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The Committees of The First International Conference on Combinatorics, Graph Theory and Network Topology (ICCGANT)

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Preface

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On the local edge antimagicness of m-splitting graphs

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On the local edge antimagicness of m-splitting graphs

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Abstract. Let G be a connected and simple graph. A split graph is a graph derived by adding new vertex v' in every vertex v such that v' adjacent to v in graph G. An m-splitting graph is a graph which has m v'-vertices, denoted by ${}_mSpl(G)$. A local edge antimagic coloring in $G = (V, E)$ graph is a bijection $f : V(G) \longrightarrow \{1, 2, 3, ..., |V(G)|\}$ in which for any two adjacent edges e_1 and e_2 satisfies $w(e_1) \neq w(e_2)$, where $e = uv \in G$. The color of any edge $e = uv$ are assigned by $w(e)$ which is defined by sum of label both end vertices $f(u)$ and $f(v)$. The chromatic number of local edge antimagic labeling $\gamma_{lea}(G)$ is the minimal number of color of edge in G graph which has local antimagic coloring. We present the exact value of chromatic number γ_{lea} of m-splitting graph and some special graphs.

Keywords: *Local edge antimagic coloring, chromatic number of graph, m-splitting graph*

1. Introduction

This paper uses connected, simple and undirected graph. We denote by $V(G)$ and $E(G)$ the set of vertices and the set of edges of a graph. It can be seen at [4], [3] and [7]. Let G be a graph with p vertices and q edges. By *labeling* , we mean one-to-one mapping which carries a set of graph elements into a set of numbers (integers), called *labels*. The labeling on graph can be done on edge or vertex. In this research, we label the edge with its own uniqueness, it is *antimagic*. The *edge-antimagic labeling* is defined as labeling that all the weights of the edge have different values, which can be seen at [6] and [9]. Agustin I. H. et all [1] introduced a concept of an local edge antimagic of graph. For a graph G of a size q, *local edge antimagic* is defined as follows:

Definition 1.1 *A local edge antimagic coloring in $G = (V, E)$ graph is a bijection $f : V(G) \longrightarrow \{1, 2, 3, ..., |V(G)|\}$ in which for any two adjacent edges e_1 and e_2 satisfies $w(e_1) \neq w(e_2)$, where $e = uv \in G$.*

Lemma 1.1 *[1] If $\delta(G)$ is maximum degree of G, then we have $\gamma_{lea}(G) \geq \delta(G)$*

Hartsfield and Ringel [8] explain that the color of any edge $e = uv$ are assigned by $w(e)$ which is defined by sum of label both end vertices $f(u)$ and $f(v)$. The chromatic number of local edge antimagic labeling $\gamma_{lea}(G)$ is the minimal number of color of edge in G graph which has local antimagic coloring.

Some new result can be seen in Arumugam et al [2] about local antimagic vertex coloring which be a basic idea about local antimagic coloring. Then, there are another research about

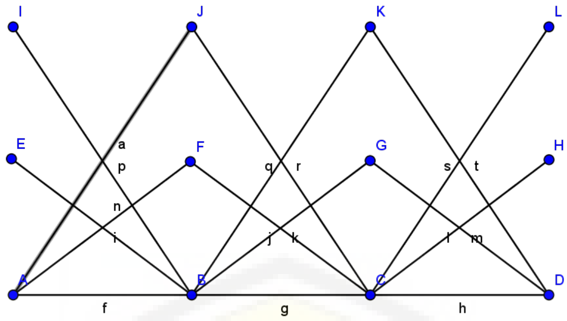

Figure 1. ${}_2Spl(P_4)$ Graph

super antimagic labeling which be found by Dafik et al [5]. The newest one is research about local edge antimagic of graph by Agustin et al [1]. Based on the previous research on it, we decide to do research in same topic such that local-edge-antimagic. But we choose different graph, it is split graph.

Definition 1.2 *Split graph is graph which be gotten by adding new vertex v' in every vertex v such that v' adjacent v in G graph. m-splitting graph is graph which has the number of vertex v' as m. m-splitting graph is denoted by ${}_mSpl(G)$.*

Based on Figure 1, we know that alphabet set $\{A, B, \ldots, L\}$ is the vertex set and $\{a, f, \ldots, t\}$ is the edge set. $\{A, B, C, D\}$ is the vertex set of basic graph or Path graph (P_4). $\{E, F, G, H\}$ is the vertex set of $1-$splitting of P_4 and $\{I, J, K, L\}$ is the vertex set of $2-$splitting of P_4.

2. Result

Some interesting theorem which be found by us such that,

Theorem 2.1 *For $n \geq 3$, the local edge antimagic of $m-$Splitting of P_n graph is*

$$\gamma_{lae}({}_mSpl(P_n)) = \gamma_{lae}(P_n) + 2m$$

Proof. The ${}_mSpl(P_n)$ graph has vertex set $V(G) = \{x_i : 1 \leq i \leq n\} \cup \{x_i^j : 1 \leq i \leq n, 1 \leq j \leq m\}$ and edge set $E(G) = \{x_i x_{i+1} : 1 \leq i \leq n-1\} \cup \{x_i x_{i+1}^j : 1 \leq i \leq n-1, 1 \leq j \leq m\} \cup \{x_i^j x_{i+1} : 1 \leq i \leq n-1, 1 \leq j \leq m\} \cup \{x_i^j x_{i+1}^{j+1} : 1 \leq i \leq n-1, 1 \leq j \leq m\} \cup \{x_i^j x_{i-1}^{j+1} : 1 \leq i \leq n-1, 1 \leq j \leq m\}$. The cardinality $|V(G)| = m(n+1)$ and $|E(G)| = 2m(n-1) + (n-1)$. Let split graph has set of vertices and edges. The set of vertices consists of set of inner and outter vertices. Inner vertex is the vertex set in center graph and outter vertex is the vertex set in split of graph. This also applies to inner and outer edge definitions. Define a bijection $f : V(G) \to \{1, 2, 3, \ldots, |V(G)|\}$ by

$$f(x_i) = \begin{cases} \frac{i+1}{2} & 1 \leq i \leq n,\ i \text{ odd;} \\ n - \frac{i-2}{2} & 1 \leq i \leq n,\ i \text{ even.} \end{cases}$$

$$f(x_i^j) = \begin{cases} \frac{2nj+i+1}{2} & 1 \leq j \leq m,\ i \text{ odd;} \\ \frac{2n(j+1)-(i-2)}{2} & 1 \leq j \leq m,\ i \text{ even.} \end{cases}$$

It is easy to see that f is a local antimagic labeling of ${}_mSpl(P_n)$ and the edge weights are as follows

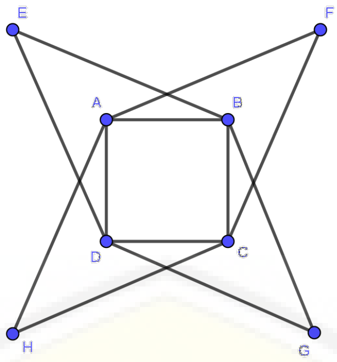

Figure 2. ${}_1Spl(C_4)$ Graph

$$w(x_ix_{i+1}) = \begin{cases} n+1 & 1 \le j \le m,\ i \text{ odd;} \\ n+2 & 1 \le j \le m,\ i \text{ even.} \end{cases}$$

$$w(x_ix^j_{i+1}) = \begin{cases} (j+1)n+1 & 1 \le j \le m,\ i \text{ odd;} \\ (j+1)n+2 & 1 \le j \le m,\ i \text{ even.} \end{cases}$$

$$w(x^j_ix_{i+1}) = \begin{cases} (j+1)n+1 & 1 \le j \le m,\ i \text{ odd;} \\ (j+1)n+2 & 1 \le j \le m,\ i \text{ even.} \end{cases}$$

Hence, from the above the edge weights, it is easy to see that f induces a proper edge colouring of ${}_mSpl(P_n)$ and it gives $\gamma_{lae}({}_mSpl(P_n)) \le \gamma_{lae}(P_n)+2m$. Then it will be showed that $\gamma_{lae}({}_mSpl(P_n)) \ge \gamma_{lae}(P_n)+2m$.

Let $\gamma(G) \ge \delta$. We will show that $\gamma(G) \ge 2m+2$. Based on Lemma [1] that $\gamma(G) \ge \delta$, we know that $\delta(G) = 2m+2$. So, $\gamma(G) \ge \delta = 2m+2$, the lower bound of local edge antimagic of G. It conclude that $\gamma = 2m+2$. So $\gamma_{lae}({}_mSpl(P_n)) = \gamma_{lae}(P_n)+2m$. □

Theorem 2.2 *For $n \ge 4$, the local edge antimagic of $m-$Splitting of C_n graph is*

$$\gamma_{lae}({}_mSpl(C_n)) = \gamma_{lae}(C_n) + 3m$$

Proof. The ${}_mSpl(C_n)$ graph has vertex set $V(G) = \{x_i : 1 \le i \le n\} \cup \{x^j_i : 1 \le i \le n, 1 \le j \le m\}$ and edge set $E(G) = \{x_ix_{i+1} : 1 \le i \le n-1\} \cup \{x_ix^j_{i+1} : 1 \le i \le n-1, 1 \le j \le m\} \cup \{x^j_ix_{i+1} : 1 \le i \le n-1, 1 \le j \le m\}$. The cardinality $|V(G)| = mn$ and $|E(G)| = 2mn$. Let split graph has set of vertices and edges. The set of vertices consists of set of inner and outter vertices. Inner vertex is the vertex set in center graph and outter vertex is the vertex set in split of graph. This also applies to inner and outer edge definitions. The illustration can be seen at Figure 2. Define a bijection $f : V(G) \to \{1, 2, 3, \ldots, |V(G)|\}$ by

$$f(x_i) = \begin{cases} \frac{i+1}{2} & 1 \le i \le n,\ i \text{ odd;} \\ n - \frac{i-2}{2} & 1 \le i \le n,\ i \text{ even.} \end{cases}$$

$$f(x^j_i) = \begin{cases} \frac{2nj+i+1}{2} & 1 \le j \le m,\ i \text{ odd;} \\ \frac{2n(j+1)-(i-2)}{2} & 1 \le j \le m,\ i \text{ even.} \end{cases}$$

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It is easy to see f is a local antimagic coloring of ${}_mSpl(C_n)$ and the edge weights as follows. For n even,

$$w(e) = \begin{cases} n+1 & \text{for } e = x_i x_{i+1},\ 1 \le i \le n-1,\ i \text{ odd;} \\ n+2 & \text{for } e = x_i x_{i+1},\ 1 \le i \le n-1,\ i \text{ even;} \\ \frac{n+2}{2} & \text{for } e = x_n x_1; \end{cases}$$

For n odd,

$$w(e) = \begin{cases} n+1 & \text{for } e = x_i x_{i+1},\ 1 \le i \le n-1,\ i \text{ odd;} \\ n+2 & \text{for } e = x_i x_{i+1},\ 1 \le i \le n-1,\ i \text{ even;} \\ \frac{n+3}{2} & \text{for } e = x_n x_1; \end{cases}$$

$$w(x_i x_{i+1}^j) = \begin{cases} (j+1)n+1 & 1 \le j \le m,\ i \text{ odd;} \\ (j+1)n+2 & 1 \le j \le m,\ i \text{ even.} \end{cases}$$

$$w(x_i^j x_{i+1}) = \begin{cases} (j+1)n+1 & 1 \le j \le m,\ i \text{ odd;} \\ (j+1)n+2 & 1 \le j \le m,\ i \text{ even.} \end{cases}$$

Hence, from the above the edge weights, it easy to see that f induces a proper edge colouring of ${}_mSpl(C_n)$ and it gives $\gamma_{lae}({}_mSpl(C_n)) \le \gamma_{lae}(C_n) + 3m$. Then it will be showed that $\gamma_{lae}({}_mSpl(C_n)) \ge \gamma_{lae}(C_n) + 3m$.

We will show that $\gamma(G) \ge 3m+3$. Based on Lemma that $\gamma(G) \ge \delta = 2m+2$. *Case 1:* Assume that $\gamma(G) < 3m+3$, we take $\gamma = 3m+2$. Let c_1 and c_2 be the color of inner edge. Then $\frac{n(c_1+c_2)}{2} = \frac{n(2n+2)}{2}$ and hence $c_1 + c_2 = 2n+2$. However, if x_i is the vertex with $f(x_i) = n$, then the colors received by inner edges are at least $n+1$ and one of them is at least $n+2$. Thus $c_1 + c_2 = (n+1)+(n+2) = 2n+3$. *Case 2:* Assume that $\gamma(G) < 3m+3$, we take $\gamma = 3m+2$. Assume that each outer edge:

- for $m = 1$, then γ of outer edge is equal to 3
- for $m = 2$, then γ of outer edge is equal to 6

  $\vdots$
- for any m, then γ of outer edge is equal to $3m$

Based on *Case 1 and Case 2*, then it is contradiction. Hence $\gamma_{lae}({}_mSpl(C_n)) \ge 2+3m$. Since $\gamma_{lae}({}_mSpl(C_n)) \le 3+3m$ and $\gamma_{lae}({}_mSpl(C_n)) \ge \gamma_{lae}(C_n)+3m$. □

Theorem 2.3 *For $n \ge 4$, the local edge antimagic of $m-$Splitting of K_n graph is*

$$\gamma_{lae}({}_mSpl(K_n)) = \gamma_{lae}(K_n) + 4m$$

Proof. The ${}_mSpl(K_n)$ graph has vertex set $V(G) = \{x_i, x_i^j : 1 \le i \le n, 1 \le j \le m\}$ and edge set $E(G) = \{x_i x_{i+k} : 1 \le i \le n, 1 \le k \le n-i\} \cup \{x_i x_{i+k}^j : 1 \le i \le n, 1 \le j \le m, 1 \le k \le n-i\} \cup \{x_i^j x_{i+k} : 1 \le i \le n, 1 \le j \le m, 1 \le k \le n-i\}$. The cardinality $|V(G)| = mn$ and $|E(G)| = 3mn$. Let split graph has set of vertices and edges. The set of vertices consists of set of inner and outter vertices. Inner vertex is the vertex set in center graph and outter vertex is the vertex set in split of graph. This also applies to inner and outer edge definitions. The illustration can be seen at Figure 3. Define a bijection $f : V(G) \to \{1, 2, 3, \ldots, |V(G)|\}$ by

$$f(x_i) = \{\ i \quad 1 \le i \le n.$$

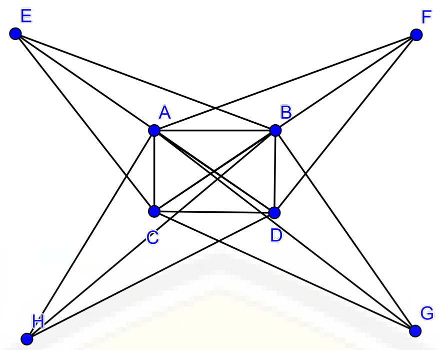

Figure 3. ${}_1Spl(K_4)$ Graph

$$f(x_i^j) = \begin{cases} \frac{2nj+i+1}{2} & 1 \leq j \leq m,\ i \text{ odd;} \\ nj+i & 1 \leq j \leq m,\ i \text{ even.} \end{cases}$$

It is easy to see f is a local antimagic coloring of ${}_mSpl(K_n)$ and the edge weights as follows.

$$w(e) = \begin{cases} i+k+1 & \text{for } e = x_i x_{i+k},\ 2 \leq i \leq n,\ 1 \leq k \leq n-1; \\ k+2 & \text{for } e = x_1 x_{1+k},\ 1 \leq k \leq n-2; \\ n+1 & \text{for } e = x_i x_n. \end{cases}$$

For $1 \leq k \leq n-i$,

$$w(x_i x_{i+k}^j) = \begin{cases} (j+1)n+1 & 1 \leq j \leq m,\ i \text{ odd;} \\ (j+1)n+2 & 1 \leq j \leq m,\ i \text{ even.} \end{cases}$$

$$w(x_i^j x_{i+k}) = \begin{cases} (j+1)n+1 & 1 \leq j \leq m,\ i \text{ odd;} \\ (j+1)n+2 & 1 \leq j \leq m,\ i \text{ even.} \end{cases}$$

Hence, from the above the edge weights, it is easy to see that f induces a proper edge colouring of ${}_mSpl(K_n)$ and it gives $\gamma_{lae}({}_mSpl(K_n)) \leq \gamma_{lae}(K_n) + 4m$. Then it will be showed that $\gamma_{lae}({}_mSpl(K_n)) \geq \gamma_{lae}(K_n) + 4m$.

We will show that $\gamma(G) \geq 4m+4$. *Case 1:* Assume that $\gamma(G) < 4m+4$, we take $\gamma = 4m+3$. Let c_1 and c_2 be the color of inner edge. Then $\frac{n(c_1+c_2)}{2} = \frac{n(2k+4)}{2}$ and hence $c_1 + c_2 = 2k+4$. However, if x_i is the vertex with $f(x_i) = n$, then the colors received by inner edges are at least $k+2$ and one of them is at least $n+1$. Thus $c_1 + c_2 = (n+1)+(n+2) = k+n+3$.
Case 1: Assume that $\gamma(G) < 4m+4$, we take $\gamma = 4m+3$. Assume that each outer edge:

- for $m = 1$, then γ of outer edge is equal to 4
- for $m = 2$, then γ of outer edge is equal to 8

  $\vdots$
- for any m, then γ of outer edge is equal to $4m$

Based on *Case 1 and Case 2*, then it is contradiction. Hence $\gamma_{lae}({}_mSpl(K_n)) \geq 3+4m$. Since $\gamma_{lae}({}_mSpl(K_n)) \leq 4+4m$ and $\gamma_{lae}({}_mSpl(K_n)) \geq \gamma_{lae}(K_n)+4m$. □

There are some theorem of special graph which be found as follows:

Theorem 2.4 *The local edge antimagic of $_mSpl(P_4)$ graph is $\gamma_{lae}(_mSpl(P_4)) = 2(1+m)$.*

Proof. The $_mSpl(P_4)$ graph has vertex set $V(G) = \{x_i : 1 \le i \le 4\} \cup \{x_i^j : 1 \le i \le 4, 1 \le j \le m\}$ and edge set $E(G) = \{x_i x_{i+1} : 1 \le i \le 3\} \cup \{x_i x_{i+1}^j : 1 \le i \le 3, 1 \le j \le m\} \cup \{x_i^j x_{i+1} : 1 \le i \le 3, 1 \le j \le m\} \cup \{x_i^j x_{i+1}^{j+1} : 1 \le i \le 3, 1 \le j \le m\} \cup \{x_i^j x_{i-1}^{j+1} : 1 \le i \le 3, 1 \le j \le m\}$. Let split graph has set of vertices and edges. The set of vertices consists of set of inner and outter vertices. Inner vertex is the vertex set in center graph and outter vertex is the vertex set in split of graph. This also applies to inner and outer edge definitions. Define a bijection $f : V(G) \to \{1, 2, 3, \dots, |V(G)|\}$ by

$$f(x_i) = \begin{cases} \frac{i+1}{2} & 1 \le i \le 4,\ i \text{ odd;} \\ n - \frac{i-2}{2} & 1 \le i \le 4,\ i \text{ even.} \end{cases}$$

For $1 \le i \le 4$,

$$f(x_i^j) = \begin{cases} \frac{8j+i+1}{2} & 1 \le j \le m,\ i \text{ odd;} \\ \frac{8(j+1)-(i-2)}{2} & 1 \le j \le m,\ i \text{ even.} \end{cases}$$

It is easy to see that f is a local antimagic labeling of $_mSpl(P_4)$ and the edge weights are as follows. For $1 \le i \le 4$,

$$w(x_i x_{i+1}) = \begin{cases} 5 & 1 \le j \le m,\ i \text{ odd;} \\ 6 & 1 \le j \le m,\ i \text{ even.} \end{cases}$$

$$w(x_i x_{i+1}^j) = \begin{cases} (j+1)4+1 & 1 \le j \le m,\ i \text{ odd;} \\ (j+1)4+2 & 1 \le j \le m,\ i \text{ even.} \end{cases}$$

$$w(x_i^j x_{i+1}) = \begin{cases} (j+1)4+1 & 1 \le j \le m,\ i \text{ odd;} \\ (j+1)4+2 & 1 \le j \le m,\ i \text{ even.} \end{cases}$$

Hence, from the above the edge weights, it is easy to see that f induces a proper edge colouring of $_mSpl(P_4)$ and it gives $\gamma_{lae}(_mSpl(P_4)) \le 2(1+m)$. Then it will be showed that $\gamma_{lae}(_mSpl(P_4)) \ge 2(1+m)$.

Let $\gamma(G) \ge \delta$. We will show that $\gamma(G) \ge 10$. Based on Lemma [1] that $\gamma(G) \ge \delta$, we know that $\delta(G) = 10$. So, $\gamma(G) \ge \delta = 10$, the lower bound of local edge antimagic of G. It conclude that $\gamma = 10$. So $\gamma_{lae}(_mSpl(P_4)) = 2(1+m)$. □

Theorem 2.5 *The local edge antimagic of $_2Spl(P_n)$ graph is $\gamma_{lae}(_2Spl(P_n)) = 6$.*

Proof. The $_2Spl(P_n)$ graph has vertex set $V(G) = \{x_i : 1 \le i \le n\} \cup \{x_i^j : 1 \le i \le n, 1 \le j \le 2\}$ and edge set $E(G) = \{x_i x_{i+1} : 1 \le i \le n-1\} \cup \{x_i x_{i+1}^j : 1 \le i \le n-1, 1 \le j \le 2\} \cup \{x_i^j x_{i+1} : 1 \le i \le n-1, 1 \le j \le 2\} \cup \{x_i^j x_{i+1}^{j+1} : 1 \le i \le n-1, 1 \le j \le 2\} \cup \{x_i^j x_{i-1}^{j+1} : 1 \le i \le n-1, 1 \le j \le 2\}$. The cardinality $|V(G)| = 2(n+1)$ and $|E(G)| = 4(n-1) + (n-1)$. Let split graph has set of vertices and edges. The set of vertices consists of set of inner and outter vertices. Inner vertex is the vertex set in center graph and outter vertex is the vertex set in split of graph. This also applies to inner and outer edge definitions. Define a bijection $f : V(G) \to \{1, 2, 3, \dots, |V(G)|\}$ by

$$f(x_i) = \begin{cases} \frac{i+1}{2} & 1 \le i \le n,\ i \text{ odd;} \\ n - \frac{i-2}{2} & 1 \le i \le n,\ i \text{ even.} \end{cases}$$

$$f(x_i^j) = \begin{cases} \frac{2nj+i+1}{2} & 1 \le j \le 2,\ i \text{ odd;} \\ \frac{2n(j+1)-(i-2)}{2} & 1 \le j \le 2,\ i \text{ even.} \end{cases}$$

It is easy to see that f is a local antimagic labeling of ${}_2Spl(P_n)$ and the edge weights are as follows

$$w(x_i x_{i+1}) = \begin{cases} n+1 & 1 \le j \le 2,\ i \text{ odd;} \\ n+2 & 1 \le j \le 2,\ i \text{ even.} \end{cases}$$

$$w(x_i x_{i+1}^j) = \begin{cases} (j+1)n+1 & 1 \le j \le 2,\ i \text{ odd;} \\ (j+1)n+2 & 1 \le j \le 2,\ i \text{ even.} \end{cases}$$

$$w(x_i^j x_{i+1}) = \begin{cases} (j+1)n+1 & 1 \le j \le 2,\ i \text{ odd;} \\ (j+1)n+2 & 1 \le j \le 2,\ i \text{ even.} \end{cases}$$

Hence, from the above the edge weights, it easy to see that f induces a proper edge colouring of ${}_mSpl(P_n)$ and it gives $\gamma_{lae}({}_2Spl(P_n)) \le 6$. Then it will be showed that $\gamma_{lae}({}_2Spl(P_n)) \ge 6$.

Assume that $\gamma_{lae}({}_2Spl(P_n)) = 6 - 1$. Since $\gamma_{lae}(P_n) = 2$, then $\gamma_{lae}({}_2Spl(P_n)) = 5$. Let c_1 till c_2 be the color of inner edge. Then $\frac{n(c_1+c_2)}{2} = \frac{n(2n+2)}{2}$ and hence $c_1 + c_2 = 2n + 2$. However, if xi is the vertex with $f(xi) = n$, then the colors received by inner edges are at least $n + 1$ and one of them is at least $n + 2$. Thus $c_1 + c_2 = (n + 1) + (n + 2) = 2n + 3$. It's contradiction. Hence $\gamma_{lae}({}_2Spl(P_n)) \ge 6$. Since $\gamma_{lae}({}_2Spl(P_n)) \le 6$ and $\gamma_{lae}({}_2Spl(P_n)) \ge 6$, it completes the proof. □

Theorem 2.6 *The local edge antimagic of ${}_mSpl(C_4)$ graph is $\gamma_{lae}({}_mSpl(C_4)) = 3(1 + m)$.*

Proof. The ${}_mSpl(C_4)$ graph has vertex set $V(G) = \{x_i : 1 \le i \le 4\} \cup \{x_i^j : 1 \le i \le 4, 1 \le j \le m\}$ and edge set $E(G) = \{x_i x_{i+1} : 1 \le i \le 3\} \cup \{x_i x_{i+1}^j : 1 \le i \le 3, 1 \le j \le m\} \cup \{x_i^j x_{i+1} : 1 \le i \le 3, 1 \le j \le m\}$. The cardinality $|V(G)| = 4m$ and $|E(G)| = 8m$. Let split graph has set of vertices and edges. The set of vertices consists of set of inner and outter vertices. Inner vertex is the vertex set in center graph and outter vertex is the vertex set in split of graph. This also applies to inner and outer edge definitions. Define a bijection $f : V(G) \to \{1, 2, 3, \ldots, |V(G)|\}$ by

$$f(x_i) = \begin{cases} \frac{i+1}{2} & 1 \le i \le 4,\ i \text{ odd;} \\ n - \frac{i-2}{2} & 1 \le i \le 4,\ i \text{ even.} \end{cases}$$

$$f(x_i^j) = \begin{cases} \frac{8j+i+1}{2} & 1 \le j \le m,\ i \text{ odd;} \\ \frac{8(j+1)-(i-2)}{2} & 1 \le j \le m,\ i \text{ even.} \end{cases}$$

It is easy to see f is a local antimagic coloring of ${}_mSpl(C_4)$ and the edge weights as follows. For n even,

$$w(e) = \begin{cases} 5 & \text{for } e = x_i x_{i+1},\ 1 \le i \le 3,\ i \text{ odd;} \\ 6 & \text{for } e = x_i x_{i+1},\ 1 \le i \le 3,\ i \text{ even;} \\ 3 & \text{for } e = x_4 x_1; \end{cases}$$

For n odd,

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$$w(e) = \begin{cases} n+1 & \text{for } e = x_i x_{i+1},\ 1 \le i \le n-1,\ i \text{ odd;} \\ n+2 & \text{for } e = x_i x_{i+1},\ 1 \le i \le n-1,\ i \text{ even;} \\ \frac{n+3}{2} & \text{for } e = x_n x_1; \end{cases}$$

$$w(x_i x_{i+1}^j) = \begin{cases} (j+1)4+1 & 1 \le j \le m,\ i \text{ odd;} \\ (j+1)4+2 & 1 \le j \le m,\ i \text{ even.} \end{cases}$$

$$w(x_i^j x_{i+1}) = \begin{cases} (j+1)4+1 & 1 \le j \le m,\ i \text{ odd;} \\ (j+1)4+2 & 1 \le j \le m,\ i \text{ even.} \end{cases}$$

Hence, from the above the edge weights, it easy to see that f induces a proper edge colouring of ${}_mSpl(C_4)$ and it gives $\gamma_{lae}({}_mSpl(C_n)) \le 3(1+m)$. Then it will be showed that $\gamma_{lae}({}_mSpl(C_n)) \ge 3(1+m)$.

Assume that $\gamma_{lae}({}_mSpl(C_4)) = (\gamma_{lae}(C_4)-1)+3m$. Since $\gamma_{lae}(C_4) = 3$, then $\gamma_{lae}({}_mSpl(C_4)) = 3(1+m)-1m$. Let c_1 till c_2 be the color of inner edge. Then $\frac{n(c_1+c_2)}{2} = \frac{n(2n+2)}{2}$ and hence $c_1 + c_2 = 2n+2$. However, if xi is the vertex with $f(xi) = n$, then the colors received by inner edges are at least $n+1$ and one of them is at least $n+2$. Thus $c_1+c_2 = (n+1)+(n+2) = 2n+3$. It's contradiction. Hence $\gamma_{lae}({}_mSpl(C_4)) \ge 3(1+m)-1$. Since $\gamma_{lae}({}_mSpl(C_4)) \le 3(1+m)$ and $\gamma_{lae}({}_mSpl(C_4)) \ge 3(1+m)$, it completes the proof. □

Theorem 2.7 *The local edge antimagic of ${}_3Spl(C_n)$ graph is $\gamma_{lae}({}_3Spl(C_n)) = 12$.*

Proof. The ${}_3Spl(C_n)$ graph has vertex set $V(G) = \{x_i : 1 \le i \le n\} \cup \{x_i^j : 1 \le i \le n, 1 \le j \le 3\}$ and edge set $E(G) = \{x_i x_{i+1} : 1 \le i \le n-1\} \cup \{x_i x_{i+1}^j : 1 \le i \le n-1, 1 \le j \le 3\} \cup \{x_i^j x_{i+1} : 1 \le i \le n-1, 1 \le j \le 3\}$. The cardinality $|V(G)| = 3n$ and $|E(G)| = 6n$. Let split graph has set of vertices and edges. The set of vertices consists of set of inner and outter vertices. Inner vertex is the vertex set in center graph and outter vertex is the vertex set in split of graph. This also applies to inner and outer edge definitions. It also admits in edge set. Define a bijection $f : V(G) \to \{1, 2, 3, \ldots, |V(G)|\}$ by

$$f(x_i) = \begin{cases} \frac{i+1}{2} & 1 \le i \le n,\ i \text{ odd;} \\ n - \frac{i-2}{2} & 1 \le i \le n,\ i \text{ even.} \end{cases}$$

$$f(x_i^j) = \begin{cases} \frac{2nj+i+1}{2} & 1 \le j \le 3,\ i \text{ odd;} \\ \frac{2n(j+1)-(i-2)}{2} & 1 \le j \le 3,\ i \text{ even.} \end{cases}$$

It is easy to see f is a local antimagic coloring of ${}_3Spl(C_n)$ and the edge weights as follows. For n even,

$$w(e) = \begin{cases} n+1 & \text{for } e = x_i x_{i+1},\ 1 \le i \le n-1,\ i \text{ odd;} \\ n+2 & \text{for } e = x_i x_{i+1},\ 1 \le i \le n-1,\ i \text{ even;} \\ \frac{n+2}{2} & \text{for } e = x_n x_1; \end{cases}$$

For n odd,

$$w(e) = \begin{cases} n+1 & \text{for } e = x_i x_{i+1},\ 1 \le i \le n-1,\ i \text{ odd;} \\ n+2 & \text{for } e = x_i x_{i+1},\ 1 \le i \le n-1,\ i \text{ even;} \\ \frac{n+3}{2} & \text{for } e = x_n x_1; \end{cases}$$

$$w(x_i x_{i+1}^j) = \begin{cases} (j+1)n+1 & 1 \leq j \leq 3,\ i \text{ odd;} \\ (j+1)n+2 & 1 \leq j \leq 3,\ i \text{ even.} \end{cases}$$

$$w(x_i^j x_{i+1}) = \begin{cases} (j+1)n+1 & 1 \leq j \leq 3,\ i \text{ odd;} \\ (j+1)n+2 & 1 \leq j \leq 3,\ i \text{ even.} \end{cases}$$

Hence, from the above the edge weights, it easy to see that f induces a proper edge colouring of ${}_mSpl(C_n)$ and it gives $\gamma_{lae}({}_mSpl(C_n)) \leq 12$. Then it will be showed that $\gamma_{lae}({}_mSpl(C_n)) \geq 12$.

Assume that $\gamma_{lae}({}_3Spl(C_n)) = (\gamma_{lae}(C_n) - 1) + 9$. Since $\gamma_{lae}(C_n) = 3$, then $\gamma_{lae}({}_mSpl(C_n)) = 11$. Let c_1 till c_2 be the color of inner edge. Then $\frac{n(c_1+c_2)}{2} = \frac{n(2n+2)}{2}$ and hence $c_1 + c_2 = 2n + 2$. However, if xi is the vertex with $f(xi) = n$, then the colors received by inner edges are at least $n+1$ and one of them is at least $n+2$. Thus $c_1+c_2 = (n+1)+(n+2) = 2n+3$. It's contradiction. Hence $\gamma_{lae}({}_mSpl(C_n)) \geq 11$. Since $\gamma_{lae}({}_3Spl(C_n)) \leq 12$ and $\gamma_{lae}({}_3Spl(C_n)) \geq 12$, it completes the proof. □

Theorem 2.8 *The local edge antimagic of* ${}_mSpl(K_4)$ *graph is* $\gamma_{lae}({}_mSpl(K_4)) = 4m + 5$.

Proof. The ${}_mSpl(K_4)$ graph has vertex set $V(G) = \{x_i, x_i^j : 1 \leq i \leq 4, 1 \leq j \leq m\}$ and edge set $E(G) = \{x_i x_{i+k} : 1 \leq i \leq 4, 1 \leq k \leq 4-i\} \cup \{x_i x_{i+k}^j : 1 \leq i \leq 4, 1 \leq j \leq m, 1 \leq k \leq 4-i\} \cup \{x_i^j x_{i+k} : 1 \leq i \leq 4, 1 \leq j \leq m, 1 \leq k \leq 4-i\}$. The cardinality $|V(G)| = 4m$ and $|E(G)| = 12m$. Let split graph has set of vertices and edges. The set of vertices consists of set of inner and outter vertices. Inner vertex is the vertex set in center graph and outter vertex is the vertex set in split of graph. This also applies to inner and outer edge definitions. Define a bijection $f : V(G) \to \{1, 2, 3, \ldots, |V(G)|\}$ by

$$f(x_i) = \{\ i \quad 1 \leq i \leq 4.$$

$$f(x_i^j) = \begin{cases} \frac{8j+i+1}{2} & 1 \leq j \leq m,\ i \text{ odd;} \\ 4j+i & 1 \leq j \leq m,\ i \text{ even.} \end{cases}$$

It is easy to see f is a local antimagic coloring of ${}_mSpl(K_4)$ and the edge weights as follows.

$$w(e) = \begin{cases} i+k+1 & \text{for } e = x_i x_{i+k},\ 2 \leq i \leq n,\ 1 \leq k \leq 3; \\ k+2 & \text{for } e = x_1 x_{1+k},\ 1 \leq k \leq 2; \\ 4+1 & \text{for } e = x_i x_4. \end{cases}$$

For $1 \leq k \leq 4 - i$,

$$w(x_i x_{i+k}^j) = \begin{cases} (j+1)4+1 & 1 \leq j \leq m,\ i \text{ odd;} \\ (j+1)4+2 & 1 \leq j \leq m,\ i \text{ even.} \end{cases}$$

$$w(x_i^j x_{i+k}) = \begin{cases} (j+1)4+1 & 1 \leq j \leq m,\ i \text{ odd;} \\ (j+1)4+2 & 1 \leq j \leq m,\ i \text{ even.} \end{cases}$$

Hence, from the above the edge weights, it easy to see that f induces a proper edge colouring of ${}_mSpl(K_4)$ and it gives $\gamma_{lae}({}_mSpl(K_4)) \leq 4m + 5$. Then it will be showed that $\gamma_{lae}({}_mSpl(K_4)) \geq 4m + 5$.

Assume that $\gamma_{lae}({}_mSpl(K_4)) = 4m+5-1$. Since $\gamma_{lae}(K_4) = 5$, then $\gamma_{lae}({}_mSpl(K_4)) = 4m+4$. Let c_1 till c_2 be the color of inner edge. Then $\frac{n(c_1+c_2)}{2} = \frac{n(2k+4)}{2}$ and hence $c_1 + c_2 = 2k + 4$.

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However, if xi is the vertex with $f(xi) = n$, then the colors received by inner edges are at least $k+2$ and one of them is at least $n+1$. Thus $c_1+c_2 = (n+1)+(n+2) = k+n+3$. It's contradiction. Hence $\gamma_{lae}({}_mSpl(K_4)) \geq 4m+4$. Since $\gamma_{lae}({}_mSpl(K_n)) \leq 4m+5$ and $\gamma_{lae}({}_mSpl(K_n)) \geq 4m+5$, it completes the proof. □

Theorem 2.9 *The local edge antimagic of ${}_2Spl(K_n)$ graph is $\gamma_{lae}({}_2Spl(K_n)) = 2n + 5$.*

Proof. The ${}_2Spl(K_n)$ graph has vertex set $V(G) = \{x_i, x_i^j : 1 \leq i \leq n, 1 \leq j \leq 2\}$ and edge set $E(G) = \{x_ix_{i+k} : 1 \leq i \leq n, 1 \leq k \leq n-i\} \cup \{x_ix_{i+k}^j : 1 \leq i \leq n, 1 \leq j \leq 2, 1 \leq k \leq n-i\} \cup \{x_i^jx_{i+k} : 1 \leq i \leq n, 1 \leq j \leq 2, 1 \leq k \leq n-i\}$. The cardinality $|V(G)| = 2n$ and $|E(G)| = 6n$. Let split graph has set of vertices and edges. The set of vertices consists of set of inner and outter vertices. Inner vertex is the vertex set in center graph and outter vertex is the vertex set in split of graph. This also applies to inner and outer edge definitions. Define a bijection $f : V(G) \to \{1, 2, 3, \ldots, |V(G)|\}$ by

$$f(x_i) = \{\ i \quad 1 \leq i \leq n.$$

$$f(x_i^j) = \begin{cases} \frac{2nj+i+1}{2} & 1 \leq j \leq 2,\ i \text{ odd;} \\ nj + i & 1 \leq j \leq 2,\ i \text{ even.} \end{cases}$$

It is easy to see f is a local antimagic coloring of ${}_2Spl(K_n)$ and the edge weights as follows.

$$w(e) = \begin{cases} i+k+1 & \text{for } e = x_ix_{i+k},\ 2 \leq i \leq n,\ 1 \leq k \leq n-1; \\ k+2 & \text{for } e = x_1x_{1+k},\ 1 \leq k \leq n-2; \\ n+1 & \text{for } e = x_ix_n. \end{cases}$$

For $1 \leq k \leq n-i$,

$$w(x_ix_{i+k}^j) = \begin{cases} (j+1)n+1 & 1 \leq j \leq 2,\ i \text{ odd;} \\ (j+1)n+2 & 1 \leq j \leq 2,\ i \text{ even.} \end{cases}$$

$$w(x_i^jx_{i+k}) = \begin{cases} (j+1)n+1 & 1 \leq j \leq 2,\ i \text{ odd;} \\ (j+1)n+2 & 1 \leq j \leq 2,\ i \text{ even.} \end{cases}$$

Hence, from the above the edge weights, it easy to see that f induces a proper edge colouring of ${}_2Spl(K_n)$ and it gives $\gamma_{lae}({}_2Spl(K_n)) \leq 2n+5$. Then it will be showed that $\gamma_{lae}({}_2Spl(K_n)) \geq 2n+5$.

Assume that $\gamma_{lae}({}_2Spl(K_n)) = 2n+5-1$. Since $\gamma_{lae}(K_n) = 2n-3$, then $\gamma_{lae}({}_2Spl(K_n)) = 2n+4$. Let c_1 till c_2 be the color of inner edge. Then $\frac{n(c_1+c_2)}{2} = \frac{n(2k+4)}{2}$ and hence $c_1+c_2 = 2k+4$. However, if xi is the vertex with $f(xi) = n$, then the colors received by inner edges are at least $k+2$ and one of them is at least $n+1$. Thus $c_1+c_2 = (n+1)+(n+2) = k+n+3$. It's contradiction. Hence $\gamma_{lae}({}_2Spl(K_n)) \geq 2n+4$. Since $\gamma_{lae}({}_2Spl(K_n)) \leq 2n+5$ and $\gamma_{lae}({}_2Spl(K_n)) \geq 2n+5$. □

3. Concluding Remarks

We have discussed about the local antimagic edge coloring of some related $m-$splitting graphs for several sets of value (m, n) in this paper. Three basic theorem is about complete graph, circle graph, and path graph which has any solutions for being a basic graph of operation $m-$splitting.

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Open Problem

Find local edge antimagic coloring of ${}_mSpl(H_n)$ graph for any n and m where H is any graph.

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