H \cong K_1$, $G \odot H$ is equal to the graph produced which add one pendant edge to every vertex in a graph G. Generally, if $H \cong mK_1$ where mK_1 is union of trivial graph K_1, $G \odot H$ is equal to the graph produced which add m pendant edges to every vertex of G which is called graphs with homogeneous pendant edges.

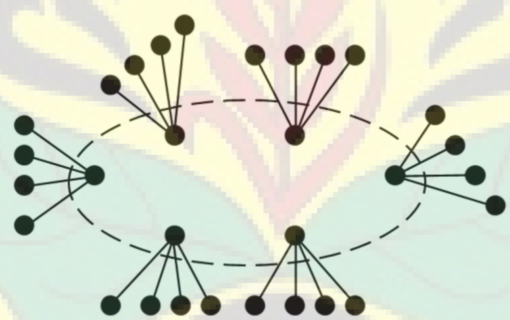

FIGURE 1. Example of Graph with Homogeneous pendant edges or $G \odot mK_1$

THE RESULTS

In this research, we invertigated the non isolated resolving number of graphs with homogeneous pendant edges in $G \odot mK_1$ for any graph G and $m \geq 2$. Next, we will use the idea of Saenpholphat and Zhang in [7] studied the concept of distance similar in a graph G. The open neigbourhood $N(v)$ of a vertex $v \in V(G)$, is all vertices in graph G which adjacent to vertex v and the close neigbourhood $N[v]$ of a vertex $v \in V(G)$ is $N(v) \cup \{v\}$. For two any veretx $u, v \in V(G)$ are defined distance similar if $d(u,x) = d(v,x)$ for all $x \in V(G) - \{u, v\}$. It can be found some of their properties in the following observasion.

Observation 2.1. ([7]) If U is a distance similar equivalence class of a connected graph G with $|U| = p \geq 2$, then every resolving set of G contains at least $p - 1$ vertices from U

The following theorem about the non isolated resolving number of graphs with homogeneous pendant edges, similar with graph with m pendant edges or $G \odot mK_1$.

Lemma 2.1. Let G be a nontrivial graph of order $|V(G)|$ and mK_1 be union of trivial graph of order $m \geq 2$, the non isolated resolving number of $G \odot mK_1$ is $nr(G \odot mK_1) \geq |V(G)|m$.

Proof. Let G be a graph of order $n \geq 2$ and vertex set $V(G) = \{u_i: 1 \leq i \leq n\}$ and $V((mK_1)_i) = \{v_{i,1}, v_{i,2}, \dots, v_{i,m}\}$ of mK_1 that joining with u_i. Let $G \odot mK_1$ be a connected graph with vertex set $V(G \odot mK_1) = V(G) \cup \{v_{i,j}: 1 \leq i \leq n, 1 \leq j \leq m\}$ and the edge set $E(G \odot mK_1) = E(G) \cup \{u_i v_{i,j}: 1 \leq i \leq n, 1 \leq j \leq m\}$.

For every $i \in \{1,2,\dots,n\}$, every pair vertices $u, v \in (mK_1)_i$ holds $d(u,x) = d(v,x)$ where $x \in V(G \odot mK_1) = \{u, v\}$. Thus, every subgraph $(mK_1)_i$ is a distance similar equivalence class of $G \odot mK_1$. Based on Observation 2.1, we have resolving set W at least $n(m-1)$ where $W = \bigcup_{i \in V(G)} W_i, W_i \subset V((mK_1)_i)$ and all vertices in W is not connected such as W is not resolving set with non-isolated vertex. We show that W is resolving set with non-isolated vertex, we need to show that every vertices $v \in W_i \subset V((mK_1)_i)$ connected to each i-th vertex in graph G by edge set $\{u_i v_{i,j}\}: 1 \leq i \leq n, 1 \leq j \leq m\}$ such that

$$|W| \geq |\bigcup_{i \in V(G)} (W_i)| + |V(G)| \geq n(m-1) + n = nm = |V(G)|m$$

It is clearly that non-isolated resolving set W at least $|V(G)|m$. Thus, $nr(G \odot mK_1) \geq |V(G)|m$. ■

For corona product $G \odot mK_1$, if $G \cong K_1$, $G \odot mK_1$ is equal to the star graph S_m with m pendant edges, then we have this following corollary

Corollary 2.2. Let $G \cong mK_1$ be a union trivial graph and $m \geq 2$, we have $nr(K_1 \odot mK_1) = m$

Proof. Based on definition corona product between union trivial graph K_1 and mK_1 that the vertex in K_1 joining to the all vertices in union trivial graph mK_1 such that we know that degree of vertex in K_1 is m and all vertices in union trivial graph mK_1 which have degree one. Thus, the corona product of $K_1 \odot mK_1$ isomorphic to star graph S_m. We know that $nr(S_m) = m$, then we obtain the non isolated resolving number of $K_1 \odot mK_1$ is $nr(K_1 \odot mK_1) = m$ ■

These results of the non isolated resolving number of graphs with homogeneous pendant edges of a graph G with a path P_n, cycle C_n, and complete graph K_n, by using the lower bound in Lemma 2.1, we have the some theorem as follows.

Theorem 2.3. Let $G \cong K_n$ be a complete graph of order n and $m \geq 2$, we have $nr(K_n \odot mK_1) = nm$

Proof. Let $K_n \odot mK_1$ be a graphs resulting corona product of complete graph K_n and union trivial graph mK_1 with vertex set $V(K_n \odot mK_1) = V(K_n) \cup \bigcup_{i \in V(K_n)} V((mK_1)_i) = \{v_i, v_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m\}$ and the cardinality of vertex set is $|V(K_n)| = n$. Based on Lemma 2.1 that $nr(K_n \odot mK_1) \geq |V(K_n)|m = nm$. Furthermore, we will prove that $nr(K_n \odot mK_1) \leq nm$. We choose $W = V(K_n) \cup \bigcup_{i \in V(K_n)} V(((m-1)K_1)_i) = \{v_i, v_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m-1\}$. The representation of vertices $v \in V(K_n \odot mK_1) - W$ are

$$r(v_{i,m}|W) = (\underbrace{\underbrace{3,\dots,3}_{m-1},\dots,\underbrace{3,\dots,3}_{m-1}}_{i-1}\ \underbrace{2,2,\dots,2}_{i-1}, 1, \underbrace{2,2,\dots,2}_{n-i}, \underbrace{2,2,\dots,2}_{m-1}, \underbrace{\underbrace{3,\dots,3}_{m-1},\dots,\underbrace{3,\dots,3}_{m-1}}_{n-i})$$

It can be seen that all representation of vertices $v \in V(K_n \odot mK_1) - W$ respect to $W$are distinct and also we need show that there is not exists the isolated-vertex in W. The vertices $v \in (mK_1)_i$ are connected which the vertices $v - v_i - w$, where $v, w \in V((mK_1)_i)$. Such that, we have $nm \leq nr(K_n \odot mK_1) \leq nm$. Thus, $nr(K_n \odot mK_1) = nm$. ■

If $G \cong P_n$, $G \odot mK_1$ is isomorphic to a caterpillar $C_{n,m}$

Theorem 2.4. Let $G \cong P_n$ be a path of order $n \geq 2$ and $m \geq 2$, we have $nr(P_n \odot mK_1) = nm$

Proof. Let $P_n \odot mK_1$ be a graphs resulting corona product of path P_n and union trivial graph mK_1 with vertex set $V(P_n \odot mK_1) = V(P_n) \cup \bigcup_{i\in V(P_n)} V((mK_1)_i) = \{v_i, v_{i,j}; 1 \le i \le n, 1 \le j \le m\}$ and the cardinality of vertex set is $|V(P_n)| = n$. Based on Lemma 2.1 that $nr(P_n \odot mK_1) \ge |V(P_n)|m = nm$. Furthermore, we will prove that $nr(P_n \odot mK_1) \le nm$. We choose $W = V(P_n) \cup \bigcup_{i\in V(P_n)} V(((m-1)K_1)_i) = \{v_i, v_{i,j}; 1 \le i \le n, 1 \le j \le m-1\}$. There are some condition of representation of vertices $v \in V(P_n \odot mK_1) - W$ are

- We know that vertices in path P_n and $m-1$ vertices in every subgraphs $(mK_1)_i$ are element of W
- Based observation 2.1 that $m-1$ vertices in every subgraphs $(mK_1)_i$, thus there is one vertex in every subgraphs $(mK_1)_i$ where not element of W
- Let u_i be a vertex of pendants which isnot element of W. We take $v_1 \in W$ and also $v_1 \in P_n$ such that $d(u_i, v_1) = d(u_i, v_i) + d(v_i, v_{i-1}) + d(v_{i-1}, v_{i-2}) + \cdots + d(v_2, v_1) = i$ such that $r(u_1|W) = \left(1, \dots, d(u_i, v_n), d\left(u_i, v_{i,1}\right), \dots, d\left(u_i, v_{i,m-1}\right)\right)$ $r(u_2|W) = \left(2, \dots, d(u_i, v_n), d\left(u_i, v_{i,1}\right), \dots, d\left(u_i, v_{i,m-1}\right)\right)$

$$\cdots$$

$$r(u_i|W) = (i, \dots, d(u_i, v_n), d\left(u_i, v_{i,1}\right), \dots, d(u_i, v_{i,m-1}))$$

It is clearly that $r(u_1|W) \ne r(u_2|W) \ne \cdots \ne r(u_i|W)$. Thus, all representation of vertices $v \in V(P_n \odot mK_1) - W$ respect to $W$are distinct and also we need show that there is not exists the isolated-vertex in W. The vertices $v \in (mK_1)_i$ are connected which the vertices $v - v_i - w$, where $v, w \in V((mK_1)_i)$. Such that, we have $nm \le nr(P_n \odot mK_1) \le nm$. Thus, $nr(P_n \odot mK_1) = nm$. ■

If $G \cong C_n$, $G \odot mK_1$ is isomorphic to a sun generalized graph $M_{n,m}$

Theorem 2.5. Let $G \cong C_n$ be a connected graph of order n and $m \ge 2$, we have $nr(C_n \odot mK_1) = nm$

Proof. Let $C_n \odot mK_1$ be a graphs resulting corona product of corona graph P_n and union trivial graph mK_1 with vertex set $V(C_n \odot mK_1) = V(C_n) \cup \bigcup_{i\in V(C_n)} V((mK_1)_i) = \{v_i, v_{i,j}; 1 \le i \le n, 1 \le j \le m\}$ and the cardinality of vertex set is $|V(C_n)| = n$. Based on Lemma 2.1 that $nr(C_n \odot mK_1) \ge |V(C_n)|m = nm$. Furthermore, we will prove that $nr(C_n \odot mK_1) \le nm$. We choose $W = V(C_n) \cup \bigcup_{i\in V(C_n)} V(((m-1)K_1)_i) = \{v_i, v_{i,j}; 1 \le i \le n, 1 \le j \le m-1\}$. There are some condition of representation of vertices $v \in V(C_n \odot mK_1) - W$ are

- We know that vertices in cycle C_n and $m-1$ vertices in every subgraphs $(mK_1)_i$ are element of W.
- Based observation 2.1 that $m-1$ vertices in every subgraphs $(mK_1)_i$, thus there is one vertex in every subgraphs $(mK_1)_i$ where not element of W
- Let u_i be a vertex of pendants which isnot element of W. We take $v_1 \in W$ and also $v_1 \in C_n$ such that $d(u_i, v_1) = d(u_i, v_i) + d(v_i, v_{i-1}) + d(v_{i-1}, v_{i-2}) + \cdots + d(v_2, v_1) = i$ for $1 \le i \le \lfloor\frac{n}{2}\rfloor$ such that

$$r(u_1|W) = \left(1, \dots, d(u_i, v_n), d\left(u_i, v_{i,1}\right), \dots, d\left(u_i, v_{i,m-1}\right)\right)$$
$$r(u_2|W) = \left(2, \dots, d(u_i, v_n), d\left(u_i, v_{i,1}\right), \dots, d\left(u_i, v_{i,m-1}\right)\right)$$
$$\cdots$$
$$r(u_i|W) = (i, \dots, d(u_i, v_n), d\left(u_i, v_{i,1}\right), \dots, d(u_i, v_{i,m-1}))$$

For $i \in [\left\lfloor\frac{n}{2}\right\rfloor + 1, n]$, the distance of u_i and v_1 have distance same with $d(u_i, v_1)$ for $1 \le i \le \lfloor\frac{n}{2}\rfloor$. But $r(u_1|W) \ne r(u_2|W) \ne \cdots \ne r(u_i|W)$ since $d\left(u_k, v_{k,j}\right) \ne d\left(u_l, v_{l,j}\right)$ for $1 \le k, l \le n$ specially for $d(u_i, v_1)$, $1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor$ and $i \in [\left\lfloor\frac{n}{2}\right\rfloor + 1, n]$ which have the distance same.

It is clearly that $r(u_1|W) \ne r(u_2|W) \ne \cdots \ne r(u_i|W)$. Thus, all representation of vertices $v \in V(C_n \odot mK_1) - W$ respect to $W$are distinct and also we need show that there is not exists the isolated-vertex in W. The vertices $v \in (mK_1)_i$ are connected which the vertices $v - v_i - w$, where $v, w \in V((mK_1)_i)$. Such that, we have $nm \le nr(C_n \odot mK_1) \le nm$. Thus, $nr(C_n \odot mK_1) = nm$. ■

CONCLUSION

We have shown the non isolated resolving number of graph with homogeneous pendant edges and the results can be attain the best lower bound by Lemma 2.1, therefore we have some open problem as follows.

Open Problem 1. Fing $nr(G \odot mK_1)$, with G is connected graph of order n and $m = 1$.

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