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On the degrees of a strongly vertex-magic graph

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Abstract

Let $G=(V,E)$ be a finite graph, where $|V|=n\geqslant 2$ and $|E|=e\geqslant 1$. A vertex-magic total labeling is a bijection λ from $V\cup E$ to the set of consecutive integers $\{1,2,\ldots,n+e\}$ with the property that for every $v\in V$, $\lambda(v)+\sum_{w\in N(v)}\lambda(vw)=h$ for some constant h. Such a labeling is strong if $\lambda(V)=\{1,2,\ldots,n\}$. In this paper, we prove first that the minimum degree of a strongly vertex-magic graph is at least two. Next, we show that if $2e\geqslant\sqrt{10n^2-6n+1}$, then the minimum degree of a strongly vertex-magic graph is at least three. Further, we obtain upper and lower bounds of any vertex degree in terms of n and e. As a consequence we show that a strongly vertex-magic graph is maximally edge-connected and hamiltonian if the number of edges is large enough. Finally, we prove that semi-regular bipartite graphs are not strongly vertex-magic graphs, and we provide strongly vertex-magic total labeling of certain families of circulant graphs.

Keywords: Graph; Labeling; Supervertex-magic; Degree

1. Introduction

All graphs considered in this paper are finite, simple and undirected. The graph G has vertex set $V=V(G)$ and edge set $E=E(G)$ and we let $n=|V|$ and $e=|E|$. Throughout the paper we will assume that $e\geqslant 1$. The degree of a vertex v is the number of edges that have v as an endpoint and the set of neighbors of v is denoted by $N(v)$.

A one-to-one map $\lambda : V\cup E\rightarrow\{1,2,\ldots,n+e\}$ is a *vertex-magic total labeling* of G if there is a constant h so that for every vertex x

$$w_\lambda(x)=\lambda(x)+\sum_{y\in N(x)}\lambda(xy)=h.$$

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