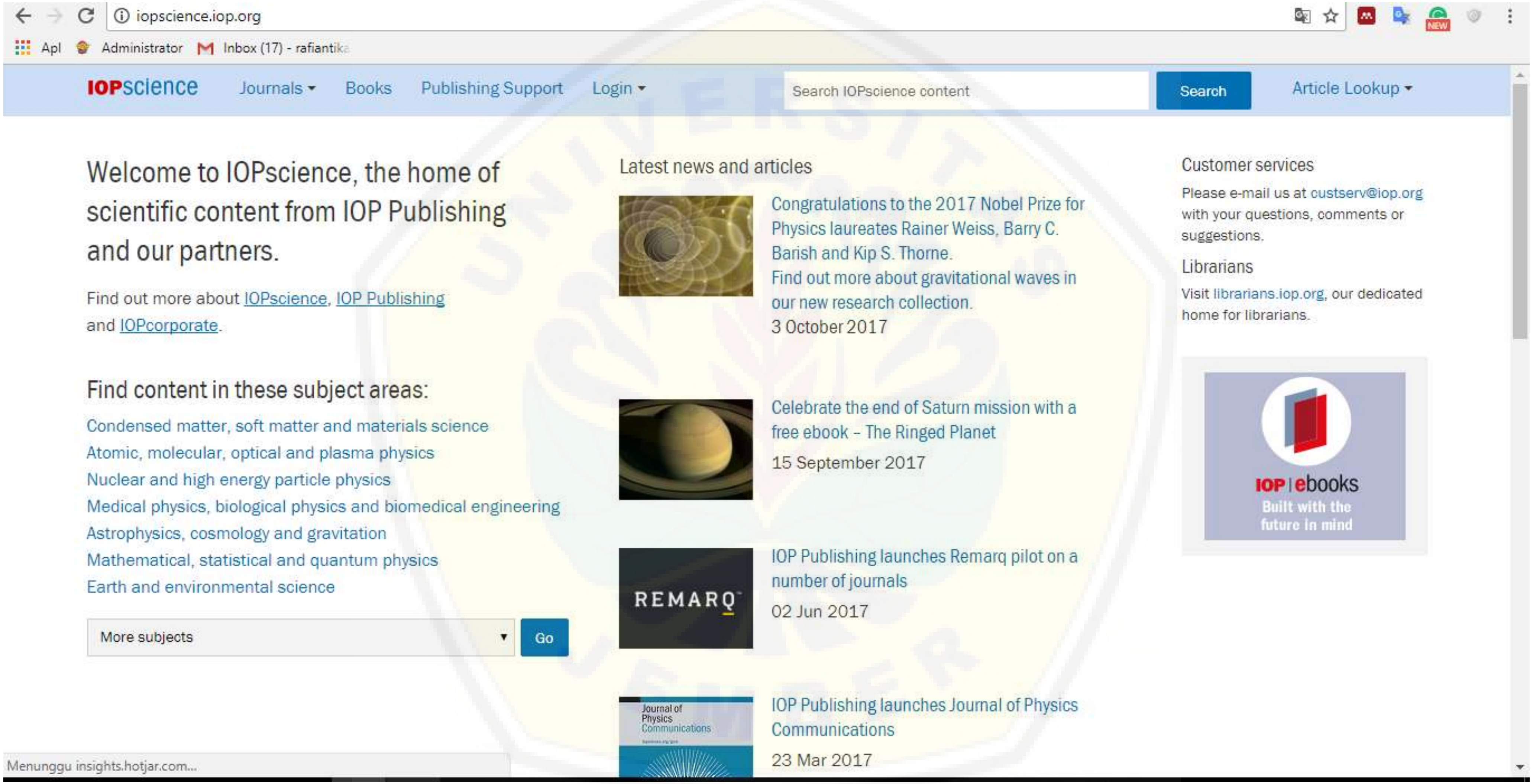
Digital Repository Universitas Jember



Digital Repository Universitas Jember

Journal of Physics: Conference Series

Country	United Kingdom
Subject Area and Category	Physics and Astronomy Physics and Astronomy (miscellaneous)
Publisher	Institute of Physics
Publication type	Journals
ISSN	17426588
Coverage	2005-ongoing
Scope	From 1 January 2010, IOP Publishing's open access proceedings titles no longer require authors to sign and submit copyright forms. For the following titles •Journal of Physics: Conference Series •IOP Conference Series: Materials Science and Engineering •IOP Conference Series: Earth and Environmental Science assignment of copyright forms are being replaced by a publishing licence under which authors retain their copyright. Please note that our regular journals are

52

H Index

Digital Repository Universitas Jember

Digital Repository Universitas Jember

Local Edge Antimagic

ORIGINALITY REPORT

39%	12%	39%	19%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	Ika Hesti Agustin, Moh. Hasan, Dafik, Ridho Alfarisi, A.I. Kristiana, R. M. Prihandini. "Local Edge Antimagic Coloring of Comb Product of Graphs", Journal of Physics: Conference Series, 2018 Publication	35%
2	Submitted to Universitas Jember Student Paper	2%
3	E R Albirri, Dafik, Slamin, I H Agustin, R Alfarisi. "On the local edge antimagicness of m-splitting graphs", Journal of Physics: Conference Series, 2018 Publication	1%
4	Submitted to iGroup Student Paper	<1%
5	hal.archives-ouvertes.fr Internet Source	<1%

Digital Repository Universitas Jember

Exclude quotes	On	Exclude matches	Off
Exclude bibliography	On		

Digital Repository Universitas Jember
Journal of Physics: Conference Series

PAPER • **OPEN ACCESS**

Local Edge Antimagic Coloring of Comb Product of Graphs

To cite this article: Ika Hesti Agustin *et al* 2018 *J. Phys.: Conf. Ser.* **1008** 012038

View the article online for updates and enhancements.

Related content

- Super local edge antimagic total coloring of ${P}_{n}\vartriangleright H$
Elsa Yuli Kurniawati, Ika Hesti Agustin, Dafik et al.

- On face antimagic labeling of double duplication of graphs
L Shobana and R Kuppan

- The Construction of ${P}_{2}\vartriangleright H$-antimagic graph using smaller edge-antimagic vertex labeling
Rafiantika M. Prihandini, I.H. Agustin and Dafik

This content was downloaded from IP address 103.241.205.66 on 23/05/2018 at 05:48

Digital Repository Universitas Jember

IOP Conf. Series: Journal of Physics: Conf. Series **1008** (2018) 012038 doi:10.1088/1742-6596/1008/1/012038

Local Edge Antimagic Coloring of Comb Product of Graphs

Ika Hesti Agustin[1,2], **Moh. Hasan**[1,2], **Dafik**[1,3], **Ridho Alfarisi**[1,4], **A.I. Kristiana**[1,3], **R. M. Prihandini**[1,4]

[1]CGANT-University of Jember, Jember, Indonesia
[3]Department of Mathematics Education, University of Jember, Jember, Indonesia
[2]Department of Mathematics, University of Jember, Jember, Indonesia
[4]Department of Elementary School Teacher Education, University of Jember, Jember, Indonesia

E-mail: `d.dafik@unej.ac.id`, `ikahesti.fmipa@unej.ac.id`, `alfarisi.fkip@unej.ac.id`

Abstract. All graph in this paper are finite, simple and connected graph. Let $G(V, E)$ be a graph of vertex set V and edge set E. A bijection $f : V(G) \longrightarrow \{1, 2, 3, ..., |V(G)|\}$ is called a local edge antimagic labeling if for any two adjacent edges e_1 and e_2, $w(e_1) \neq w(e_2)$, where for $e = uv \in G$, $w(e) = f(u) + f(v)$. Thus, any local edge antimagic labeling induces a proper edge coloring of G if each edge e is assigned the color $w(e)$. The local edge antimagic hromatic number $\gamma_{lea}(G)$ is the minimum number of colors taken over all colorings induced by local edge antimagic labelings of G. In this paper, we have found the lower bound of the local edge antimagic coloring of $G \triangleright H$ and determine exact value local edge antimagic coloring of $G \triangleright H$.

Keywords: Antimagic labeling, Local antimagic edge coloring, Local antimagic edge chromatic number, comb product.

1. Introduction

All graphs in this paper are finite, simple and connected graph, for detail definition of graph see [1, 2]. A bijection mapping that assigns natural number to vertices of a graph is called a graph labeling. In this type of labeling, we consider all weights associated with each edge. If all the edge weights have the different value then we call the labeling as an antimagic.

Hartsfield and Ringel [3] introduced the concept of antimagic labeling of a graph. A bijection $f : V(G) \longrightarrow \{1, 2, 3, ..., |V(G)|\}$ is called a local edge antimagic labeling if for any two adjacent edges e_1 and e_2, $w(e_1) \neq w(e_2)$, where for $e = uv \in G$, $w(e) = f(u) + f(v)$. Thus, any local edge antimagic labeling induces a proper edge coloring of G if each edge e is assigned the color $w(e)$. The local edge antimagic chromatic

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
Published under licence by IOP Publishing Ltd

Digital Repository Universitas Jember

number $\gamma_{lea}(G)$ is the minimum number of colors taken over all colorings induced by local edge antimagic labelings of G.

There are some results related to local antimagic coloring. Arumugam *et al.* [4] firstly introduced a new notion of local antimagic vertex coloring of a graph G. They give an lower bound and upper bound of local antimagic vertex coloring of joint graph and also give the exact value of local antimagic vertex coloring for path, cycle, complete graph, friendship, wheel, bipartite and complete bipartite. Furthermore, we get the relation between local edge antimagic edge chromatic number and local antimagic vertex chromatic number studied in [4] . In their paper also showed the following observation.

Observation 1.1. [4] *For any graph G, $\chi_{lea}(G) \geq \chi(G)$, where $\chi(G)$ is a chromatic number of vertex coloring of G.*

Dafik, *et. al.* [5, 6] determined Super edge-antimagic total labelings of $mK_{n,n}$ and Super edge-antimagicness for a class of disconnected graphs, respectively. The research related to the coloring of a graph who has researched by ika, *et.al* [8] about On r-dynamic coloring of some graph operations. Marr *et, al* [11] stated the magic rectangles are a generalization of magic squares. Let local antimagic labeling with $\gamma_{lae}(G) = 2$ be the same as bimagic labeling. For more details on bimagic labelings we refer to Marr *et. al.* [10].

Agustin *et. al.* [9] studied a different type of local antimagic coloring, namely local edge antimagic coloring. Their research studied the existence of local edge antimagic coloring of some special graphs and also analyse the lower bound of its local edge antimagic chromatic number. On this research, they have found the local edge antimagic chromatic number of path graph P_n, cycle graph C_n, friendship graph $\mathcal{F}_n$, ladder graph L_n, star graph S_n, wheel graph W_n, complete graph K_n, prism graph Pr_n and the graphs $C_n \odot mK_1$ and $G \odot mK_1$. They also give a lower bound of local edge antimagic chromatic number its $\gamma_{lea} \geq \Delta(G)$. The chromatic number of local edge antimagic for path graph P_n, cycle graph C_n, friendship graph $\mathcal{F}_n$, ladder graph L_n, star graph S_n, wheel graph W_n, complete graph K_n, prism graph Pr_n by the following: $\gamma_{lea}(P_n) = 2$, $\gamma_{lea}(C_n) = 3$, $\gamma_{lea}(\mathcal{F}_n) = 2n + 1$, $\gamma_{lea}(L_n) = 3$, $\gamma_{lea}(S_n) = n$, $\gamma_{lea}(W_n) = n + 2$, $\gamma_{lea}(K_n) = 2n - 3$, and $\gamma_{lea}(Pr_n) = 5$.

In this paper, we investigate the local edge antimagic coloring of comb product graphs can be found in [7], [12]. Let G and H be two connected graphs. Let o be a vertex of H. The comb product between G and H, denoted by $G \rhd H$, is a graph obtained by taking one copy of G and $|V(G)|$ copies of H and grafting the i-th copy of H at the vertex o to the i-th vertex of G. By the definition of comb product, we can say that $V(G \rhd H) = \{(a, u)|a \in V(G), u \in V(H)\}$ and $(a, u)(b, v) \in E(G \rhd H)$ whenever $a = b$ and $uv \in E(H)$, or $ab \in E(G)$ and $u = v = o$.

2. Main Results

In this paper, we have studied the existence of local edge antimagic coloring of comb product of graphs. We have found the local edge antimagic chromatic number of path comb path, path comb cycle, cycle and path, cycle and cycle, path and star, cycle and star. We also analyse the lower bound of its local edge antimagic coloring of $G \rhd H$.

Digital Repository Universitas Jember

IOP Conf. Series: Journal of Physics: Conf. Series **1008** (2018) 012038 doi:10.1088/1742-6596/1008/1/012038

Theorem 2.1. *Let G and H be a connected graph, the local edge antimagic chromatic number of $G \triangleright H$ is $\gamma_{lae}(G \triangleright H) \geq \gamma_{lea}(G) + \gamma_{lea}(H)$.*

Proof. The comb product between G and H is a graph obtained by one copy of G and $\mid V(G) \mid$ copies of H, and grafting the i vertex of G to the u_i in i-th copy of H. Suppose G admits a local edge antimagic coloring with $\gamma_{lae}(G)$. Furthermore, the subgraph H_i admits a local edge antimagic coloring with $\gamma_{lae}(H_i)$. By claiming that edge weight in G and H_i are distict and edge weight in every subgraph H_i induces a local edge antimagic of H so that $\gamma_{lea}(H_1) = \gamma_{lea}(H_2) = \cdots = \gamma_{lea}(H_n) = \gamma_{lea}(H)$. Thus, we obtain that a local antimagic labeling of $G \triangleright H$.

$$\begin{aligned}
\gamma_{lea}(G \triangleright H) &\geq |w_G(e), e \in V(G)| + |w_{H_i}(e), e \in V(H_i)| \\
&= \gamma_{lea}(G) + \gamma_{lea}(H_i) \\
&= \gamma_{lea}(G) + \gamma_{lea}(H)
\end{aligned}$$

Hence, from the above edge weight it is easy to see that the lower bound local antimagic labeling of $G \triangleright H$ is $\gamma_{lea}(G \triangleright H) \geq \gamma_{lea}(G) + \gamma_{lea}(H)$. □

Theorem 2.2. *For $n, m \geq 3$, the local edge antimagic chromatic number of $P_n \triangleright P_m$ with grafting pendant vertex $x \in V(P_m)$ is $\gamma_{lea}(P_n \triangleright P_m) = 4$*

Proof. The graph $P_n \triangleright P_m$ is a connected graph with vertex set $V(P_n \triangleright P_m) = \{x_{i,j} : 1 \leq i \leq n, 1 \leq j \leq m\}$ and edge set $E(P_n \triangleright P_m) = \{x_{i,1}x_{i+1,1} : 1 \leq i \leq n-1\} \cup \{x_{i,j}x_{i,j+1}; 1 \leq i \leq n, 1 \leq j \leq m-1\}$. Hence $|V(P_n \triangleright P_m)| = mn$ and $|E(P_n \triangleright P_m)| = mn - 1$. Define a bijection $f : V(P_n \triangleright P_m) \longrightarrow \{1, 2, 3, ..., |V(P_n \triangleright P_m)|\}$ by the following

$$f(x_{i,j}) = \begin{cases} \frac{1-n+i+nj}{2}, & \text{if } i \equiv 1(mod\ 2) \text{ and } j \equiv 1(mod\ 2) \\ \frac{n+2+nj-i}{2}, & \text{if } i \equiv 0(mod\ 2) \text{ and } j \equiv 1(mod\ 2) \\ n(m+1) + \frac{1-i-nj}{2}, & \text{if } i \equiv 1(mod\ 2) \text{ and } j \equiv 0(mod\ 2) \\ mn + \frac{i-nj}{2}, & \text{if } i \equiv 0(mod\ 2) \text{ and } j \equiv 0(mod\ 2) \end{cases}$$

And the edge weight are as follows

$$w(x_{i,1}x_{i+1,1}) = \begin{cases} n+2 & i \equiv 0(mod\ 2); \\ n+1 & i \equiv 1(mod\ 2); \end{cases}$$

$$w(x_{i,j}x_{i,j+1}) = \begin{cases} nm+n+1 & j \equiv 0(mod\ 2); \\ nm+1 & j \equiv 1(mod\ 2); \end{cases}$$

Hence, from the above edge weights, it easy to see that f induces a proper edge colouring of $P_n \triangleright P_m$ and it gives $\gamma_{lea}(P_n \triangleright P_m) \leq 4$. Based on Theorem 2.1, the lower bound is $\gamma_{lea}(P_n \triangleright P_m) \geq \gamma_{lea}(P_n) + \gamma_{lea}(P_m) = 4$. It concludes that $\gamma_{lea}(P_n \triangleright P_m) = 4$. □

Digital Repository Universitas Jember

IOP Conf. Series: Journal of Physics: Conf. Series **1008** (2018) 012038 doi:10.1088/1742-6596/1008/1/012038

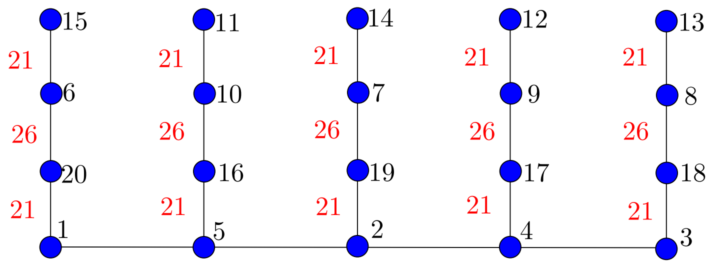


Figure 1. Example of Local antimagic edge coloring of $P_5 \triangleright P_4$

Theorem 2.3. *For $n \geq 3$ and m be even positive integers, the local edge antimagic chromatic number of $P_n \triangleright C_m$ is $\gamma_{lea}(P_n \triangleright C_m) = 5$*

Proof. The graph $P_n \triangleright C_m$ is a connected graph with vertex set $V(P_n \triangleright C_m) = \{x_{i,j} : 1 \leq i \leq n, 1 \leq j \leq m\}$ and edge set $E(P_n \triangleright C_m) = \{x_{i,1}x_{i+1,1} : 1 \leq i \leq n-1\} \cup \{x_{i,j}x_{i,j+1}, x_{i,m}x_{i,1}; 1 \leq i \leq n, 1 \leq j \leq m-1\}$. Hence $|V(P_n \triangleright C_m)| = mn$ and $|E(P_n \triangleright C_m)| = mn + n - 1$. For $n \geq 3$. Define a bijection $f : V(P_n \triangleright C_m) \longrightarrow \{1, 2, 3, ..., |V(P_n \triangleright C_m)|\}$ by the following

$$f(x_{i,j}) = \begin{cases} \frac{1-n+i+nj}{2}, & \text{if } i \equiv 1(mod\ 2) \text{ and } j \equiv 1(mod\ 2) \\ \frac{n+2+nj-i}{2}, & \text{if } i \equiv 0(mod\ 2) \text{ and } j \equiv 1(mod\ 2) \\ n(m+1) + \frac{1-i-nj}{2}, & \text{if } i \equiv 1(mod\ 2) \text{ and } j \equiv 0(mod\ 2) \\ mn + \frac{i-nj}{2}, & \text{if } i \equiv 0(mod\ 2) \text{ and } j \equiv 0(mod\ 2) \end{cases}$$

And the edge weight are as follows

$$w(x_{i,1}x_{i+1,1}) = \begin{cases} n+2 & i \equiv 0(mod\ 2); \\ n+1 & i \equiv 1(mod\ 2); \end{cases}$$

$$w(x_{i,j}x_{i,j+1}) = \begin{cases} nm+n+1 & j \equiv 0(mod\ 2); \\ nm+1 & j \equiv 1(mod\ 2); \end{cases}$$

$$w(x_{i,1}x_{i,m}) = \frac{mn}{2} + n + 1$$

Hence, from the above edge weights, it easy to see that f induces a proper edge colouring of $P_n \triangleright C_m$ and it gives $\gamma_{lea}(P_n \triangleright C_m) \leq 5$. Based on Theorem 2.1, the lower bound is $\gamma_{lea}(P_n \triangleright C_m) \geq \gamma_{lea}(P_n) + \gamma_{lea}(C_m) = 5$. It concludes that $\gamma_{lea}(P_n \triangleright C_m) = 5$.
□

Digital Repository Universitas Jember

IOP Conf. Series: Journal of Physics: Conf. Series **1008** (2018) 012038 doi:10.1088/1742-6596/1008/1/012038

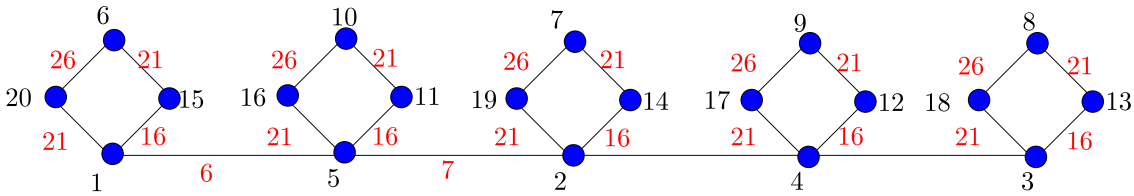


Figure 2. Example of Local antimagic edge coloring of $P_5 \triangleright C_4$

Theorem 2.4. *For $n, m \geq 3$, the local edge antimagic chromatic number of $C_n \triangleright P_m$ with grafting pendant vertex $x \in V(P_m)$ is $\gamma_{lea}(C_n \triangleright P_m) = 5$*

Proof. The graph $C_n \triangleright P_m$ is a connected graph with vertex set $V(C_n \triangleright P_m) = \{x_{i,j} : 1 \leq i \leq n, 1 \leq j \leq m\}$ and edge set $E(C_n \triangleright P_m) = \{x_{i,1}x_{i+1,1} : 1 \leq i \leq n-1\} \cup \{x_{i,j}x_{i,j+1}; 1 \leq i \leq n, 1 \leq j \leq m-1\} \cup \{x_{i,m}x_{i,1}\}$. Hence $|V(C_n \triangleright P_m)| = mn$ and $|E(C_n \triangleright P_m)| = mn$. For $n, m \geq 3$. Define a bijection $f : V(C_n \triangleright P_m) \longrightarrow \{1, 2, 3, ..., |V(C_n \triangleright P_m)|\}$ by the following

$$f(x_{i,j}) = \begin{cases} \frac{1-n+i+nj}{2}, & \text{if } i \equiv 1(mod\ 2) \text{ and } j \equiv 1(mod\ 2) \\ \frac{n+2+nj-i}{2}, & \text{if } i \equiv 0(mod\ 2) \text{ and } j \equiv 1(mod\ 2) \\ n(m+1) + \frac{1-i-nj}{2}, & \text{if } i \equiv 1(mod\ 2) \text{ and } j \equiv 0(mod\ 2) \\ mn + \frac{i-nj}{2}, & \text{if } i \equiv 0(mod\ 2) \text{ and } j \equiv 0(mod\ 2) \end{cases}$$

And the edge weight are as follows

$$w(x_{i,1}x_{i+1,1}) = \begin{cases} n+2 & i \equiv 0(mod\ 2); \\ n+1 & i \equiv 1(mod\ 2); \end{cases}$$

$$w(x_{1,1}x_{n,1}) = \begin{cases} \frac{n+4}{2} & \text{if } n \text{ is even;} \\ \frac{n+3}{2} & \text{if } n \text{ is odd;} \end{cases}$$

$$w(x_{i,j}x_{i,j+1}) = \begin{cases} nm+n+1 & j \equiv 0(mod\ 2); \\ nm+1 & j \equiv 1(mod\ 2); \end{cases}$$

Hence, from the above edge weights, it easy to see that f induces a proper edge colouring of $C_n \triangleright P_m$ and it gives $\gamma_{lea}(C_n \triangleright P_m) \leq 5$. Based on Theorem 2.1, the lower bound is $\gamma_{lea}(C_n \triangleright P_m) \geq \gamma_{lea}(C_n) + \gamma_{lea}(P_m) = 5$. It concludes that $\gamma_{lea}(C_n \triangleright P_m) = 5$. $\square$

Digital Repository Universitas Jember

IOP Conf. Series: Journal of Physics: Conf. Series **1008** (2018) 012038 doi:10.1088/1742-6596/1008/1/012038

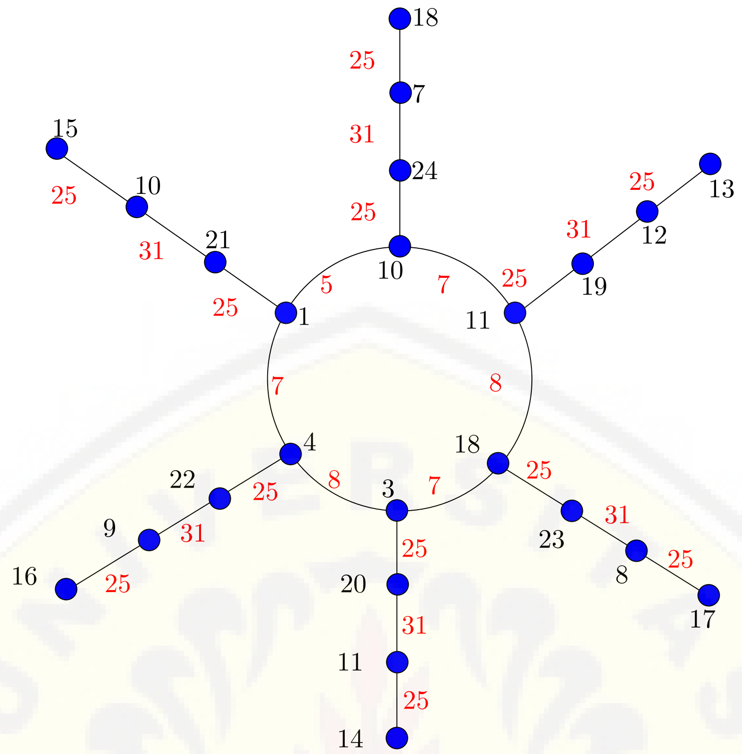


Figure 3. Example of Local antimagic edge coloring of $C_6 \triangleright P_4$

Theorem 2.5. *For $n \geq 3$ and m be even positive integers, the local edge antimagic chromatic number of $C_n \triangleright C_m$ is $\gamma_{lea}(C_n \triangleright C_m) = 6$*

Proof. The graph $C_n \triangleright C_m$ is a connected graph with vertex set $V(C_n \triangleright C_m) = \{x_{i,j} : 1 \leq i \leq n, 1 \leq j \leq m\}$ and edge set $E(C_n \triangleright C_m) = \{x_{i,1}x_{i+1,1} : 1 \leq i \leq n-1\} \cup \{x_{i,j}x_{i,j+1}; 1 \leq i \leq n, 1 \leq j \leq m-1\} \cup \{x_{1,1}x_{1,n}\} \cup \{x_{i,1}x_{i,m}; 1 \leq i \leq n\}$. Hence $|V(C_n \triangleright C_m)| = mn$ and $|E(C_n \triangleright C_m)| = mn + n$. For $n \geq 3$ and m be even positive integers. Define a bijection $f : V(C_n \triangleright C_m) \longrightarrow \{1, 2, 3, ..., |V(C_n \triangleright C_m)|\}$ by the following

$$f(x_{i,j}) = \begin{cases} \frac{1-n+i+nj}{2}, & \text{if } i \equiv 1(mod\ 2) \text{ and } j \equiv 1(mod\ 2) \\ \frac{n+2+nj-i}{2}, & \text{if } i \equiv 0(mod\ 2) \text{ and } j \equiv 1(mod\ 2) \\ n(m+1) + \frac{1-i-nj}{2}, & \text{if } i \equiv 1(mod\ 2) \text{ and } j \equiv 0(mod\ 2) \\ mn + \frac{i-nj}{2}, & \text{if } i \equiv 0(mod\ 2) \text{ and } j \equiv 0(mod\ 2) \end{cases}$$

And the edge weight are as follows

$$w(x_{i,1}x_{i+1,1}) = \begin{cases} n+2 & i \equiv 0(mod\ 2); \\ n+1 & i \equiv 1(mod\ 2); \end{cases}$$

Digital Repository Universitas Jember

IOP Conf. Series: Journal of Physics: Conf. Series **1008** (2018) 012038 doi:10.1088/1742-6596/1008/1/012038

$$w(x_{1,1}x_{n,1}) = \begin{cases} \frac{n+4}{2} & \text{if } n \text{ is even;} \\ \\ \frac{n+3}{2} & \text{if } n \text{ is odd;} \end{cases}$$

$$w(x_{i,j}x_{i,j+1}) = \begin{cases} nm+n+1 & j \equiv 0(mod\ 2); \\ nm+1 & j \equiv 1(mod\ 2); \end{cases}$$

$$w(x_{i,1}x_{i,m}) = \frac{mn}{2} + n + 1$$

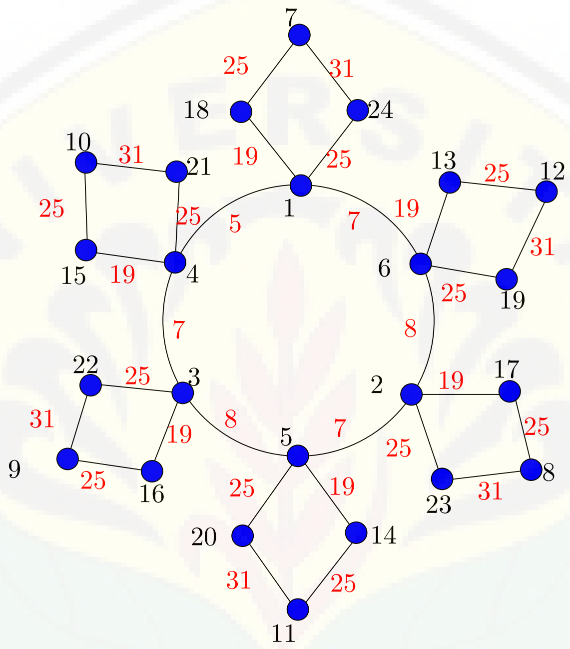


Figure 4. Example of Local antimagic edge coloring of $C_6 \triangleright C_4$

Hence, from the above edge weights, it easy to see that f induces a proper edge colouring of $C_n \triangleright C_m$ and it gives $\gamma_{lea}(C_n \triangleright C_m) \leq 6$. Based on Theorem 2.1, the lower bound is $\gamma_{lea}(C_n \triangleright C_m) \geq \gamma_{lea}(C_n) + \gamma_{lea}(C_m) = 6$. It concludes that $\gamma_{lea}(C_n \triangleright C_m) = 6$. $\square$

Theorem 2.6. *For $n, m \geq 3$, the local edge antimagic chromatic number of $P_n \triangleright S_m$ with grafting central vertex $x \in V(S_m)$ is $\gamma_{lea}(P_n \triangleright S_m) = 2 + m$*

Proof. The graph $P_n \triangleright S_m$ is a connected graph with vertex set $V(P_n \triangleright S_m) = \{x_i : 1 \leq i \leq n\} \cup \{x_{i,j} : 1 \leq i \leq n, 1 \leq j \leq m\}$ and edge set $E(P_n \triangleright S_m) = \{x_{i,1}x_{i+1,1} : 1 \leq i \leq n-1\} \cup \{x_i x_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m\}$. Hence $|V(P_n \triangleright S_m)| = n + mn$ and $|E(P_n \triangleright S_m)| = n + mn - 1$. For $n, m \geq 3$. Define a bijection $f : V(P_n \triangleright S_m) \longrightarrow \{1, 2, 3, ..., |V(P_n \triangleright S_m)|\}$ by the following

Digital Repository Universitas Jember

IOP Conf. Series: Journal of Physics: Conf. Series **1008** (2018) 012038 doi:10.1088/1742-6596/1008/1/012038

$$f(x_i) = \begin{cases} n+1-\frac{i}{2}, & \text{if } i \equiv 0(mod\ 2) \\ \frac{i+1}{2}, & \text{if } i \equiv 1(mod\ 2) \end{cases}$$

$$f(x_{i,j}) = \begin{cases} \frac{2nj+i}{2}, & \text{if } i \equiv 0(mod\ 2) \\ \frac{2nj+2n-i+3}{2}, & \text{if } i \equiv 1(mod\ 2) \end{cases}$$

And the edge weight are as follows

$$w(x_i x_{i+1}) = \begin{cases} n+2 & i \equiv 0(mod\ 2); \\ n+1 & i \equiv 1(mod\ 2); \end{cases}$$

$$w(x_i x_{i,j}) = nj + 1$$

Hence, from the above edge weights, it easy to see that f induces a proper edge colouring of $P_n \triangleright S_m$ and it gives $\gamma_{lea}(P_n \triangleright S_m) \leq 2+m$. Based on Theorem 2.1, the lower bound is $\gamma_{lea}(P_n \triangleright S_m) \geq \gamma_{lea}(P_n) + \gamma_{lea}(S_m) = 2+m$. It concludes that $\gamma_{lea}(P_n \triangleright S_m) = 2+m$. □

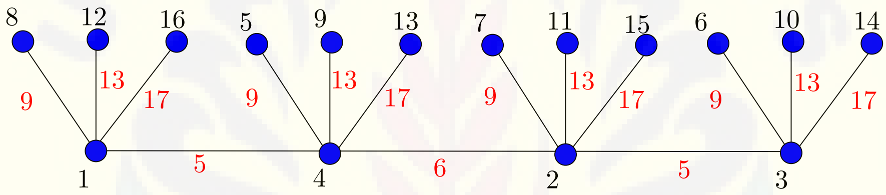


Figure 5. Example of Local antimagic edge coloring of $P_4 \triangleright S_3$

Theorem 2.7. *For $n, m \geq 3$, the local edge antimagic chromatic number of $C_n \triangleright S_m$ with grafting central vertex $x \in V(S_m)$ is $\gamma_{lea}(C_n \triangleright S_m) = 3+m$*

Proof. The graph $C_n \triangleright S_m$ is a connected graph with vertex set $V(C_n \triangleright S_m) = \{x_i : 1 \leq i \leq n\} \cup \{x_{i,j} : 1 \leq i \leq n, 1 \leq j \leq m\}$ and edge set $E(C_n \triangleright S_m) = \{x_{i,1}x_{i+1,1} : 1 \leq i \leq n-1\} \cup \{x_1 x_n\} \cup \{x_i x_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m\}$. Hence $|V(C_n \triangleright S_m)| = n + mn$ and $|E(C_n \triangleright S_m)| = n + mn$. For $n, m \geq 3$. Define a bijection $f : V(C_n \triangleright S_m) \longrightarrow \{1, 2, 3, ..., |V(C_n \triangleright S_m)|\}$ by the following

$$f(x_i) = \begin{cases} n+1-\frac{i}{2}, & \text{if } i \equiv 0(mod\ 2) \\ \frac{i+1}{2}, & \text{if } i \equiv 1(mod\ 2) \end{cases}$$

$$f(x_{i,j}) = \begin{cases} \frac{2nj+i}{2}, & \text{if } i \equiv 0(mod\ 2) \\ \frac{2nj+2n-i+3}{2}, & \text{if } i \equiv 1(mod\ 2) \end{cases}$$

And the edge weight are as follows

Digital Repository Universitas Jember

IOP Conf. Series: Journal of Physics: Conf. Series **1008** (2018) 012038 doi:10.1088/1742-6596/1008/1/012038

$$w(x_i x_{i+1}) = \begin{cases} n+2 & i \equiv 0(mod\ 2); \\ n+1 & i \equiv 1(mod\ 2); \end{cases}$$

$$w(x_{1,1}x_{n,1}) = \begin{cases} \frac{n+4}{2} & \text{if } n \text{ is even;} \\ \frac{n+3}{2} & \text{if } n \text{ is odd;} \end{cases}$$

$$w(x_i x_{i,j}) = nj + 1$$

Hence, from the above edge weights, it easy to see that f induces a proper edge colouring of $C_n \triangleright S_m$ and it gives $\gamma_{lea}(C_n \triangleright S_m) \leq 3 + m$. Based on Theorem 2.1, the lower bound is $\gamma_{lea}(C_n \triangleright S_m) \geq \gamma_{lea}(C_n) + \gamma_{lea}(S_m) = 3 + m$. It concludes that $\gamma_{lea}(C_n \triangleright S_m) = 3 + m$. □

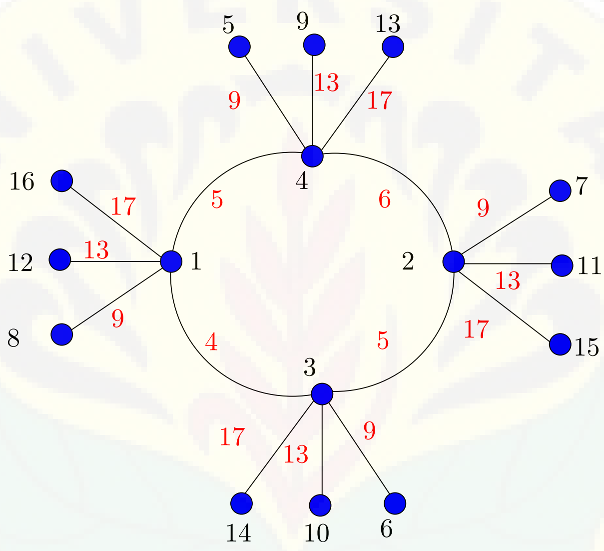


Figure 6. Example of Local antimagic edge coloring of $C_4 \triangleright S_3$

3. Conclusion

In this paper we have given an asymptotically tight result on local edge antimagic coloring of comb product of special graphs, namely path, cycle, and star. We also determine the lower bound of local edge antimagic coloring of comb product of any two graphs. Hence the following problem aries naturally.

Open Problem 3.1. *Determine exact value local edge antimagic coloring of comb product for another family graphs?*

Open Problem 3.2. *Determine exact value local edge antimagic total coloring of comb product $P_n \triangleright C_m$, if m is odd integer?*

Digital Repository Universitas Jember

Acknowledgement
We gratefully acknowledge the support from CGANT - University of Jember of year 2017.

References

[1] Gross J L, Yellen J and Zhang P 2014 *Handbook of graph Theory* Second Edition CRC Press Taylor and Francis Group
[2] Chartrand G and Lesniak L 2000 *Graphs and digraphs 3rd* ed (London: Chapman and Hall)
[3] Hartsfield N dan Ringel G 1994 *Pearls in Graph Theory* Academic Press. United Kingdom
[4] Arumugam S, Premalatha K, Baca M and Semanicova-Fenovcikova A 2017 Local Antimagic Vertex Coloring of a Graph, *Graphs and Combinatorics* **33** 275-285
[5] Dafik, Miller M, Ryan J, and Baca M 2011 Super edge-antimagic total labelings of $mK_{n,n}$ *Ars Combinatoria* 101 97-107.
[6] Dafik, Mirka M, Ryan J, and Baca M 2006 Super edge-antimagicness for a class of disconnected graphs.
[7] Agustin I H, Dafik, Prihandini R M 2018 $P_2 \rhd H$-super antimagic total labeling of comb product of graph *AKCE International Journal of Graphs and Combinatorics In Press*
[8] Agustin I H, Dafik and Harsya A Y 2016 On r-dynamic coloring of some graph operations *Indonesian Journal of Combinatorics* 1.1 22-30.
[9] Agustin I H, Dafik, Moh. Hasan, Alfarisi R, Prihandini R M 2017 Local Edge Antimagic Coloring of Graphs *Far East Journal of Mathematical Sciences.*
[10] Marr A, Phillips N C K, Wallis W D 2009 Bimagic labelings *AKCE Int. J. Graphs. Combin.* 6(1) 155-160.
[11] Marr A, Wallis W D 2013 *Magic Graphs 2nd* edn. Birkhauser, Switzerland
[12] Saputro S W, Mardiana N and Purwasi I A 2013 The metric dimension of comb product graph *In Graph Theory Conference in Honor of Egawas 60th Birthday* September **10**