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On locating independent domination number of amalgamation graphs

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Abstract. An independent set or stable set is a set of vertices in a graph in which no two of vertices are adjacent. A set D of vertices of graph G is called a dominating set if every vertex $u \in V(G) - D$ is adjacent to some vertex $v \in D$. A set S of vertices in a graph G is an independent dominating set of G if D is an independent set and every vertex not in D is adjacent to a vertex in D. By locating independent dominating set of graph G, we mean that an independent dominating set D of G with the additional properties that for $u, v \in (V - D)$ satisfies $N(u) \cap D \neq N(v) \cap D$. A minimum locating independent dominating set is a locating independent dominating set of smallest possible size for a given graph G. This size is called the locating independent dominating number of G and denoted $\gamma_{Li}(G)$. In this paper, we analyze the locating independent domination number of graph operations.

1. Introduction

Let G be a nontrivial, finite, simple, undirected and connected graphs, with vertex set $V(G)$, edge set $E(G)$ and with no isolated vertex, for more detail definition of graph see [1, 2]

A set D of vertices of a graph $G = (V, E)$ is dominating if every vertex in $V(G) - D$ is adjacent to some vertex in D. The domination number of G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set in G. A locating-dominating set is a dominating set D that locates all the vertices in the sense that every vertex not in D is uniquely determined by its neighborhood in D. The locating domination number of G, denoted by $\gamma_L(G)$, is the minimum cardinality of a locating dominating set in G. A locating-dominating set of order $\gamma_L(G)$ is called an $\gamma_L(G)$-set The concept of a locating dominating set was introduced and first studied by Slater [3, 4, 5, 6] and also Waspodo *et. al.* [8] studied the bound of distance domination number of edge comb product.

For definition and notation of locating dominating set in [7] explained that the open neighborhood of a vertex $v \in V(G)$ is $N_G(v) = \{u \in V(G); uv \in E(G)\}$ and its closed neighborhood is the set $N_G[v] = N_G(v) \cup \{v\}$. For a set D of vertex set of G, $N_G[D]$ is the union of all closed neighborhoods of vertices in D. The degree of v is $d_G(v) = |N_G(v)|$. If the graph G is a connected graph, we simply write $V(G), E(G), N(v), N[v], N[D]$ and $d(v)$ rather than $V(G), E(G), N_G(v), N_G[v], N_G[D]$ and $d_G(v)$, respectively.

A dominating set of G and denoted by D, if $N[v] \cap D \neq \emptyset$ for all vertex $v \in G$, or equivalently, $N[D] = V(G)$. Any two vertices u and $v \in V(G)\ D$ are located by D if they have distinct neighbors in D that is, $N(u) \cap D \neq N(v) \cap D$. If a vertex $u \in V(G)\ D$ is located from every other vertex in $V(G)\ D$, we simply say that u is located by D. A set D is a locating set of G if any two distinct vertices outside D are located by D. In particular, if S is both a dominating set and a locating set, then S is a locating dominating set. We remark that the only difference between a locating set and a locating dominating set in G is that a locating set might have a unique non-dominated vertex.

A set D of vertices in a graph G is an independent dominating set of G if D is an independent set and every vertex not in D is adjacent to a vertex in D. The definition and notation of locating independent dominating set of graph G similarly the definition and notation of locating dominating set, we mean that a locating independent dominating set D of G with the additional properties that D is an independent set and every vertex not in D is adjacent to a vertex in D. The locating independent dominating number of a graph G, denoted by $\gamma_{LI}(G)$, is the minimum cardinality of a locating independent dominating set of graph G. A locating independent dominating set of order $\gamma_{LI}(G)$ is called an $\gamma_{Li}(G)$-set.

2. Main Results

The definition of amalgamation of graph is taken from [9]. Let G_i be a simple connected graph, for $i \in \{1, 2, \ldots, t\}$ and $t \in \mathcal{N}$ and $|V(G_i)| = k_i \geq 2$ for some $k_i \in \mathcal{N}$. For $t \geq 2$ let $\{G_1, G_2, \ldots, G_t\}$ be a finite collection of graphs and each G_i, $i \in \{1, 2, \ldots, t\}$, has a fixed vertex v_{oi} called a *terminal*. The amalgamation denote by $Amal(G_i, v_{oi})$.

In this section, we determine the exact values of locating independent dominating number of some special graphs and its operations namely star graph S_n, $Amal(S_n, v, m)$, path graph P_n, $Amal(P_n, v, m)$, wheel graph W_n, $Amal(W_n, v, m)$, ladder graph L_n, $Amal(L_n, v, m)$.

Lemma 2.1. *For any graph G of order n, the lower bound of locating independent domination number of amalgamation graph $Amal(G, v, m)$ is $\gamma_{Li}\ (Amal(G, v, m)) \geq m(\gamma_{Li}(G) - 1) + 1$.*

Proof. The graph $Amal(G, v, m)$ is a connected graph of order $|V(Amal(G, v, m))| = (p(G) - 1)m + 1$ and size $|E(Amal(G, v, m))| = (q(G))m$.

To prove the lemma above, we claim that $\gamma_{Li}\ (Amal(G, v, m)) \geq m(\gamma_{Li}(G) - 1) + 1$. To convince this, assume that $\gamma_{Li}\ (Amal(G, v, m)) < m(\gamma_{Li}(G) - 1) + 1$. The intersection between the neighborhood $N(v)$ with $v \in V(G) - D$ will be empty set. Thus, it is a contradiction. See Figure 1 for illustration. □

Theorem 2.2. *For $n \geq 3$, the locating independent domination number of S_n is $\gamma_{Li}(S_n) = n$.*

Proof. Star graph S_n is a connected graph with vertex set $V(S_n) = \{A\} \cup \{x_i; 1 \leq i \leq n\}$ and edge set $E(S_n) = \{Ax_i; 1 \leq i \leq n\}$. The order and size of S_n are $|V(S_n)| = n + 1$ and $|E(S_n)| = n$.

We claim that $\gamma_{Li}\ (S_n) \geq n$. To convince the proof, assume that $\gamma_{Li}(S_n) < n$. Let the dominator vertex set of S_n, for $n \geq 3$, be $D = \{x_i; 1 \leq i \leq n - 1\}$, thus $|D| = n - 1$, and let non-dominator vertex set of S_n, for $n \geq 3$, be $V - D = \{A\} \cup \{x_n\}$. Then we get the intersection of the neighborhood $N(v)$ with $v \in V(G) - D$ and dominator set D, in the following.

$$N(A) \cap D = \{x_i; 1 \leq i \leq n - 1\}$$
$$N(x_n) \cap D = \emptyset$$

It can be seen that the intersection between the neighborhood $N(v)$ with $v \in V(G) - D$, for $N(x_n) \cap D = \emptyset$. Thus, the dominator set D do not dominate all vertices in $V(S_n)$. It concludes that, by assuming $\gamma_{Li}(S_n) < n$, it will not comply the condition of locating independent dominating set. Therefore, the lower bound of locating independent domination number of

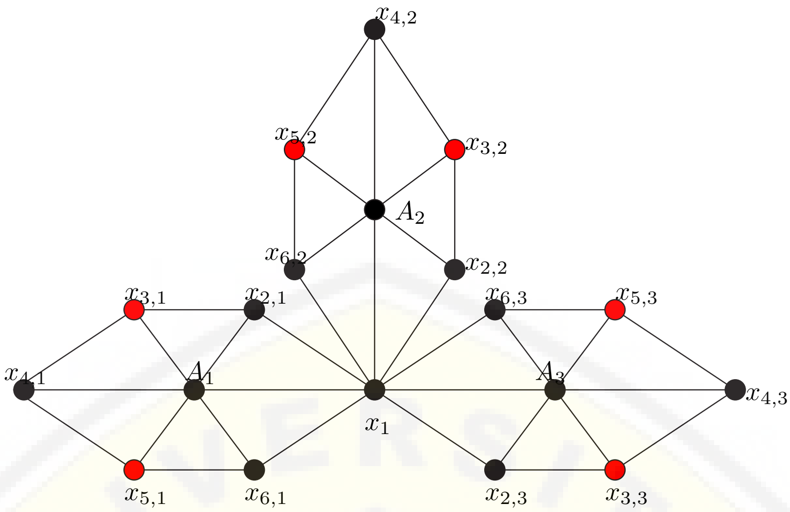

Figure 1. Amalgamation of Wheel Graph $Amal(W_n, v, m)$

S_n is $\gamma_{Li}(S_n) \geq n$. Furthermore, we will show that the upper bound of locating independent domination number of S_n is $\gamma_{Li}(S_n) \leq n$. Choose $D = \{x_i; 1 \leq i \leq n\}$ as the dominator set of S_n, for $n \geq 3$, thus $|D| = n$. Choose $V - D = \{A\}$ as the non-dominator set of S_n for $n \geq 3$. We will get the intersection between the neighborhood $N(v)$ with $v \in V(G) - D$ and dominator set D, in the following.

$$N(A) \cap D = \{x_i; 1 \leq i \leq n\}$$

It can be seen that the intersection between the neighborhood $N(v)$ with $v \in V(G) - D$ are all different, and it is not empty set. The dominator set D does not dominate all vertices in $V(S_n)$. It can be concluded that, for $\gamma_{Li}(S_n) \leq n$, it will comply the condition of locating independent dominating set. Thus $\gamma_{Li}(S_n) \leq n$. Hence, then the locating independent domination number of S_n is $\gamma_{Li}(S_n) = n$. □

Theorem 2.3. *Let G be an amalgamation graph of star S_n with $n \geq 3$ and $m \geq 3$. Then locating independent domination number of $Amal(S_n, v, m)$ is $\gamma_{Li}\ (Amal(S_n, v, m)) = m(n-1)+1$.*

Proof. The graph $Amal(S_n, v, m)$ is a connected graph with $V(Amal(S_n, v, m)) = \{x\} \cup \{A_i; 1 \leq i \leq m\} \cup \{x_{i,j}; 1 \leq i \leq m; 1 \leq j \leq n-1\}$ and $E(Amal(S_n, v, m)) = \{xA_i; 1 \leq i \leq m\} \cup \{A_i x_{i,j}; 1 \leq i \leq m; 1 \leq j \leq n-1\}$. The order of this graph is $|V(Amal(S_n, v, m))| = nm + 1$ and the size is $|E(Amal(S_n, v, m))| = nm$. To prove the above theorem $\gamma_{Li}(Amal(S_n, v, m)) = m(n-1)+1$, we will show that the lower bound $\gamma_{Li}(Amal(S_n, v, m)) \geq m(n-1)+1$ and the upper bound $\gamma_{Li}(Amal(S_n, v, m)) \leq m(n-1)+1$.

Firstly, we will show that $\gamma_{Li}\ (Amal(S_n, v, m)) \geq m(n-1)+1$. By Lemma 2.1, we have $\gamma_{Li}(Amal(S_n, v, m)) = m(\gamma_{Li}(S_n) - 1) + 1$. Since by Theorem 2.2 we have $\gamma_{Li}(S_n) = n$, thus we get so $\gamma_{Li}(Amal(S_n, v, m)) \geq m(n-1)+1$. Furthermore, we will show that the upper bound of locating independent domination number of $\gamma_{Li}(Amal(S_n, v, m)) \leq m(n-1)+1$. We consider $D(Amal(S_n, v, m)) = \{x\} \cup \{x_{i,j}; 1 \leq i \leq m; 1 \leq j \leq n-1\}$ as the dominator set of $Amal$ (S_n, v, m) for $n \geq 3$ and $m \geq 3$. It is clearly to see that $|D| = m(n-1)+1$, and the dominator set D dominates all vertices of $G = Amal(S_n, v, m)$. By definition, we have the non-dominator set of $Amal(S_n, v, m)$ for $n \geq 3$ and $m \geq 3$ is $V(Amal(S_n, v, m)) - D(Amal(S_n, v, m)) = \{A_i; 1 \leq i \leq m\}$. The intersection between the neighborhood $N(v)$ with $v \in V(G) - D(G)$ and dominator set $D(G)$ is as follows.

$N(A_i) \cap D = \{x\}, \{x_{i,j}; 1 \le i \le m; 1 \le j \le n-1\}$

It can be seen intersection between the neighborhood $N(v)$ with $v \in V(G) - D(G)$ and the obtained dominator set D are uniques and it is not empty set. Thus, it can be concluded that $\gamma_{Li}(Amal(S_n, v, m)) \le m(n-1)+1$. D also complies the condition of locating independent dominating set. Hence, the lower bound and upper bound of locating independent domination number respectively, are $\gamma_{Li} \ge m(n-1)+1$ and $\gamma_{Li} \le m(n-1)+1$. It conludes the locating independent domination number of $Amal(S_n, v, m)$ is $\gamma_{Li}(Amal(S_n, v, m)) = m(n-1)+1$. □

Theorem 2.4. *For $n \ge 4$, locating independent domination number of P_n is $\gamma_{Li}(P_n) = \lceil \frac{2n}{5} \rceil$.*

Proof. Path graph P_n is a connected graph with vertex set $V(P_n) = \{x_i; 1 \le i \le n\}$ and edge set $E(P_n) = \{x_i x_{i+1}; 1 \le i \le n-1\}$. The order and size of P_n are $|V(P_n)| = n$ and $|E(P_n)| = n-1$.

We claim that $\gamma_{Li}\ (P_n) \ge \lceil \frac{2n}{5} \rceil$. To convince the proof, assume that $\gamma_{Li}(P_n) < \lceil \frac{2n}{5} \rceil$. Let the dominator vertex set of P_n, for $n \ge 4$, $D = \{x_i; i \equiv 0 \bmod 2; i > 2\}$ those $|D| = \lceil \frac{2n}{5} \rceil$ and non-dominator vertex set of P_n for $n \ge 4$ is $V - D = \{x_2\} \cup \{x_i; i \equiv 1 \bmod 2\}$. Then we get the intersection of the neighborhood $N(v)$ with $v \in V(G) - D$ and dominator set D, in the following.

$N(x_i) \cap D = \{x_i; i \equiv 0 \bmod 2; i > 2\}$
$N(x_1) \cap D = \emptyset$
$N(x_2) \cap D = \emptyset$

It can be seen that the intersection between the neighborhood $N(v)$ with $v \in V(G) - D$, for $N(x_1), N(x_2) \cap D = \emptyset$. Thus, the dominator set D do not dominate all vertices in $V(P_n)$. It concludes that, by assuming $\gamma_{Li}(P_n) < \lceil \frac{2n}{5} \rceil$, it will not comply the condition of locating independent dominating set. Therefore, the lower bound of locating independent domination number of P_n is $\gamma_{Li}(P_n) \ge \lceil \frac{2n}{5} \rceil$. Furthermore, we will show that the upper bound of locating independent domination number of P_n is $\gamma_{Li}(P_n) \le \lceil \frac{2n}{5} \rceil$. Choose $D = \{x_i; i \equiv 0 \bmod 2\}$ as the dominator set of P_n, for $n \ge 4$, thus $|D| = \lceil \frac{2n}{5} \rceil$. Choose $V - D = \{x_i; i \equiv 1 \bmod 2\}$ as the non-dominator set of P_n for $n \ge 4$. We will get the intersection between the neighborhood $N(v)$ with $v \in V(G) - D$ and dominator set D, in the following.

$N(x_i) \cap D = \{x_i; i \equiv 0 \bmod 2\}$

It can be seen that the intersection between the neighborhood $N(v)$ with $v \in V(G) - D$ are all different, and it is not empty set. The dominator set D does not dominate all vertices in $V(P_n)$. It can be concluded that, for $\gamma_{Li}(P_n) \le \lceil \frac{2n}{5} \rceil$, it will comply the condition of locating independent dominating set. Thus $\gamma_{Li}(P_n) \le \lceil \frac{2n}{5} \rceil$. Hence, then the locating independent domination number of P_n is $\gamma_{Li}(P_n) = \lceil \frac{2n}{5} \rceil$. □

Theorem 2.5. *Let G be a amalgamation graph of path (P_n) with $n \ge 4$ and $m \ge 3$. Then locating independent domination number of $Amal(P_n, v, m)$ is $\gamma_{Li}\ (Amal(P_n, v, m)) = m(\lceil \frac{2n}{5} \rceil - 1) + 1$.*

Proof. The graph $Amal(P_n, v, m)$ is a connected graph with $V(Amal(P_n, v, m)) = \{x\} \cup \{x_{i,j}; 1 \leq i \leq m; 1 \leq j \leq n-1\}$ and $E(Amal(P_n, v, m)) = \{xx_{i,1}; 1 \leq i \leq m\} \cup \{x_{i,j}x_{i,j+1}; 1 \leq i \leq m; 1 \leq j \leq n-2\}$. The order of this graph is $|V(Amal(P_n, v, m))| = nm - m + 1$ and the size is $|E(Amal(P_n, v, m))| = nm - m$. To proof the above theorem $\gamma_{Li}(Amal(P_n, v, m)) = m(\lceil\frac{2n}{5}\rceil - 1) + 1$, we will show that the lower bound $\gamma_{Li}(Amal(P_n, v, m)) \geq m(\lceil\frac{2n}{5}\rceil - 1) + 1$ and the upper bound $\gamma_{Li}(Amal(P_n, v, m)) \leq m(\lceil\frac{2n}{5}\rceil - 1) + 1$.

Firstly, we will show that γ_{Li} $(Amal(P_n, v, m)) \geq m(\lceil\frac{2n}{5}\rceil - 1) + 1$. By Lemma 2.1, we have $\gamma_{Li}(Amal(P_n, v, m)) = m(\gamma_{Li}(P_n) - 1) + 1$. Since by Theorem 2.4 we have $\gamma_{Li}(P_n) = \lceil\frac{2n}{5}\rceil$, thus we get so $\gamma_{Li}(Amal(P_n, v, m)) \geq m(\lceil\frac{2n}{5}\rceil - 1) + 1$. Furthermore, we will show that the upper bound of locating independent domination number of $\gamma_{Li}(Amal(P_n, v, m)) \leq m(\lceil\frac{2n}{5}\rceil - 1) + 1$. We consider $D(Amal(P_n, v, m)) = \{x\} \cup \{x_{i,j}; 1 \leq i \leq m; j \equiv 0 \bmod 2\}$ as the dominator set of $Amal$ (P_n, v, m) for $n \geq 4$ and $m \geq 3$. It is clearly to see that $|D| = m(\lceil\frac{2n}{5}\rceil - 1) + 1$, and the dominator set D dominates all vertices of $G = Amal(P_n, v, m)$. By definition, we have the non-dominator set of $Amal(P_n, v, m)$ for $n \geq 4$ and $m \geq 3$ is $V(Amal(P_n, v, m)) - D(Amal(P_n, v, m)) = \{x_{i,j}; 1 \leq i \leq m; j \equiv 1 \bmod 2\}$. The intersection between the neighborhood $N(v)$ with $v \in V(G) - D(G)$ and dominator set $D(G)$ is as follows.

$$N(x_{i,j}) \cap D = \{x\}, \{x_{i,j}; 1 \leq i \leq m; j \equiv 0 \bmod 2\}$$

It can be seen intersection between the neighborhood $N(v)$ with $v \in V(G) - D(G)$ and the obtained dominator set D are uniques and it is not empty set. Thus, it can be concluded that $\gamma_{Li}(Amal(P_n, v, m)) \leq m(\lceil\frac{2n}{5}\rceil - 1) + 1$. D also complies the condition of locating independent dominating set. Hence, the lower bound and upper bound of locating independent domination number respectively, are $\gamma_{Li} \geq m(\lceil\frac{2n}{5}\rceil - 1) + 1$ and $\gamma_{Li} \leq m(\lceil\frac{2n}{5}\rceil - 1) + 1$. It conludes the locating independent domination number of $Amal(P_n, v, m)$ is $\gamma_{Li}(Amal(P_n, v, m)) = m(\lceil\frac{2n}{5}\rceil - 1) + 1$. □

Theorem 2.6. *For $n \geq 3$, the locating independent domination number of L_n is $\gamma_{Li}(L_n) = n$.*

Proof. Ladder graph L_n is a connected graph with vertex set $V(L_n) = \{x_i; 1 \leq i \leq n\} \cup \{y_i; 1 \leq i \leq n\}$ and edge set $E(L_n) = \{x_ix_{i+1}; 1 \leq i \leq n-1\} \cup \{y_iy_{i+1}; 1 \leq i \leq n-1\} \cup \{x_iy_i; 1 \leq i \leq n\}$. The order and size of L_n are $|V(L_n)| = 2n$ and $|E(L_n)| = 3n - 2$.

We claim that γ_{Li} $(L_n) \geq n$. To convince the proof, assume that $\gamma_{Li}(L_n) < n$. Let the dominator vertex set of L_n, for $n \geq 3$, be $D = \{x_i; 1 \leq i \leq n-1; i = \text{odd}\} \cup \{y_i; 1 \leq i \leq n-1; i = \text{even}\}$, thus $|D| = n - 1$, and let non-dominator vertex set of L_n, for $n \geq 3$, be $V - D = \{x_i; 1 \leq i \leq n; i = \text{even}\} \cup \{y_i; 1 \leq i \leq n; i = \text{odd}\} \cup \{x_n; n = \text{odd}\} \cup \{y_n, n = \text{even}\}$. Then we get the intersection of the neighborhood $N(v)$ with $v \in V(G) - D$ and dominator set D, in the following.

$$N(x_i) \cap D = \{x_{i-1}, x_{i+1}, y_i\};\ 2 \leq i \leq n-2; i = \text{even}$$
$$N(x_n) \cap D = \{x_{n-1}\};\ n = \text{even}$$
$$N(x_{n-1}) \cap D = \{x_{n-2}, y_{n-1}\};\ n = \text{odd}$$
$$N(x_n) \cap D = \emptyset;\ n = \text{odd}$$
$$N(y_1) \cap D = \{x_1, y_2\}$$
$$N(y_i) \cap D = \{x_i, y_{i-1}, y_{i+1}\};\ 3 \leq i \leq n-2; i = \text{odd}$$

$N(y_n) \cap D = \{y_{n-1}\}$; $n =$ odd
$N(y_{n-1}) \cap D = \{x_{n-1}, y_{n-2}\}$; $n =$ even
$N(y_n) \cap D = \emptyset$; $n =$ even

It can be seen that the intersection between the neighborhood $N(v)$ with $v \in V(G) - D$, for $N(x_n) \cap D = \emptyset$ for n odd and $N(y_n) \cap D = \emptyset$ for n even. Thus, the dominator set D do not dominate all vertices in $V(L_n)$. It concludes that, by assuming $\gamma_{Li}(L_n) < n$, it will not comply the condition of locating independent dominating set. Therefore, the lower bound of locating independent domination number of L_n is $\gamma_{Li}(L_n) \geq n$. Furthermore, we will show that the upper bound of locating independent domination number of L_n is $\gamma_{Li}(L_n) \leq n$. Choose $D = \{x_i; 1 \leq i \leq n; i = \text{odd}\} \cup \{y_i; 1 \leq i \leq n; i = \text{even}\}$ as the dominator set of L_n, for $n \geq 3$, thus $|D| = n$. Choose $V - D = \{A\}$ as the non-dominator set of S_n for $n \geq 3$. We will get the intersection between the neighborhood $N(v)$ with $v \in V(G) - D$ and dominator set D, in the following.

$N(x_i) \cap D = \{x_{i-1}, x_{i+1}, y_i\}$; $2 \leq i \leq n-2; i =$ even
$N(x_n) \cap D = \{x_{n-1}, y_n\}$; $n =$ even
$N(y_1) \cap D = \{x_1, y_2\}$
$N(y_i) \cap D = \{x_i, y_{i-1}, y_{i+1}\}$; $3 \leq i \leq n; i =$ odd
$N(y_n) \cap D = \{x_n, y_{n-1}\}$; $n =$ odd

It can be seen that the intersection between the neighborhood $N(v)$ with $v \in V(G) - D$ are all different, and it is not empty set. The dominator set L does not dominate all vertices in $V(L_n)$. It can be concluded that, for $\gamma_{Li}(L_n) \leq n$, it will comply the condition of locating independent dominating set. Thus $\gamma_{Li}(L_n) \leq n$. Hence, then the locating independent domination number of L_n is $\gamma_{Li}(L_n) = n$. □

Theorem 2.7. *Let G be a amalgamation graph of ladder (L_n) with $n \geq 2$ and $m \geq 2$, then locating independent domination number of $Amal(L_n, v, m)$ is $\gamma_{Li}\ (Amal(L_n, v, m)) = nm$.*

Proof. The graph $Amal(L_n, v, m)$ is a connected graph with $V(Amal(L_n, v, m)) = \{y_1\} \cup \{x_i^j; 1 \leq i \leq n; 1 \leq j \leq m\} \cup \{y_{i+1}^j; 1 \leq i \leq n-1; 1 \leq j \leq m\}$ and $E(Amal(L_n, v, m)) = \{y_1 x_1^j; 1 \leq j \leq m\} \cup \{y_1 y_2^j; 1 \leq j \leq m\} \cup \{x_{i+1}^j y_{i+1}^j; 1 \leq i \leq n-1; 1 \leq j \leq m\} \cup \{x_i^j x_{i+1}^j; 1 \leq i \leq n-1; 1 \leq j \leq m\} \cup \{y_{i+1}^j y_{i+2}^j; 1 \leq i \leq n-2; 1 \leq j \leq m\}$. The order of this graph is $|V(Amal(L_n, v, m))| = 2nm - m + 1$ and the size is $|E(Amal(L_n, v, m))| = 3nm - 2m$. To prove the above theorem $\gamma_{Li}(Amal(L_n, v, m)) = nm$, we will show that the lower bound $\gamma_{Li}(Amal(L_n, v, m)) \geq nm$ and the upper bound $\gamma_{Li}(Amal(L_n, v, m)) \leq nm$.

Firstly, we will show that $\gamma_{Li}\ (Amal(L_n, v, m)) \geq nm$. By Lemma 2.1, we have $\gamma_{Li}(Amal(L_n, v, m)) = m(\gamma_{Li}(L_n) - 1) + 1$. Since by Theorem 2.6 we have $\gamma_{Li}(L_n) = n$, thus we get so $\gamma_{Li}(Amal(L_n, v, m)) \geq m(n-1) + 1$. Furthermore, we will show that the upper bound of locating independent domination number of $\gamma_{Li}(Amal(L_n, v, m)) \leq nm$. We consider $D(Amal(L_n, v, m)) = \{x_i^j; 1 \leq i \leq n; 1 \leq j \leq m; i = \text{odd}\} \cup \{y_i^j; 1 \leq i \leq n; 1 \leq j \leq m; i = \text{even}\}$ as the dominator set of $Amal\ (L_n, v, m)$ for $n \geq 2$ and $m \geq 2$. It is clearly to see that $|D| = nm$, and the dominator set D dominates all vertices of $G = Amal(L_n, v, m)$. By definition, we have the non-dominator set of $Amal(L_n, v, m)$ for $n \geq 2$ and $m \geq 2$ is $V(Amal(L_n, v, m)) - D(Amal(L_n, v, m)) = \{y_1\} \cup \{x_i^j; 1 \leq i \leq n; 1 \leq j \leq m; i = \text{even}\} \cup \{y_i^j; 3 \leq i \leq n; 1 \leq j \leq m; i = \text{odd}\}$. The intersection between the neighborhood $N(v)$ with $v \in V(G) - D(G)$ and dominator set $D(G)$ is as follows.

$N(y_1) \cap D = \{x_1^j, y_2^j; 1 \leq j \leq m\}$
$N(x_2^j) \cap D = \{x_1^j, x_3^j, y_2^j\}$; $1 \leq j \leq m$

$N(x_i^j) \cap D = \{x_{i-1}^j, x_{i+1}^j, y_i^j\}$; $4 \leq i \leq n-2; 1 \leq j \leq m; i = \text{even}$
$N(x_n^j) \cap D = \{x_{n-1}^j, y_n^j\}$; $1 \leq j \leq m; n = \text{even}$
$N(y_i^j) \cap D = \{x_i^j, y_{i-1}^j, y_{i+1}^j\}$; $3 \leq i \leq n; 1 \leq j \leq m; i = \text{odd}$
$N(y_n^j) \cap D = \{x_n^j, y_{n-1}^j\}$; $1 \leq j \leq m; n = \text{odd}$

It can be seen intersection between the neighborhood $N(v)$ with $v \in V(G) - D(G)$ and the obtained dominator set D are uniques and it is not empty set. Thus, it can be concluded that $\gamma_{Li}(Amal(L_n, v, m)) \leq nm$. D also complies the condition of locating independent dominating set. Hence, the lower bound and upper bound of locating independent domination number respectively, are $\gamma_{Li} \geq nm$ and $\gamma_{Li} \leq nm$. It conludes the locating independent domination number of $Amal(L_n, v, m)$ is $\gamma_{Li}(Amal(L_n, v, m)) = nm$. □

3. Concluding Remarks

In this paper, we have determined the exact values of locating independent dominating number of some graph operations, namely ladder graph S_n, $Amal(S_n, v, m)$, path graph P_n, $Amal(P_n, v, m)$, wheel graph W_n, $Amal(W_n, v, m)$, ladder graph L_n, $Amal(L_n, v, m)$. As we have mentioned in introduction, to prove weather an locating independent dominating number is a hard problem. Thus, it still gives the following open problem.

Open Problem 3.1. *Let G be any connected graph, determine sharper lower bounds of $\gamma_{Li}(G)$ in term of the degrees of the graph?*

3.1. Acknowledgments

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