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# CAUCHY

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CAUCHY is a mathematical journal published twice a year on May and November by the Mathematics Department, Faculty of Science and Technology, Maulana Malik Ibrahim State Islamic University of Malang. This journal includes research papers, literature studies, analysis, and problem solving in Mathematics (Algebra, Analysis, Statistics, Computing and Applied Mathematics).

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# CAUCHY

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## PREFACE

**Cauchy** is a national journal published by Mathematics Department, Science and Technology Faculty, Maulana Malik Ibrahim State Islamic University of Malang. This is the second issue of this year. It contains 6 (six) article from all over the country, not only from local area of East Java. Those articles covered graph theory, numerical analysis, applied mathematics, statistics, economic mathematics, and algebra. This issue was authored by 13 authors and co-authors.

In the first article, the author discussed the existence and uniqueness of fixed point in complex valued b-metric spaces. She considers some fixed-point theorems in b-metric spaces to be extended to fixed point theorems in complex valued b-metric spaces. Some fixed-point theorems in complex-valued b-metric spaces which are derived from the corresponding theorems in b-metric spaces are obtained in this research.

The second article focuses on graph theory and its applications. The paper showed that super $(a,d)-B_m$-antimagic total labeling of $Amal(F_n,P_n,m)$ and $sAmal(F_n,P_n,m)$ for some feasible $d$. The author also suggests to do some other research concerning to this field since there are a lot of things to determine yet the obtained results are still very limited.

The third article, entitled "*Application of Modified Spatial K'luster Analysis by Tree Edge Removal (SKATER) Method on the level of Crime in Way Kanan district, Lampung*" elaborated the modification of spatial k'luster analysis by tree edge removal. The result shows that the spread pattern of the spoliation, robberies and gambling cases in Way Kanan district showed their spatial clustering. It also explained that modification of SKATER clustering method produce the 4-rise cluster, it is because the mean level of crime is $k_1 \geq k_2 \geq k_3 \geq k_4$.

The fourth article emphasized the discussion of graph theory especially on harmonious labeling on pleated of the Dutch windmill graphs. It showed that the pleated of the Dutch windmill graphs and the union pleated of the Dutch windmill graphs admitted odd harmonious labeling. The authors suggest an open problem for further research including the question if there exists odd harmonious labeling on another kind of pleated of the Dutch windmill graph.

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The fifth article covered the discussion of almost surjective epsilon-isometry in the reflexive Banach spaces. In this paper, the author discussed some applications of almost surjective $\varepsilon$-isometry mapping, one of them is in Lorentz space ($L_{p,q} -$ space).

The last article entitled "*on the spectra of commuting and non-commuting graph on dihedral group*". This paper integrates the Algebraic field of study with the application of graph theory. In this paper, the author investigates adjacency spectrum, Laplacian spectrum, signless Laplacian spectrum, and detour spectrum of commuting and non-commuting graph of dihedral group $D_{2n}$.

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# A Super (A,D)-Bm-Antimagic Total Covering Of Ageneralized Amalgamation Of Fan Graphs

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## ABSTRACT

We assume finite, simple and undirected graphs in this study. Let *G, H* be two graphs. By an (*a*,*d*)-*H*-antimagic total graph, we mean any obtained bijective function $f: V(G) \cup E(G) \rightarrow \{1, 2, 3, \dots, |V(G)| + |E(G)|\}$ such that for each subgraph *H'* which is isomorphic to *H*, their total *H*-weights $w(H) = \sum_{v\in E(H')} f(v) + \sum_{v\in E(H')} f(e)$ show an arithmetic sequence $\{a, a+d, a+2d, \dots., a+(m-1)d\}$ where *a, d* > 0 are integers and *m* is the cardinality of all subgraphs *H'* isomorphic to *H*. An (*a, d*)-*H*-antimagic total labeling *f* is called super if the smallest labels are assigned in the vertices. In this paper, we will study a super (*a, d*)-$B_m$-antimagicness of a connected and disconnected generalized amalgamation of fan graphs in which a path is a terminal.

**Keywords**: Super (*a, d*)-$B_m$-antimagic total covering, generalized amalgamation of fan graphs, connected and disconnected

## INTRODUCTION

In [1], Dafik *et.al.* defined an amalgamation of graphs as follows: Let $G_i$ be a finite collection of graphs and suppose each $G_i$ has a fixed vertex $v_j$ called a terminal. The amalgamation $G_i$ where $v_j$ as a terminal is formed by taking all the $G_i$'s and identifying their terminal. When $G_i$ are all isomorphic connected graphs, for any positive integer *m*, we denote such amalgamation by *Amal*(*G, m*), where *m* denotes the number of copies of *G*. If we replace the terminal vertex $v_j$ by a subgraph $P \subset G$ then such amalgamation is said to be a generalized amalgamation of *G* and denoted by *amal*(*G, P, m*).

Furthermore, Baca *et. al.* in [2] and Dafik *et, el.* [3] defined an (*a, d*)-edge-antimagic total labeling of *G* as a mapping $f: V(G) \cup E(G) \rightarrow \{1, 2, 3, \dots, |V(G)| + |E(G)|\}$, such that the set of edge-weights $\{f(u) + f(uv) + f(v) |\, uv \in E(G)\}$ is equal to the set $\{a, a+d, a+2d, \dots, a+(|E(G)|-1)d\}$ for some positive integers *a* and *d*. Combining the two previous labelings, [1], [4], [5], [6], [7] introduced the (*a*,*d*)-*H*- antimagic total labeling. A graph *G* is said to be an (*a, d*)-*H*-antimagic total graph if there exist a bijective function $f: V(G) \cup E(G) \rightarrow \{1, 2, 3, \dots, |V(G)| + |E(G)|\}$ such that for all subgraphs *H'* isomorphic to *H*, the total *H*-weights $w(H) = \sum_{v\in E(H')} f(v) + \sum_{v\in E(H')} f(e) = \gamma$ form an arithmetic

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progression $\{a, a+d, a+2d, \ldots, a+(m-1)d\}$, where *a, d* > 0 are integers and *m* is the number of all subgraphs *H'* isomorphic to *H*. An (*a, d*)-*H*-antimagic total labeling *f* is called super if the smallest labels are assigned in the vertices.

There are many results show the existence of the (*a, d*)-*H*-antimagic total labeling, see [1], [4], [7], [8], [9], and [10]. In this paper, we will study a super (*a, d*)-$B_m$-antimagicness of an amalgamation of fans of order *m* when a path of order *n* is a terminal, denoted by *Amal*($F_n$, $P_n$, *m*) as well as the disjoint union of multiple *s* copies of *Amal*($F_n$, $P_n$, *m*). The cover *H'* is a book of order two, thus $H = B_m$. In other word, we will show the existence of super (*a, d*)-$B_m$-antimagic total labeling of *Amal*($F_n$, $P_n$, *m*) and disjoint union of multiple *s* copies of *Amal*($F_n$, $P_n$, *m*) denoted by *sAmal*($F_n$, $P_n$, *m*).

## LITERATURE REVIEW

Prior to showing the research result on the existence of super (*a,d*)-$B_m$-antimagic total labeling *sAmal*($F_n$, $P_n$, *m*), we will rewrite a known lemma excluding the proof that will be useful for determining the necessary condition for a graph to be super (a,d)-$B_m$-antimagic total labeling. This lemma proved by [2] provides an upper bound for feasible value of *d*, and it is a sharp.

**Lemma1.** [2] *Let G be a simple graph of order $p_G$ and size $q_G$. If G is super (a, d)-H- antimagic total labeling then* $d \le \frac{(p_G - p_{H'})p_{H'} + (q_G - q_{H'})q_{H'}}{t-1}$, *for H' are subgraphs isomorphic to H.* $|V(G)| = p_G, |E(G)| = q_G, |V(H')| = p_{H'}, |E(H')| = q_{H'}$, *and* $t = |H'_j|$.

## RESULTS AND DISCUSSIONS

**The Connected Graph**. An amalgamation of fan graphs, denoted by *Amal*($F_n$, $P_n$, *m*), is a connected graph with vertex set *V* (*Amal*($F_n$, $P_n$,*m*)) $=\{A_j, x_i ; 1 \le j \le m, 1 \le i \le n\}$ and *E*(*Amal*($F_n$, $P_n$, *m*)) $= \{A_j, x_i ; 1 \le j \le m, 1 \le i \le n\} \cup \{x_i x_{i+1}; 1 \le i \le n-1\}$. Since we study a super (*a, d*)-*H*- antimagic total labeling for $H' = B_m$ isomorphic to *H*, thus $p_G$ = |*V* (*Amal*($F_n$, $P_n$, *m*)) | = *m* + *n*, $q_G$ = |*E*(*Amal*($F_n$, $P_n$, *m*))| =*mn* + *n* - 1, $p_{H'}$ = |*V* ($B_m$)| = *m* + 2, $q_{H'}$ = |*E*($B_m$)| = 2*m* + 1, *t* = |$H'_j$ | =|$B_m$| = *n* - 1.

If amalgamation of fan graphs *Amal*($F_n$, $P_n$, *m*) has a super (*a, d*)-$B_m$- antimagic total labeling then for $p_G$ = |*V*(*Amal*($F_n$, $P_n$, *m*))|= *m*+*n*, $q_G$ = |*E*(*Amal*($F_n$, $P_n$,*m*))|= *mn*+*n*-1, $p_{H'}$ = |*V*($F_n$, $P_n$, *m*)|= *m*+2, $q_{H'}$ = |*E* ($F_n$, $P_n$, *m*)|= 2*m* + 1, *t* =|$H'_j$ | = *n* - 1, it follows from Lemma 1.1 the upper bound of $d \le 2m^2 + 4m + 3$.

Now we start to describe the result of the super (*a,d*)-*H*-antimagic total labeling of amalgamation of fan graph with the following theorems. Figure. 1 shows an illustrasion of graph *Amal*($F_n$, $P_n$,*m*).

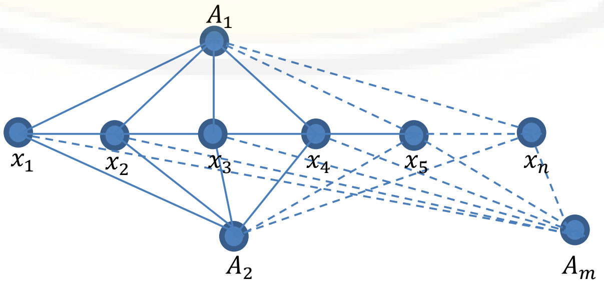

Figure. 1 illustrasion of graph *Amal* ($F_n$, $P_n$,*m*)

**Theorem 2.1**. *For m, n ≥2, the graph Amal($F_n$, $P_n$, m) admits a super* $\left(\left(n+\frac{5}{2}\right)m^2+\left(2n+\frac{9}{2}\right)m+n+2m+3+1, 2m+3\right)$*-$B_m$-antimagic total labeling.*

**Proof.** For $G = Amal(F_n, P_n, m)$, define the vertex labeling $f_1$, as follow: $f_1\,(A_j) = j$ and $f_1(x_i)\ = m+i;\ 1 \le j \le m, 1 \le i \le n$, and the edge labeling as follows:

$$f_1(A_j x_i\ ) = m+n+(j-1)n+i;\ 1 \le j \le m, 1 \le i \le n$$
$$f_1\left(x_i\ \ x_{i+1}\right) = m+n+nm+i+i; 1 \le i \le n-1$$

The vertex and edge labelings $f_1$ are a bijective function $f_1\colon V\,(G) \cup E(G) \to \{1,2,3,\dots,3mn-m+1\}$. The *H*-weights of *Amal*($F_n$, $P_n$,*m*), for $1 \le j \le m, 1 \le i \le n$ under the labeling $f_1$, constitute the following sets $w_{f_1} = \bigcup_{i=1}^{n-1}\{f_1\left(A_j\right)+f_1(x_i)\} = \{\bigcup_{i=1}^{n-1}\{(2m+2i+1+\left(\frac{m^2+m}{2}\right)\}$, and the total *H*-weights of *Amal*($F_n$, $P_n$,*m*) constitute the following sets $W_{f_1} = \bigcup_{i=1}^{n-1}\{w_{f_1}+\sum_{j=1}^{m} f_1(A_j\, x_i)+f_1(x_i x_{i+1})\} = \bigcup_{i=1}^{n-1}\{\left(n+\frac{5}{2}\right)m^2+\left(2n+\frac{9}{2}\right)m+n+(2m+3)i+1\}$. It is easy to observe that the set $W f_1 = \{\left(n+\frac{5}{2}\right)m^2+\left(2n+\frac{9}{2}\right)m+n+(2m+4), \left(n+\frac{5}{2}\right)m^2+\left(2n+\frac{9}{2}\right)m+n+4m+7, \left(n+\frac{5}{2}\right)m^2+\left(2n+\frac{9}{2}\right)m+n+6m+10, \dots, \left(n+\frac{5}{2}\right)m^2+\left(4n+\frac{5}{2}\right)m+4n-2\}$. It gives the desired proof.

■

**Theorem 2.2**. *For m, n ≥2, the graph Amal($F_n$, $P_n$,m) admits a super* $\left(\left(n+\frac{5}{2}\right)m^2+\left(2n+\frac{5}{2}\right)m+2n+2, 2m+1\right)$*-$B_m$-antimagic total labeling.*

**Proof.** For $G = Amal(F_n, P_n, m)$, define the vertex labeling $f_2$, as follow: $f_2\,(A_j) = \{n+j;\ 1 \le j \le m\}$ *and* $f_2(x_i)\ =\ i;\ 1 \le i \le n$, and the edge labeling as follows:

$$f_2(A_j x_i\ ) = 2n+m-1+(j-1)n+i;\ 1 \le j \le m, 1 \le i \le n$$
$$f_2\left(x_i\ \ x_{i+1}\right) = 2n+m-i; 1 \le i \le n-1$$

The vertex and edge labelings $f_2$ are a bijective function $f_2\colon V\,(G) \cup E(G) \to \{1,2,3,\dots,3mn-m+1\}$. The *H*-weights of *Amal*($F_n$, $P_n$,*m*), for $1 \le j \le m, 1 \le i \le n$ under the labeling $f_2$, constitute the following sets $w_{f_2} = \bigcup_{i=1}^{n-1}\{f_2(x_i)+f_2(x_{i+1})+\sum_{j=1}^{m} f_2(A_j)\} = \{\bigcup_{i=1}^{n-1}\{\frac{1}{2}m^2+j+1\}$, and the total *H*-weights of *Amal*($F_n$, $P_n$,*m*) constitute the following sets $W_{f_2} = \bigcup_{i=1}^{n-1}\{w_{f_2}+\sum_{j=1}^{m} f_2(A_j)+f_2(x_i x_{i+1})\} = \bigcup_{i=1}^{n-1}\{\left(n+\frac{5}{2}\right)m^2+4nm+\frac{1}{2}m+2n+1+i(2m+1)\}$. It is easy to observe that the set $W_{f_2}\ = \{\left(n+\frac{5}{2}\right)m^2+\left(4n+\frac{5}{2}\right)m+2n+2, \left(n+\frac{5}{2}\right)m^2+\left(4n+\frac{9}{2}\right)m+2n+3, \left(n+\frac{5}{2}\right)m^2+\left(4n+\frac{13}{2}\right)m+2n+4, .., \left(n+\frac{5}{2}\right)m^2+\left(6n-\frac{3}{2}\right)m+3n\}$. Therefore, the graph *Amal*($F_n$, $P_n$,*m*) admits a super $\left(\left(n+\frac{5}{2}\right)m^2+\left(2n+\frac{5}{2}\right)m+2n+2, 2m+1\right)$-$B_m$- antimagic total labeling, *For* $m, n\ \ge 2$ ■

**Theorem 2.3**. *For m, n ≥2, the graph Amal($F_n$, $P_n$,m) admits a super* $\left(\frac{5}{2}(m^2+m)+4nm+6+2m^2, 2m^2+3\right)$*-$B_m$-antimagic total labeling.*

**Proof.** For $G = Amal(F_n, P_n, m)$, define the vertex labeling $f_3$, as follow: $f_3\ (A_1) = 1,\ f_3(x_i) = i + 1;\ 1 \le i \le n$ and $f_3(xA_j)\ = n + j;\ 2 \le j \le m$ and the edge labeling as follows:

$$f_3(A_j x_i\ ) = n + mi + j;\ 1 \le j \le m, 1 \le i \le n$$
$$f_3\big(x_i\ \ x_{i+1}\big) = m + n + nm + i; 1 \le i \le n - 1$$

The vertex and edge labelings $f_3$ are a bijective function $f_3: V\ (G) \cup E(G) \rightarrow \{1, 2, 3, \ldots, 3mn - m + 1\}$. The $H$-weights of $Amal(F_n, P_n, m)$, for $1 \le j \le m, 1 \le i \le n$ under the labeling $f_3$, constitute the following sets $w_{f_3} = \bigcup_{i=1}^{n-1}\{\sum_{j=2}^{m} f_3\big(A_j\big) + f_3(x_i) + f_3(x_{i+1}) + f_3(A_1)\} = \bigcup_{i=1}^{n-1}\{\frac{1}{2}m^2 + \frac{1}{2}m + (m-1)n + 2i + 3\}$, and the total $H$-weights of $Amal(F_n, P_n, m)$ constitute the following sets $W_{f_3} = \bigcup_{i=1}^{n-1}\{w_{f_3} + f_3(x_i x_{i+1}) + \sum_{j=1}^{m} f_3(A_j\, x_i) + f_3(A_j x_{i+1})\} = \bigcup_{i=1}^{n-1}\{\frac{5}{2}m^2 + \frac{5}{2}m + 4nm + 3 + (2m^2 + 3)i\}$. It is easy to observe that the set $Wf_3 = \left\{\frac{5}{2}(m^2 + m\right) + 4nm + 2m^2 + 6, \frac{5}{2}(m^2 + m) + 4nm + 4m^2 + 9, \ldots, 3n^2\left(2m - \frac{1}{2}\right) + n\left(\frac{15}{2} - 6m\right) + 5m - 5\}$. It gives the desired proof ■

**Theorem 2.4**. *For n ≥2, the graph* $Amal(F_n, P_n, 2)$ *admits a super* $\left(\frac{29n+32}{2}, 0\right)$-$B_2$*-antimagic total labeling for n even and super* $\left(\frac{29n+32}{2}, 0\right) - B_2$*-antimagic total labeling for n odd.*

**Proof.** Define the vertex and edge labeling $f_4$ as follows:

$$f_4(a) = 1; f_4(b) = 2$$
$$f_4(x_i) = \begin{cases} \dfrac{i+5}{2}, for\ 1 \le i \le n, i\ odd \\ \dfrac{n+i+4}{2}, for\ 1 \le i \le n, i\ even, n\ even \\ \dfrac{n+i+5}{2}, for\ 1 \le i \le n, i\ even, n\ odd \end{cases}$$
$$f_4(x_i x_{i+1}) = 2n - i + 2, for\ 1 \le i \le n - 1$$

$$f_4(bx_i) = 2n - i + 1, for\ 1 \le i \le n$$
$$f_4(ax_i) = 4n - i + 2, for\ 1 \le i \le n$$

The vertex and edge labelings $f_4$ are a bijective function $f_4: V\ (Amal(F_n, P_n, 2)) \cup E\big(Amal(F_n, P_n, 2)\big) \rightarrow \{1, 2, 3, \ldots, 4n + 1\}$. The $H$-weights of $Amal(F_n, P_n, 2)$, for $1 \le i \le n$ under the labeling $f_4$, constitute the following sets $w_{f_4} = f_4(a) + f_4(b) + f_4(x_i) + f_4(x_{i+1}) = \frac{n+2i+16}{2}$, for $n$ even and $w_{f_4} = f_4(a) + f_4(b) + f_4(x_i) + f_4(x_{i+1}) = \frac{n+2i+17}{2}$ for $n$ odd and the total $H$-weights of $Amal(F_n, P_n, 2)$ constitute the following sets $W_{f_4} = wf_4 + f_4(x_i x_{i+1}) + f_4(bx_i) + f_4(bx_{i+1}) + f_4(ax_i) + f_4(ax_{i+1}) = \frac{29n+32}{2}$, for $n$ even and $W_{f_4} = wf_4 + f_4(x_i x_{i+1}) + f_4(bx_i) + f_4(bx_{i+1}) + f_4(ax_i) + f_4(ax_{i+1}) = \frac{29n+25}{2}$ for $n$ odd. It is easy to observe that the set $Wf_4 = \{\frac{29n+32}{2}, \frac{29n+32}{2}, \ldots, \frac{29n+32}{2}\}$ for $n$ even and $Wf_4 = \left\{\frac{29n+25}{2}, \frac{29n+25}{2}, \ldots, \frac{29n+25}{2}\right\}$ for $n$ odd. Therefore, the graph $Amal(F_n, P_n, 2)$ admits a super $\left(\frac{29n+32}{2}, 0\right) - B_2$- antimagic total labeling for $n \ge 2$ for $n$ even, and the graph $Amal(F_n, P_n, 2)$ admits a super $\left(\frac{29n+25}{2}, 0\right) - B_2$ -antimagic total labeling for $n \ge 2$ for $n$ odd It gives the desired proof . ■

**Theorem 2.5**. *For $n \geq 2$, the graph Amal($F_n$, $P_n$, 2) admits a super* $(13n+19,1)$-*$B_2$ - antimagic total labeling.*

**Proof.** Define the vertex and edge labeling $f_5$ as follows:

$$f_5(a) = 1; f_5(b) = n+2$$

$$f_5(x_i) = i+2, for\ 1 \leq i \leq n$$

$$f_5(bx_i) = 2n - i + 3, for\ 1 \leq i \leq n$$
$$f_5(ax_i) = 2n + i + 2, for\ 1 \leq i \leq n$$
$$f_5(x_i x_{i+1}) = 4n - i + 2, for\ 1 \leq i \leq n-1$$

The vertex and edge labelings $f_5$ are a bijective function $f_5\colon V\ (Amal(F_n, P_n, 2)) \cup E\big(Amal(F_n, P_n, 2)\big) \rightarrow \{1, 2, 3, \ldots, 4n+1\}$. The *H*-weights of *Amal*($F_n$, $P_n$, *2*), for $1 \leq i \leq n$ under the labeling $f_5$, constitute the following sets $w_{f_5} = f_5(a) + f_5(b) + f_5(x_i) + f_5(x_{i+1}) = n + 2i + 6$, and the total *H*-weights of *Amal*($F_n$, $P_n$, *2*) constitute the following sets $W_{f_5} = wf_5 + f_5(x_i x_{i+1}) + f_5(bx_i) + f_5(bx_{i+1}) + f_5(ax_i) + f_5(ax_{i+1}) = 13n + i + 18$. It is easy to observe that the set $Wf_5 = \{\frac{29n+32}{2}, \frac{29n+32}{2}, \ldots, \frac{29n+32}{2}\}$ for *n* even and $Wf_5 = \{13n+19, 13n+20, \ldots, 14n+18\}$. Therefore, the graph $Amal(F_n, P_n, 2)$ admits a super $(13n + i + 18, 1) - B_2 -$ antimagic total labeling for $n \geq 2$ It gives the desired proof ■

**The Disconnected Graph**. A disjoint union of amalgamation of fan graphs, denoted by *sAmal*($F_n$, $P_n$,*m*), is a disconnected graph with vertex set *V* (*sAmal*($F_n$, $P_n$, *m*)) =$A_j^k, x_i^k; 1 \leq j \leq m, 1 \leq i \leq n; 1 \leq k \leq s\}$ and *E*(*sAmal*($F_n$, $P_n$, *m*)) = $A_j^k, x_i^k; 1 \leq j \leq m, 1 \leq i \leq n; 1 \leq k \leq s\}$ Since we study a super (*a, d*)-*H*- antimagic total labeling for *H'* = $B_m$ isomorphic to *H*, thus $p_G$ = |*V* (*sAmal*($F_n$, $P_n$,*m*)) | = *s*(*m* + *n*), $q_G$ = |*E*(*sAmal*($F_n$, $P_n$,*m*))| =*s*(*mn* + *n* - 1), $p_{H'}$ = |*V* ($B_m$)| = *m* + 2, $q_{H'}$ = |*E*($B_m$)| = 2*m* + 1, *t* = |*H'*$_j$ | =|$B_m$| = *s*(*n* - 1).

If amalgamation of fan graphs *sAmal*($F_n$, $P_n$,*m*) has a super (*a, d*)-$B_m$- antimagic total labeling then for $p_G$ = *s*(*m*+*n*), $q_G$ = *s*(*mn*+*n*-1), $p_{H'}$ = *m*+2, $q_{H'}$ = 2*m* + 1, *t* = *s*(*n* - 1), it follows from Lemma 1.1 the upper bound of

$$d \leq \frac{[m^2(2sn + s - 5) + 4snm + 3sn - 8m - s - 5}{s(n-1) - 1}$$

**Theorem 2.6**. *For m, n ≥2, s ≥ 2 and m is even integer, the sAmal*($F_n$, $P_n$,*m*) *admits a super* $\left((3+n)m^2 s + (2m+1)ns - 2s + \frac{m}{2} + (2m+3)(s+1), 2m+3\right)$-*$B_m$- antimagic total labeling.*

**Proof.** For *G* = *sAmal*($F_n$, $P_n$, *m*), define the vertex labeling $f_6$, for $1 \leq j \leq m, 1 \leq i \leq n$ (*m* is even integer), $1 \leq k \leq s$ as follow:

$$f_6(x_i^k) = s(m + i - 1) + k$$
$$f_6\big(A_j^k\big) = \begin{cases} k + (j-1)s; & for\ \ 1 \leq k \leq s, 1 \leq j \leq m, j\ odd \\ (m-4)s + 1 + js - k & for\ \ 1 \leq k \leq s, 1 \leq j \leq m, j\ even \end{cases}$$

and edge labeling as follow:

for $1 \leq j \leq m, 1 \leq i \leq n$ (*m* is even integer), $1 \leq k \leq s$

$$f_6\big(A_j^k x_i^k\big) = s(m + nj + i - 1) + k$$

for $1 \leq i \leq n-1, 1 \leq k \leq s$

$$f_6\big(x_i^k x_{i+1}^k\big) = s(m + n + nm + i - 1) + k$$

The vertex and edge labelings $f_6$ are a bijective function $f_6 : V(G) \cup E(G) \rightarrow \{1, 2, 3, \dots, 3mns - ms + s\}$. The $H$-weights of $sAmal(F_n, P_n, m)$, for $1 \le j \le m, 1 \le i \le n$ ($m$ is even integer), $1 \le k \le s$ under the labeling $f_6$, constitute the following sets $w_{f_6} = \bigcup_{i=1}^{n-1} \bigcup_{k=1}^{s} \{f_6(x_i^k) + f_6(x_{i+1}^k)\} + \sum_{j=1}^{m}(A_j^k) = \bigcup_{i=1}^{n-1} \bigcup_{k=1}^{s} \{s(2m + 2i - 1) + 2k + \frac{m}{2}(2ms - 4s + 1)\}$, and the total $H$-weights of $sAmal(F_n, P_n, m)$ constitute the following sets:
$W_{f_6} = \bigcup_{i=1}^{n-1} \bigcup_{k=1}^{s} \{w_{f_6} + f_6(x_i^k x_{i+1}^k) + \sum_{j=1}^{m}[f_6(A_j^k) + f_6(A_j^k x_{i+1}^k)] = \bigcup_{i=1}^{n-1} \bigcup_{k=1}^{s} \{s(3m + n + nm + 3i - 2) + 3k + \frac{m}{2}(2ms - 4s + 1) + \sum_{j=1}^{m}[s(m + jn + i - 1) + k + s(m + jn + i) + k]\} = \bigcup_{i=1}^{n-1} \bigcup_{k=1}^{s} \{(3 + n)m^2 s + (2m + 1)ns - 2s + \frac{m}{2} + (2m + 3)(si + k)\}\}$. It is easy to observe that the set $W_{f_6} = \{(3 + n)m^2 s + (2m + 1)ns - 2s + \frac{m}{2} + (2m + 3)(s + 1), (3 + n)m^2 s + (2m + 1)ns - 2s + \frac{m}{2} + (2m + 3)(s + 2), (3 + n)m^2 s + (2m + 1)ns - 2s + \frac{m}{2} + (2m + 3)(s + 3), \dots, 2ms(2n^2 - 2n + 1) - s\left(n^2 - n - \frac{5}{2}\right) - \frac{1}{2}(n^2 - n - 3) + (n^2 + 2n - 3)(ms + s)\}$. It gives the desired proof. ■

**Theorem 2.7.** *For m, n ≥2, s ≥ 2 and m is even integer, the* $sAmal(F_n, P_n, m)$ *admits a super* $\left(\frac{m^2 s}{2}(2n + 5) + (2sn - s)(2m + 1) + \frac{m}{2} + 1 + (2m + 1)(s + 1), 2m + 1\right)$*-$B_m$-antimagic total labeling.*
**Proof.** For $G = sAmal(F_n, P_n, m)$, define the vertex labeling $f_5$, for $1 \le j \le m, 1 \le i \le n, 1 \le k \le s$ as follow:

$$f_7(x_i^k) = si + k - s$$

$$f_7(A_j^k) = \begin{cases} s(j - 1) + sn + k; & for\ \ 1 \le k \le s, 1 \le j \le m, j\ odd \\ sn + 1 + js - k & for\ \ 1 \le k \le s, 1 \le j \le m, j\ even \end{cases}$$

and edge labeling as follow:
for $1 \le j \le m, 1 \le i \le n, 1 \le k \le s$

$$f_7(A_j^k x_i^k) = s(2n + m) + 1 - si - k$$

for $1 \le i \le n - 1, 1 \le k \le s$

$$f_7(x_i^k x_{i+1}^k) = s(2n + m - 2 + (j - 1)n + i) + k$$

The vertex and edge labelings $f_7$ are a bijective function $f_7 : V(G) \cup E(G) \rightarrow \{1, 2, 3, \dots, 3mns - ms + s\}$. The $H$-weights of $sAmal(F_n, P_n, m)$, for $1 \le j \le m, 1 \le i \le n$ ($m$ is even integer), $1 \le k \le s$ under the labeling $f_5$, constitute the following sets $w_{f_7} = \bigcup_{i=1}^{n-1} \bigcup_{k=1}^{s} \{f_7(x_i^k) + f_7(x_{i+1}^k)\} + \sum_{j=1}^{m}(A_j^k) = \bigcup_{i=1}^{n-1} \bigcup_{k=1}^{s} \{\frac{1}{2}(sm^2 + m) + s(mn - 1) + 2(si + k)\} + 2k + \frac{m}{2}(2ms - 4s + 1)\}$, and the total $H$-weights of $sAmal(F_n, P_n, m)$ constitute the following sets $W_{f_7} = \bigcup_{i=1}^{n-1} \bigcup_{k=1}^{s} \{w_{f_7} + f_7(x_i^k x_{i+1}^k) + \sum_{j=1}^{m}[f_7(A_j^k) + f_5(A_j^k x_{i+1}^k)] = \bigcup_{i=1}^{n-1} \bigcup_{k=1}^{s} \{\frac{m^2 s}{2}(2n + 5) + (2sn - s)(2m + 1) + \frac{m}{2} + 1 + (2m + 1)(si + k)\}$. It is easy to observe that the set $W_{f_7} = \{\frac{m^2 s}{2}(2n + 5) + (2sn - s)(2m + 1) + \frac{m}{2} + 1 + (2m + 1)(s + 1), \frac{m^2 s}{2}(2n + 5) + (2sn - s)(2m + 1) + \frac{m}{2} + 1 + (2m + 1)(s + 2), \frac{m^2 s}{2}(2n + 5) + (2sn - s)(2m + 1) + \frac{m}{2} + 1 + (2m + 1)(s + 3), \dots, \frac{m^2 s}{2}(2n + 5) + (2sn - s)(2m + 1) + \frac{m}{2} + 1 + (2m + 1)sn\}$. It gives the desired proof. ■

**Theorem 2.8**. *For $m, n \geq 2$, $s \geq 2$ and m is even integer, the sAmal$(F_n, P_n, m)$ admits a super* $\left(\frac{5}{4}m^2s + sn(4m+2) + 2s + \frac{m}{2} + 2 + (2m^2-1)s + 2m - 1, 2m - 1\right)$-*$B_m$- antimagic total labeling.*

**Proof.** For $G$ = *sAmal*$(F_n, P_n, m)$, define the vertex labeling $f_8$, for $1 \leq j \leq m, 1 \leq i \leq n$ ($m$ is even integer), $1 \leq k \leq s$ as follow:

$$f_8(A_1^k) = k$$
$$f_8(x_i^k) = s(n+2) + 1 - si - k$$
$$f_8(A_j^k) = \begin{cases} sn + 1 + js - k; & for\ \ 1 \leq k \leq s, 1 \leq j \leq m, j\ odd \\ s(n+j-1) + k; & for\ \ 1 \leq k \leq s, 1 \leq j \leq m, j\ even \end{cases}$$

and edge labeling as follow:
for $1 \leq j \leq m, 1 \leq i \leq n$ ($m$ is even integer), $1 \leq k \leq s$

$$f_8(A_j^k x_i^k) = s(n + mi + j - 1) + k$$

for $1 \leq i \leq n-1, 1 \leq k \leq s$

$$f_8(x_i^k x_{i+1}^k) = s(n + m + nm + i - 1) + k$$

The vertex and edge labelings $f_8$ are a bijective function $f_8\colon V\,(G) \cup E(G) \rightarrow \{1, 2, 3, \dots, 3mns - ms + s\}$. The $H$-weights of *sAmal*$(F_n, P_n, m)$, for $1 \leq j \leq m, 1 \leq i \leq n$ ($m$ is even integer), $1 \leq k \leq s$ under the labeling $f_8$, constitute the following sets $w_{f_8} = \bigcup_{i=1}^{n-1} \bigcup_{k=1}^{s} \{f_8(A_1^k) + f_8(x_i^k) + f_8(x_{i+1}^k) + \sum_{j=2}^{m} f_8(A_j^k) + \sum_{j=3}^{m} f_8(A_j^k)\} = \bigcup_{i=1}^{n-1} \bigcup_{k=1}^{s} \left\{s\left(\frac{m^2}{2} + n - 2i + mn + 3\right) + \frac{m}{2} + 2 - 2k\right\}$, and the total $H$-weights of *sAmal*$(F_n, P_n, m)$ constitute the following sets $W_{f_8} = \bigcup_{i=1}^{n-1} \bigcup_{k=1}^{s} \{w_{f_8} + f_8(x_i^k x_{i+1}^k) + \sum_{j=1}^{m} [f_8(A_j^k x_i^k) + f_8(A_j^k x_{i+1}^k)]\} = \bigcup_{i=1}^{n-1} \bigcup_{k=1}^{s} \{\frac{5}{2}m^2s + sn(4m+2) + 2s + \frac{m}{2} + 2 + (2m^2-1)si + (2m-1)k\}$. It is easy to observe that the set $W_{f_8} = \{\frac{5}{2}m^2s + sn(4m+2) + 2s + \frac{m}{2} + 2 + (2m^2-1)s + (2m-1),\ \frac{5}{2}m^2s + sn(4m+2) + 2s + \frac{m}{2} + 2 + (2m^2-1)s + 4m - 2,\ \frac{5}{2}m^2s + sn(4m+2) + 2s + \frac{m}{2} + 2 + (2m^2-1)s + 6m - 3, \dots, \frac{5}{2}m^2s + sn(4m+2) + 2s + \frac{m}{2} + 2 + (2m^2-1)s(n-1) + (2m-1)s\}$. It gives the desired proof. ∎

**Theorem 2.9**. *For $n \geq 2$, the graph sAmal$(F_n, P_n, 2)$ admits a super* $(12sn + 16s + 5, 1)$-*$B_2$-antimagic total labeling.*

**Proof.** Define the vertex and edge labeling $f_9$ as follows:

$$f_9(a^j) = s - j + 1, for\ 1 \leq j \leq s$$
$$f_9(b^j) = s + j, for\ 1 \leq j \leq s$$
$$f_9(x_i{}^j) = si + s + j, for\ 1 \leq i \leq n\,, 1 \leq j \leq s$$
$$f_9(a^j x_i{}^j) = 2sn + 3s - si - j + 1, for\ 1 \leq i \leq n, 1 \leq j \leq s$$
$$f_9(b^j x_i{}^j) = si + 2sn + s + j, for\ 1 \leq i \leq n, 1 \leq j \leq s$$
$$f_9(x_i{}^j x_{i+1}{}^j) = 4sn - si + 2s - j + 1, for\ 1 \leq i \leq n-1, 1 \leq j \leq s$$

The vertex and edge labelings $f_9$ are a bijective function $f_9\colon V\,(sAmal(F_n, P_n, 2)) \cup E(sAmal(F_n, P_n, 2)) \rightarrow \{1, 2, 3, \dots, 4sn + s\}$. The $H$-weights of sAmal$(F_n, P_n, 2)$, for $1 \leq i \leq n$ and $1 \leq j \leq s$ under the labeling $f_9$, constitute the following sets $w_{f_9} = f_9(a^j) + f_9(b^j) + f_9(x_i{}^j) + f_9(x_{i+1}{}^j) = 5s + 2j + 2si + 1$, and the total $H$-weights of sAmal$(F_n, P_n,$

2) constitute the following sets $W_{f_9} = w_{f_9} + f_9(a^j x_i{}^j) + f_9(a^j x_{i+1}{}^j) + f_9(b^j x_i{}^j) + f_9(b^j x_{i+1}{}^j) + f_9(x_i{}^j x_{i+1}{}^j) = 15s + j + si + 4 + 12sn$. It is easy to observe that the set $Wf_9$ = $\{12sn + 16s + 5, 12sn + 16s + 6, \dots, 13sn + 16s + 4\}$. Therefore, the graph $sAmal(F_n, P_n, 2)$ admits a super $(12sn + 16s + 5, 1) - B_2$- antimagic total labeling for $m, n \geq 2$ It gives the desired proof. ■

**Theorem 2.10**. *For n ≥2, the graph sAmal(*$F_n$*,* $P_n$*, 2) admits a super* $(11sn + 17s + 6, 3)$-*$B_2$-antimagic total labeling.*

**Proof.** Define the vertex and edge labeling $f_{10}$ as follows:

$$f_{10}(a^j) = s - j + 1, for\ 1 \leq j \leq s$$
$$f_{10}(b^j) = s + j, for\ 1 \leq j \leq s$$
$$f_{10}(x_i{}^j) = si + s + j, for\ 1 \leq i \leq n, 1 \leq j \leq s$$
$$f_{10}(a^j x_i{}^j) = 2sn + 3s - si - j + 1, for\ 1 \leq i \leq n, 1 \leq j \leq s$$
$$f_{10}(b^j x_i{}^j) = si + 2sn + s + j, for\ 1 \leq i \leq n, 1 \leq j \leq s$$
$$f_{10}(x_i{}^j x_{i+1}{}^j) = si + s + 3sn + j, for\ 1 \leq i \leq n - 1, 1 \leq j \leq s$$

The vertex and edge labelings $f_9$ are a bijective function $f_{10}: V\ (sAmal(F_n, P_n, 2)) \cup E(sAmal(F_n, P_n, 2)) \to \{1, 2, 3, \dots, 4sn + s\}$. The $H$-weights of $sAmal(F_n, P_n, 2)$, for $1 \leq i \leq n$ and $1 \leq j \leq s$ under the labeling $f_{10}$, constitute the following sets $w_{f_{10}} = f_{10}(a^j) + f_{10}(b^j) + f_{10}(x_i{}^j) + f_{10}(x_{i+1}{}^j) = 5s + 2j + 2si + 1$, and the total $H$-weights of $sAmal(F_n, P_n, 2)$ constitute the following sets $W_{f_{10}} = w_{f_{10}} + f_{10}(a^j x_i{}^j) + f_{10}(a^j x_{i+1}{}^j) + f_{10}(b^j x_i{}^j) + f_{10}(b^j x_{i+1}{}^j) + f_{10}(x_i{}^j x_{i+1}{}^j) = 3mi + 14m + 3j + 3 + 11sn$. It is easy to observe that the set $Wf_{10}$ = $\{11sn + 17s + 6, 11sn + 17s + 9, \dots, 14sn + 17s + 3\}$. Therefore, the graph $sAmal(F_n, P_n, 2)$ admits a super $(11sn + 17s + 6, 3) - B_2$ -antimagic total labeling for $m, n \geq 2$ It gives the desired proof.

## CONCLUSIONS

In this paper, the result show that super (*a, d*)-$B_m$-antimagic total labeling of $Amal(F_n$, $P_n$, $m)$ and $sAmal(F_n$, $P_n$, $m)$ for some feasible $d$ are respectively $d \in \{2m + 1, 2m + 3, 2m^3 + 3\}$ and $d \in \{2m + 3, 2m + 1, 2m - 1\}$. Apart from obtained $d$ above, we haven't found any result yet, so we propose the following open problem:

*Let sG = sAmal(*$F_n$*,* $P_n$*, m) , for m, n ≥2, s ≥ 2,* and *s odd, does sG admit a super (a, d)-*$B_m$*-antimagic total labeling for feasible d?*

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