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On (local) metric dimension of graphs with *m*-pendant points

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ABSTRACT. An ordered set of vertices *S* is called as a (local) resolving set of a connected graph $G=(V_G,E_G)$ if for any two adjacent vertices $s\neq t\in V_G$ have distinct representation with respect to *S*, that is $r(s\mid S)\neq r(t\mid S)$. A representation of a vertex in *G* is a vector of distances to vertices in *S*. The minimum (local) resolving set for *G* is called as a (local) basis of *G*. A (local) metric dimension for *G* denoted by dim(*G*), is the cardinality of vertices in a *basis* for *G*, and its local variant by $\dim_l(G)$.

Given two graphs, *G* with vertices $s_1, s_2, ..., s_p$ and edges $e_1, e_2, ..., e_q$, and *H*. An edge-corona of *G* and *H*, $G\Diamond H$ is defined as a graph obtained by taking a copy of *G* and *q* copies of *H* and for each edge $e_j = s_i s_h$ of *G* joining edges between the two end-vertices s_i, s_h of e_j and each vertex of *j*-copy of *H*.

In this paper, we determine and compare between the metric dimension of graphs with *m*-pendant points, $G\Diamond mK_1$, and its local variant for any connected graph *G*. We give an upper bound of the dimensions.

1. Introduction

Metric dimension is an attractive parameter in Graph Theory. This is shown by many applications and research results about it, in the last two decade recently. One of them is related to the structure of chemistry compound, such that there are some researchs by using notion in Chemistry such as: metric dimension of corona graph [5, 10] and its local variant [1, 6, 7, 8].

The development of the last result is local metric dimension of edge-corona graphs [9], that is for operation of graphs, *G* and *H*, with *H* of order greater than equal to two. It is important to motivate researcher in Graph Theory to produce the metric dimension of graphs.

Although edge-corona graph has appeared in [4] for case spectrum of graphs, at that time there is no result about metric dimension of this graph. Moreover, specially for the other variant.

In this paper, we go on our research about (local) metric dimension edge-corona graph *G* and *H*, with *H* of order one. We call this graph as graph with pendant points. We determine and compare between the metric dimension of graphs with *m*-pendant points and its local variant for any connected graph *G*. We give an upper bound of the dimensions.

2. Material and method

In this paper, we use a simple, finite and connected graph $G=(V_G,E_G)$, with V_G is a set of vertices in *G* and V_G is a set of edges in *G*. An ordered set of vertices $S=\{s_1,s_2,...,s_k\}\subset V_G$ is called a resolving set of G if for every two distinct vertices $u, v \in V_G$ hold $r(u \mid S) \neq r(v \mid S)$. The representation of a vertex $x \in V_G$ with respect to S, denoted by $r(x \mid S)$, is defined as $r(x \mid S) = (d(x, s_1), d(x, s_2), \ldots, d(x, s_k))$, where $d(x, s_i)$ is the distance between vertices $x, s_i \in V_G$. If $u, v \in V_G$ are adjacent then S is called a local resolving set of G. A minimal (local) resolving set of G is a (local) basis of G and its cardinality, $\dim_l(G)$, called by a (local) metric dimension of G.

We use the following results that have been obtained from some researchs before.

Theorem 2.1. [3] *Let G be a connected graph of order* $n \geq 2$.
(*i*). $\dim(G) = 1$ *if only if* $G = P_n$.
(*ii*). $\dim(G) = n - 1$ *if only if* $G = K_n$.
(*iii*). *For* $n \geq 4$, $\dim(G) = n - 2$ *if only if* $G = K_{r,s}$ $(r, s \geq 1)$, $G = K_r + \overline{K_s}$ $(r \geq 1, s \geq 2)$ *or* $G = K_r + (K_1 \cup K_s)$, $(r,s \geq 1)$.
(*iv*). *For* $n \geq 3$, $\dim(C_n) = 2$.

Theorem 2.2.[6] *Let G be a connected graph of order* $n \geq 2$ *and diameter k, then*
(*i*). $\dim_l(G) = 1$ *if only if G is a bipartite graph..*
(*ii*). $\dim_l(G) = n - 1$ *if only if* $G = K_n$.

Theorem 2.3.[9] *For any connected-graph which is not an incomplete-bipartite graph G of order* $n_1 \geq 3$ *with* $|E(G)| = m$ *and any graph complement of complete graph* $H = \overline{K_{n_2}}$ *hold* $\dim_l(G \lozenge H) = \dim_l(G)$.

In this study, we determine and compare the metric dimension of graphs with pendant points and their local variant. We use a simple, finite and connected graph $G = (V_G, E_G)$, with V_G is a set of vertices in G and V_G is a set of edges in G. We start with basic concepts and notations. Notation $u \sim v$, we mean that u adjacent to v.

Given two graphs, G with vertices $s_1, s_2, \ldots, s_p$ and edges $e_1, e_2, \ldots, e_q$, and H. An edge-corona $G \lozenge H$ graph of G and H graphs, is defined as a graph obtained by taking a copy of G and q copies of H and for each edge $e_j = s_i s_h$ of G joining edges between the two end-vertices s_i, s_h of e_j and each vertex of j-copy of H [4]. If we x take $H \cong mK_1, m \geq 1$, we called a $G \lozenge H$ graph as graph with pendant points. The set of vertices of G will denoted by $\{x_1, x_2, \ldots, x_p\}$ and the vertex of a trivial graph K_1 (that is a pendant point) will be denoted by y_j such that $y \sim x_i$ and $y \sim x_h$ too, with $e_j = x_i x_h$, $i \neq h \in \{1, 2, \ldots, p\}$, $j \in \{1, 2, \ldots, q\}$.

3. Main Results

In this part, we want to determine the metric dimension of $G \lozenge H$ for $H \cong K_1$. We determine the exact values of $\dim(G \lozenge K_1)$ for some simple connected graphs G. We obtain general bound of $\dim(G \lozenge mK_1)$.

3.1. The Value of $\dim(G \lozenge mK_1)$

Here, as G, we use P_n, C_n, K_n, S_n, and $K_{r,s}; r + s = n, r \geq 2, s \geq 1$.

3.1.1. The exact value of $\dim(G \lozenge mK_1)$ when G is a path graph

Let P_p is a path of order p and measure $q = p-1$, and K_1 is a trivial graph. A G graph, is an edge-corona graph of P_p and K_1 denoted by $G \cong P_p \lozenge K_1$, has q copies of a trivial graph K_1. For a simple way, a copy of a j^{th} trivial graph K_1 denoted by $H_j = y_j$ for some $j \in \{1,2,\ldots,q\}$. An edge-corona graph $G \cong P_p \Diamond K_1$ has a set of vertices $V_G = \{x_1, x_2, ..., x_p\} \cup \{y_1, y_2, ..., y_q\}$, and a set of edges $E_G = \{x_i x_{i+1} | 1 \le i \le p\text{-}1\} \cup \{x_i y_j, x_{i+1} y_j \,|1 \le i{=}j \le p\text{-}1\}$. We consider $\langle e_j \rangle \Diamond H_j \subset P_p \Diamond K_1$. The metric dimension of $P_p \Diamond K_1$, with $p \ge 3$ given by the following Theorem 3.1.1.

Theorem 3.1.1. *Given a path of order* $p = |V_{P_p}|$ *with measure* $q = |E_{P_p}|$, *and a trivial graph* K_1.*For* $p \ge 3$, *the metric dimension of edge-corona of* P_p *and* K_1, $\dim(P_p \Diamond K_1)$, *is two.*

Proof. 1). According to Theorem 2.1., because $G \cong P_p \Diamond K_1$ is not a path then $\dim(G) \ge 2$.
2). We will show $\dim(G) \le 2$.

Choose a set $W = \{y_1, y_q\} \subset V_G$ and a vertex $v \in V_G$. The representation of all vertices in $V_G \backslash \{y_1, y_q\}$ are:

$r(x_1|W) = (1, p\text{-}1)$
$r(x_p|W) = (p\text{-}1, 1)$
$r(x_i|W) = (i\text{-}1, p\text{-}i)$; $2 \le i \le p\text{-}1$
$r(y_h|W) = (h, q\text{-}h\text{+}1)$; $2 \le h \le q\text{-}1$

We can show that all of the representation above are distinct. For example, $(s\text{-}1, p\text{-}s) = (t, q\text{-}t\text{+}1)$ for some s and t fixed. Then $s = t\text{+}1$ and $s = n\text{-}q\text{+}t\text{-}1$, a contradiction. So W is a resolving set of G with minimal cardinality. Hence $\dim(P_p \Diamond K_1) = 2$. □

3.1.2. The exact value of $\dim(G \Diamond mK_1)$ when G is a cycle graph

Given a cycle C_p of order p and measure $q{=}p$, and a trivial graph K_1. A graph G, is an edge-corona graph of C_p and K_1 denoted by $G \cong C_p \Diamond K_1$, has q copies of a trivial graph K_1. For a simplification, a copy of a j^{th} trivial graph denoted by $H_j{=}y_j$ for some $j \in \{1,2,\ldots,q\}$.An Edge-corona graph $G \cong C_p \Diamond K_1$ has a set of vertices $V_G = \{x_1, x_2, ..., x_p\} \cup \{y_1, y_2, ..., y_p\}$and a set of edges $E_G{=}\{x_i x_{i+1} | 1 \le i \le p\text{-}1, x_p x_1\} \cup \{x_i y_h, x_{i+1} y_h \,|1 \le i \le n\text{-}1, 1 \le h \le n\}$. We consider $\langle e_j \rangle \Diamond H_j \subset C_p \Diamond K_1$. The metric dimension of $C_p \Diamond K_1$, with $p \ge 4$ given by the following Theorem 3.1.2.

Teorema 3.1.2. *Given a cycle* C_p *of order* p *and measure* q *and a trivial graph* K_1. *For* $p \ge 4$, *the metric dimension of an edge-corona graph of* C_p *and* K_1,

$$\dim(C_p \Diamond K_1) = \begin{cases} 2 & , \text{ for even } p \\ 3 & , \text{ for odd } p. \end{cases}$$

Proof. a). When $p = 2k$ for $k \in N$.
1). According to Theorem 2.1., because $G \cong C_p \Diamond K_1$ is not a path then $\dim(G) \ge 2$.
2). We will show $\dim(G) \le 2$.

Choose a set $W = \{y_1, y_k\} \subset V_G$ and a vertex $v \in V_G$. Representations of all vertices in $V_G \backslash \{y_1, y_q\}$ are:

$$r(x_i \mid W) = \begin{cases} (1,k) & ,i=1 \\ (i-1,k-i+1) & ,2 \le i \le k \\ (k,1) & ,i=k+1 \\ (p-i+2,i-k) & ,k+2 \le i \le p. \end{cases}$$

All of the above representations are distinct. We can check for some fixed s and t as follows (s-1, k-s+1) = (p-t+2, t-k). Then s-1 =p-t+2 and k-s+1= t-k, a contradiction. Thus, W is a resolving set for G and obviously from minimal cardinality. Hence, $\dim(C_p \Diamond K_1) = 2$. □

Since $K_n \Diamond K_1 \cong K_{n+1}$, then $\dim(K_n \Diamond K_1) = (n+1)-1 = n$ (Theorem 2.1.). Since $S_n \Diamond K_1 \cong f_n$ and $\dim(f_n) = n$ then $\dim(S_n \Diamond K_1) = n$. For $r=2, s=1$, $K_{2,1} \cong P_3$ so that $\dim(K_{2,1} \Diamond K_1) = 2$ (Theorem 3.1.1.). For $r=2, s=2$, $K_{2,2} \cong C_4$ so that $\dim(K_{2,2} \Diamond K_1) = 2$ (Theorema 3.1.2.). Moreover, we just need to show $\dim(K_{r,s} \Diamond K_1)$ for $r \ge 2, s \ge 3$, similar to the proof in the following theorem.

3.1.3. The exact value of $\dim(G \Diamond mK_1)$ when G is a bipartite graph

Given a bipartite graph $K_{r,s}$, $r \ge 2, s \ge 3$, of order $p=r+s$ and measure q, and a trivial graph K_1. The set of vertices of $K_{r,s}$ is $V_{K_{r,s}} = V_1 \cup V_2 = \{x_{1i} \mid i=1,2,\dots,r\} \cup \{x_{2h} \mid h=1,2,\dots,s\}$ and the set of edges is $E_{K_{r,s}} = \{x_{1i}x_{2h} \mid i=1,2,\dots,r; h=1,2,\dots,s\}$. A G graph, is an edge-corona graph of $K_{r,s}$ and K_1 denoted by $G \cong K_{r,s} \Diamond K_1$, has q copies of a trivial graph K_1. As above, a copy of a j^{th} trivial graph denoted by $H_j = y_j = y_{ih}$, $i=1,2,\dots,r; h=1,2,\dots,s$. An Edge-corona graph $G \cong K_{r,s} \Diamond K_1$ has a set of vertices $V_G = V_{K_{r,s}} \cup \bigcup_{i=1}^{r} (\bigcup_{h=1}^{s} \{y_{ih}\})$ and a set of edges $E_G = E_{K_{r,s}} \bigcup \{x_i y_h, x_{i+1} y_h \mid 1 \le i \le p\text{-}1, 1 \le h \le p\}$. We consider $\langle e_j \rangle \Diamond H_j \subset K_{r,s} \Diamond K_1$. The metric dimension of $K_{r,s} \Diamond K_1$, with $p \ge 4$ given by the following Theorem 3.1.3.

Theorem 3.1.3. *Given a bipartite graph $K_{r,s}$ of order $p=r+s$ and measure q and a trivial graph K_1.For any integer r, s with $r \ge 2, s \ge 3$, the metric dimension of an edge-corona graph of $K_{r,s}$ and K_1,* $\dim(K_{r,s} \Diamond K_1) = p-2$.

The following results is the metric dimension of graphs with m-pendant points.

Theorem 3.1.4. *Given a connected graph G of order $p \ge 4$ and measure $q \ge 2$. Then,*

$$\dim(G \Diamond mK_1) = \begin{cases} mq-1 & ,\text{for } G \cong S_n \\ q(m-1) & ,\text{otherwise.} \end{cases}$$

It is mentioned in [2] that if we add a point at a graph G such that there are greater than one edge incident to this point then metric dimension of this graph maybe fixed, decrease or increase from metric dimension of G. From this statement and the results in Theorem 3.1.1. until Theorem 3.1.3., we give the following lemma.

Lemma 3.1.5. *If G' is a graph obtained by adding a pendant point to every edge of a connected graph G of order $p \ge 2$, then* $\dim(G) \le \dim(G') \le \dim(G)+1$.

Lemma 3.1.5. useful to obtain the following theorem.

Theorem 3.1.6. *For every connected graph G of order* $p \geq 3$ with $|E(G)| = q \geq 2$, $\dim(G\Diamond mK_1) \leq q(m-1)$.

The above results is an upper bound of $\dim(G\Diamond mK_1)$. Now, we determine the local metric dimension of $G\Diamond H$ for $H \cong mK_1$.

3.2. The Exact Value of $\dim_l(G\Diamond mK_1)$

As mentioned above, a graph G that we use in this paper are P_n, C_n, K_n, S_n and $K_{r,s}$, $r \geq 2, s \geq 1$. In this part, we determine the exact values of $\dim_l(G\Diamond mK_1)$ for some simple connected graphs G. By Theorem 2.3., we only show $\dim_l(G\Diamond mK_1)$ with G is a bipartite graph that is P_n, S_n and $K_{r,s}$, $r \geq 2, s \geq 1$.

3.2.1. The exact value of $\dim_l(G\Diamond mK_1)$ when *G* is a path graph

Let P_p be a path with vertex set $V_{P_p} = \{x_i \mid 1 \leq i \leq p, i \in N\}$ and edge set $E_{P_p} = \{x_i x_h \mid 1 \leq i \leq p-1, h = i+1; i \in N\}$ and K_1 is a trivial graph. A graph $G \cong P_p\Diamond mK_1$, has mq copies of a trivial graph K_1. We denote by y_{jk}, the pendant point of edge $e_j = x_i x_h$, $1 \leq i = j \leq p\text{-}1; h = i+1$. Here, a copy of j^{th}-mK_1 denoted by $H_j = (mK_1)_j = \bigcup_{k=1}^{m}\{y_{jk}\}$ for some $j \in \{1,2,\ldots,q\}$. A graph $G \cong P_p\Diamond mK_1$ has a set of vertices $V_G = V_{P_p} \cup \bigcup_{k=1}^{m} y_{jk}$ and a set of edges $E_G = E_{P_p} \cup \{x_i y_{jk}, x_{i+1} y_{ik} \mid 1 \leq i = j \leq p-1, 1 \leq k \leq m\}$. We prove the following theorem.

Theorem 3.2.1. *Let P_p be a path graph of order $p \geq 4$, and K_1 is a trivial graph. For all integer $m \geq 1$,* $\dim_l(P_p\Diamond mK_1) = 2$.

Proof. Let $G \cong P_p\Diamond mK_1$. Since G is not a bipartite graph, then by Theorem 2.3., we obtain $\dim_l(G) \geq 2$. Moreover, we just need to show $\dim_l(G) \leq 2$. We will show that $W = \{x_1, x_m\}$ is a resolving set of G.The representation of all vertices in $V_G - W$ are

$$r(x_i \mid W) = (i-1, p-i), \quad 2 \leq i \leq p-2$$

$$r(y_{jk} \mid W) = (j, p-j), \quad 1 \leq j \leq p-1; \quad 1 \leq k \leq m.$$

All the above representations are distinct. Suppose that $(s-1, p-s) = (t, p-t)$ for some fixed s and t. Then $s = t+1$ and $s = t$, a contradiction. Thus, representation of any two adjacent vertices are distinct, too. Therefore, W is a local resolving set of G. Obviously cardinality of W is minimal. So, $\dim_l(G) \leq 2$. □

3.2.2. The exact value of $\dim_l(G\Diamond mK_1)$ when *G* is a star graph

Let S_p be a star graph with vertex set $V_{S_p} = \{c, x_i \mid 1 \leq i \leq p, i \in N\}$ and edge set $E_{S_p} = \{cx_i \mid 1 \leq i \leq p, i \in N\}$ and K_1 is a trivial graph. A graph $G \cong S_p\Diamond pK_1$, has mp copies of a trivial graph K_1. We denote by y_{jk}, the pendant point of edge $e_i = cx_i$, $1 \leq i \leq p$. We denote a copy of i^{th}-mK_1 by $H_i = (mK_1)_i = \bigcup_{k=1}^{m}\{y_{ik}\}$ for some $i \in \{1,2,\ldots,p\}$. A graph $G \cong S_p\Diamond mK_1$ has a set of vertices $V_G = V_{S_p} \cup$

$\bigcup_{k=1}^{m} y_{ik}$, $1 \le i \le p$ and a set of edges $E_G = E_{S_p} \cup \{cy_{ik}, x_i y_{ik} \mid 1 \le i \le p,\ 1 \le k \le m\}$. The following is Theorem 3.2.2. and its proof.

Theorem 3.2.2. *Let S_p be a star graph of order $n \ge 4$, and K_1 is a trivial graph. For all integer* $m \ge 1$, $\dim_l(S_p \Diamond mK_1) = p-1$.

Proof. Let $G \cong S_p \Diamond mK_1$. By Theorem 2.2., since G is not a complete graph, then $\dim_l(G) \ge p-1$. We will show that $W = \{x_i \mid 1 \le i \le p, p \in N\}$ is a local resolving set of G.The representation of all vertices in $V_G - W$ are

$$r(c \mid W) = (1,1,\ldots,1)$$
$$r(y_{ik} \mid W) = (2,2,\ldots,\underbrace{1}_{i-term},2,\ldots,2),\quad 1 \le i \le p;\quad 1 \le k \le m.$$

From direct observation, we can see that all the above representations are distinct. Such that for any two adjacent vertices in G also have distinct representations. So, W is a local resolving set of G and $|W| \le p-1$.

Suppose that B is a local basis of G with $|B| < |W|$. Because H is a null graph, if there is $z \in B \cap V_{H_i}$, for some i, then the pairs of vertices of G which are distinguished by z can be distinguised also by x_i. Let the set B_1 obtained from B by replacing x_i by $z \in B \cap V_{H_i}$, with $i \in \{1,2,\ldots,p\}$. So, B_1 is a local resolving set of G and $|B_1| \le |B| < |W| = \dim_l(G)$, a contradiction. Thus, W is a local basis of G. □

3.2.3. The exact value of $\dim_l(G \Diamond mK_1)$ when G is a bipartite graph

Let $K_{r,s}, r \ge 2, s \ge 1$ be a bipartite graph of order $p = r+s$ and measure $q = r.s$ with vertex set $V_{K_{r,s}} = V_1 \cup V_2$, where $V_1 = \{x_i \mid 1 \le i \le r,\ r \in N\}$, $V_2 = \{y_h \mid 1 \le h \le s;\ s \in N\}$, and edge set $E_{K_{r,s}} = \{x_i y_h \mid 1 \le i \le r,\ 1 \le h \le s;\ i,h \in N\}$. K_1 is a trivial graph. A graph $G \cong K_{r,s} \Diamond mK_1$ has mq copies of a trivial graph K_1. We denote by $z_{i(h(k))}$, $1 \le k \le m$; the pendant point K_1 of edge $e_{i(h)} = x_i x_h$, For some $1 \le h \le s$; $1 \le i \le r$. We denote a copy of j^{th}-mK_1 by $H_j = ((mK_1)_h)_i = \bigcup_{k=1}^{m} \{z_{i(h(k))} = z_{jk}\}$ for some $j \in \{1,2,\ldots,q\}$, with $j = \begin{cases} h & ,i=1 \\ (i-1)r+h & ,i \ge 2. \end{cases}$ A graph $G \cong K_{r,s} \Diamond mK_1$ has a set of vertices $V_G = V_{K_{r,s}} \cup \bigcup_{k=1}^{m} z_{jk}$, $1 \le j \le q$ and a set of edges $E_G = E_{K_{r,s}} \cup \{x_{1i} z_{jk}, x_{2i} z_{jk} \mid 1 \le i \le r,\ 1 \le j \le q$ $1 \le k \le m\}$. The following is Theorem 3.2.3. and its proof.

Theorem 3.2.3. *Let $K_{r,s}$ with $r \ge 2$, $s \ge 1$, is a bipartite graph of order $n \ge 4$, and K_1 is a trivial graph. For any integer $m \ge 1$,* $\dim_l(K_{r,s} \Diamond mK_1) = \max\{r,s\}$.

Proof. Let $G \cong K_{r,s} \Diamond mK_1$. We suppose that $s \ge t$. We will prove that $W = \{z_{1(h(1))} \mid 1 \le h \le s\}$ is a local resolving set of G.The representation of all vertices in $V_G - W$ are

$$r(x_i \mid W) = \begin{cases} (1,1,\ldots,1) & ,i=1 \\ (2,2,\ldots,2) & ,2 \le i \le r. \end{cases}$$

$$r(y_h \mid W) = (2,2,\ldots,\underbrace{1}_{h-term},2,\ldots,2),\quad 1 \le h \le s.$$

$$r(z_{1(h(k))}) \mid W) = (2,2,\ldots,2),\ 1 \le h \le s;\ 2 \le k \le m.$$

$$r(z_{i(h(k))}) \mid W) = (3,3,\ldots,\underbrace{2}_{h-term},3,3,\ldots,3),\ 2 \le i \le r;\ 1 \le h \le s;\ 1 \le k \le m.$$

Among pendant points $z_{i(h(k))}$ are not adjacent. For every $2 \le j \ne l \le r$, $x_j \not\sim x_l$, and for every $1 \le j \ne l \le s$, $y_j \not\sim y_l$. So, for every pairs of adjacent vertices in G have distinct representations. Therefore, W is a local resolving set of G.

Suppose that A is a local basis of G with $|A| < |W|$. Because H is a null graph, if there is $v \in A \cap V_{H_j}$, for some j, then the pairs of vertices of G which are distinguished by v can be distinguised also by x_j or y_j. Let the set A_1 obtained from A by replacing x_j or y_j by $v \in A \cap V_{H_j}$, with $j \in \{1,2,\ldots,q\}$. So, A_1 is a local resolving set of G and $|A_1| \le |A| < |W| = \dim_l(G)$, a contradiction. Thus, W is a local basis of G. □

From the results that have disscused in part 3.1 and 3.2, we can conclude as follows.

Theorem 3.2.4. *Let G be a connected graph of order $p \ge 4$. For any integer $m \ge 1$,*

$$\dim_l(G \Diamond mK_1) \le \dim(G \Diamond mK_1).$$

4. Conclussions.

We have discussed the (local) metric dimension of edge-corona of some connected graphs G and null graph $\overline{K_m}$ or mK_1 that is a graph with pendant points. As metric dimension, if we add a point at a graph G such that there are greater than one edge incident to this point, then the local metric dimension of this graph can be fixed, decrease or increase from local metric dimension of G and $\dim_l(G \Diamond mK_1) \le \dim(G \Diamond mK_1)$.

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