Diregularity of digraphs of out-degree three and order two less than Moore bound

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Abstract. It is easy to show that any digraph with out-degree at most $d \geq 2$, diameter $k \geq 2$ and order $n = d+d^2+\ldots+d^k-1$, that is, two less than Moore bound must have all vertices of out-degree d. In other words, the out-degree of the digraph is constant $(= d)$. However, establishing the diregularity or otherwise of the in-degree of such a digraph is not easy. It was proved that every digraph of out-degree at most two, diameter $k \geq 3$ and order two less than the Moore bound is diregular [8]. In this paper, we consider the diregularity of digraphs of out-degree at most three, diameter $k \geq 3$ and order two less than the Moore bound.

1 Introduction and preliminaries

A *directed graph* or *digraph* G is a pair of sets (V, A) where V is a finite nonempty set of distinct elements called *vertices*; and A is a set of ordered pairs (u, v) of distinct vertices $u, v \in V$ called *arcs*.

The *order* n of a digraph G is the number of vertices in G, that is, $n = |V|$. An *in-neighbour* (respectively *out-neighbour*) of a vertex v in G is a vertex u (respectively w) such that $(u, v) \in A$ (respectively $(v, w) \in A$). The set of all in-neighbours (respectively out-neighbours) of a vertex v is denoted by $N^-(v)$ (respectively $N^+(v)$). The *in-degree* (respectively *out-degree*) of a vertex v is the number of its in-neighbours (respectively out-neighbours). We denote by $d^-(v)$ the in-degree of v in G. If the in-degree equals the out-degree $(= d$, say) for every vertex in G, then G is called a *diregular* digraph of degree d.

A $v_0 - v_l$ *walk* of length l in a digraph G is an alternating sequence $v_0 a_1 v_1 a_2 ... a_l v_l$ of vertices and arcs in G such that $a_i = (v_{i-1}, v_i)$ for each i, $1 \leq i \leq l$. A walk is closed if $v_0 = v_l$. If all the vertices of a $v_0 - v_l$ walk are distinct, then such a walk is called a *path*. A *cycle* is a closed walk with all vertices and edges are distinct (except the first and the last vertices).

The *distance* from vertex u to v, denoted by $\delta(u, v)$, is the length of the shortest path from vertex u to vertex v. Note that in general $\delta(u, v)$ is not necessarily