Exclusive Sum Labeling of Graphs

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Abstract. A graph $G(V, E)$ is called a *sum graph* if there is an injective labeling called *sum labeling* L from V to a set of distinct positive integers S such that $xy \in E$ if and only if there is a vertex w in V such that $L(w) = L(x) + L(y) \in S$. In such a case w is called a *working vertex*. Every graph can be made into a sum graph by adding some isolated vertices, if necessary. The smallest number of isolated vertices that need to be added to a graph H to obtain a sum graph is called the *sum number* of H; it is denoted by $\sigma(H)$. A sum labeling which realizes $H \cup \overline{K_{\sigma(G)}}$ as a sum graph is called an *optimal sum labeling* of H.

Sum graph labeling offers a new method for defining graphs and for storing them digitally. Traditionally, a graph is defined as a set of vertices and a set of edges, specified by pairs of vertices which are the endpoints of an edge. To record a graph on a computer, the edges are usually stored either in the form of an adjacency matrix or as a linked list. Using sum graph labeling we only need to store the set of vertices, together with some additional isolates, if needed. While previously the edges in a graph were specified explicitly, using sum graphs, edges can be specified implicitly.
A sum labeling L is called an exclusive sum labeling with respect to a subgraph H of G if L is a sum labeling of G where H contains no working vertex. The *exclusive sum number* $\epsilon(H)$ of a graph H is the smallest number r such that there exists an exclusive sum labeling L which realizes $H \cup \overline{K_r}$ as a sum graph. A labeling L is an *optimal exclusive sum labeling* of a graph H if L is a sum labeling of $H \cup K_{\epsilon(H)}$ and H contains no working vertex. While the exclusive sum number is never smaller than the corresponding sum number of a graph, labeling graphs exclusively has other desirable features which give greater scope for combining two or more labeled graphs.

JCMCC **55** (2005), pp. 137-148