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On the Domination Number of Some Graph Operations

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Abstract

A set D of vertices of a simple graph G , that is a graph without loops and multiple edges, is called a dominating set if every vertex $u \in V(G) - D$ is adjacent to some vertex $v \in D$. The domination number of a graph G , denoted by $\gamma(G)$, is the order of a smallest dominating set of G . A dominating set D with $|D| = \gamma(G)$ is called a minimum dominating set. This research aims to characterize the domination number of some graph operations, namely joint graphs, coronation of graphs, graph compositions, tensor product of two graphs, and graph amalgamation. The results shows that most of the resulting domination numbers attain the given lower bound of $\gamma(G)$.

Keywords: Dominating set, domination number, graph operations.

Introduction

The idea of domination in graphs arose in chessboard problems. In 1862, de Jaenisch posed the problem of finding the minimum number of mutually non-attacking queens that can be placed on a chessboard so that each square of the chessboard is attacked by at least one of the queens. A graph G may be formed by an 8×8 chessboard by taking the squares as the vertices with two vertices adjacent if a queen situated on one square attacks the other square. This problem can be modeled by finding a dominating set of five queens. Inspired by this problem, dominating set are now studied either for theoretical means or for applications.

All graphs in this paper are finite, undirected, and simple. For a graph G , $V(G)$ and $E(G)$ denote the vertex-set and the edge-set, respectively. The order and size of graph, denoted by $|V(G)| = p$ and $|E(G)| = q$, are respectively number of vertices and edges. We refer the reader to [3] for all other terms and notations which are not provided in this paper. Haynes, Hedetniemi and Slater [5] defines that a set D

of vertices of graph G is called a dominating set if every vertex $u \in V(G) - D(G)$ is adjacent to some vertex $v \in D$. The domination number of a graph G , denoted by $\gamma(G)$, is the order of a smallest dominating set of G . A dominating set D with $|D| = \gamma(G)$ is called a minimum dominating set.

Cockayne *et al.* [6] proved that if G is a connected graph of order $n \geq 3$ then $\gamma(G) = 2n/3$. Haynes and Henning, in [4], showed that $\gamma(C_n) = \frac{n}{3}$, $n \equiv 0 \pmod{3}$ and $n \geq 6$ and $\gamma(C_n) = \lceil \frac{n}{3} \rceil + 1$, n otherwise. Furthermore Murugesan proved that let $G = (V, E)$ be a simple graph of order n , if G has a vertex v of degree $n - 1$ then $\gamma(G) = 1$.

This research aims to characterize the domination number of some graph operations, namely joint graphs, coronation of graphs, graph compositions, tensor product of two graphs, and graph amalgamation. The results shows that most of the resulting domination numbers attain the given lower bound of $\gamma(G)$.

A Useful Lemma

The following theorem gives a feasible $\gamma(G)$ for any graph, proved by Haynes, Hedetniemi and Slater in [4]. The theorem is very useful, therefore we reproof it for a hint to prove the resulting theorems of this research.

Theorem 1 [4] *Let $\gamma(G)$ be a domination number. For any graph G of order p , size q and maximum degrees $\Delta(G)$ will satisfies the following:*

$$\lceil \frac{p}{1+\Delta(G)} \rceil \leq \gamma(G) \leq p - \Delta(G)$$

Proof: Let S be a γ -set of G . first we consider the lower bound. Each vertex can dominate at most itself and $\Delta(G)$ other vertices. Hence, $\gamma(G) \geq \lceil \frac{p}{1+\Delta(G)} \rceil$. For the upper bound, let v be a vertex of maximum degree $\Delta(G)$ and $N[v]$ be an out-neighbor vertices of v . Then v dominates $N[v]$ and the vertices in $V - N[v]$ dominate themselves. Hence, $V - N[v]$ is the dominating set of cardinality $n - \Delta(G)$, so $\gamma(G) \leq n - \Delta(G)$. It implies that $\lceil \frac{p}{1+\Delta(G)} \rceil \leq \gamma(G) \leq p - \Delta(G)$. \square

The Results

The followings show $\gamma(G)$ when G are joint graphs $G = G_1 + G_2$, coronation of graphs $G = G_1 \odot G_2$, graph compositions $G = G_1[G_2]$. The results shows that most of the resulting domination numbers attain the given lower bound of $\gamma(G)$.

Theorem 2 *Let $n \geq 3$ and $m \geq 2$. The domination number of $C_n + Fl_m$ is $\gamma(C_n + Fl_m) = 1$.*

Proof. A joint graph $C_n + Fl_m$ is a connected with vertex set $V(C_n + Fl_m) = \{A, x_i, y_i; 1 \leq i \leq n\} \cup \{z_j; 1 \leq j \leq m\}$, and edge set $E(C_n + Fl_m) = \{Ax_i, Ay_i, x_i x_{i+1}, x_n x_1, x_i y_i; 1 \leq i \leq n\} \cup \{Az_j, x_i z_j, y_i z_j; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{z_j z_{j+1}, z_m z_1; 1 \leq j \leq m-1\}$. Thus $p = |V(C_n + Fl_m)| = 2n + m + 1$, $q = |E(C_n + Fl_m)| = 2nm + 4n + 2m$, and $\Delta(C_n + Fl_m) = 2n + m$. From Theorem 1, it follows that $\gamma(C_n + Fl_m) \geq \lceil \frac{m+2n+1}{m+2n+1} \rceil = 1$. We need to show that $\gamma(C_n + Fl_m) \leq 1$, by choosing a dominating set $D = \{A\}$. It obviously shows that every vertex in G is adjacent to $A \in D$, thus $(C_n + Fl_m) \leq |D| = 1$. It completes the proof. \square

Theorem 3 Let $n \geq 2$ and $m \geq 3$. The domination number of $F_n + L_m$ is $\gamma(F_n + L_m) = 1$.

Proof. A joint graph $F_n + L_m$ is a connected with vertex set $V(F_n + L_m) = \{x_i, y_j; 1 \leq i \leq n, 1 \leq j \leq m\}$, and edge set $E(F_n + L_m) = \{x_i x_{i+1}, x_n x_1, x_i y_j; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{y_j y_{j+1}; 1 \leq j \leq m\} \cup \{y_j y_{j+m}; 1 \leq j \leq m\}$. Thus $p = |V(F_n + L_m)| = n + 2m + 1$, $q = |E(F_n + L_m)| = 4nm + 2n + 4m$, and $\Delta(F_n + L_m) = n + 2m$. From Theorem 1, it follows that $\gamma(F_n + L_m) \geq \lceil \frac{n+2m+1}{n+2m+1} \rceil = 1$. We need to show that $\gamma(F_n + L_m) \leq 1$, by choosing a dominating set $D = \{x_1\}$. It obviously shows that every vertex in G is adjacent to $\{x_1 \in D\}$, thus $(F_n + L_m) \leq |D| = 1$. It completes the proof. \square

Theorem 4 Let $n \geq 3$ and $m \geq 2$. The domination number of $C_n[Fl_m]$ is $\gamma(C_n[Fl_m]) = \lceil \frac{n}{3} \rceil$.

Proof. A coronation of graphs $\lceil \frac{n}{3} \rceil$ is a connected with vertex set $V(C_n[Fl_m]) = \{x_{ij}; 1 \leq i \leq n, 1 \leq j \leq m\}$ and edge set $E(C_n[Fl_m]) = \{x_{ij} x_{ij+1}; 1 \leq i \leq n, 1 \leq j \leq m-1\} \cup \{x_{nm} x_{1j}; 1 \leq i \leq n, 1 \leq j \leq m-1\} \cup \{x_{ij} x_{i+1j+1}; 1 \leq i \leq n, 1 \leq j \leq m-1\} \cup \{x_{ij} x_{i-1j+1}; 2 \leq i \leq n-1, 1 \leq j \leq m\} \cup \{x_{ij} x_{i+nj+1}; 1 \leq i \leq n, 1 \leq j \leq m-1\} \cup \{x_{ij} x_{i+nj}; 1 \leq i \leq n, 1 \leq j \leq m-1\} \cup \{x_{ij} x_{i+1j}; 1 \leq i \leq n, 1 \leq j \leq m-1\}$. Thus $p = |V| = 2nm + n$, $q = |E(C_n \odot TL_m)| = 6mn - 2n$, and $\Delta C_n[Fl_m] = 6m + 2$. From Theorem 1, it follows that $\gamma(C_n[Fl_m]) \geq \lceil \frac{n}{3} \rceil$. We need to show that $\gamma(C_n[Fl_m]) \leq 9n - 26$, by choosing a dominating set $D = \{x_{1,i}, x_{1,i+1}; \text{dimana } i \text{ mod } 3\}$. It obviously shows that every vertex in G is adjacent to $\{x_{1,i}, x_{1,i+1} \in D\}$, thus $(C_n[Fl_m]) \leq |D| = \lceil \frac{n}{3} \rceil$. It completes the proof. \square

Theorem 5 Let $n \geq 2$ and $m \geq 3$. The domination number of $F_n \square P_m$ is $\gamma(F_n \square P_m) = \lceil \frac{5m}{6} \rceil$.

Proof. A coronation of graphs $F_n \square P_m$ is a connected with vertex set $V(F_n \square P_m) = \{x_{ij}; 1 \leq i \leq n; 1 \leq j \leq m\}$ and edge set $E(F_n \square P_m) = \{x_{ij} x_{ij+1}; 1 \leq i \leq n, 1 \leq$

$i \leq m\} \cup \{x_{ij}x_{i+1j}; 1 \leq i \leq n; 1 \leq j \leq m\} \cup \{x_{nj}x_{1j}; 1 \leq i \leq n; 1 \leq j \leq m-1\} \cup \{x_{nj}x_{n+j}; 1 \leq i \leq n, 1 \leq i \leq m\}$. Thus $p = |V(F_n \square P_m)| = nm + m$, $q = |E(F_n \square P_m)| = 3nm - n - 1$ and $\Delta F_n \square P_m = n + 1$. From Theorem 1, it follows that $\gamma(F_n \square P_m) \geq \lceil \frac{5m}{6} \rceil$. We need to show that $\gamma(F_n \square P_m) \leq 5m - 5$, by choosing a dominating set $\{D = x_{1,j}; \text{dimana } j \text{ adalah semua dari } m\}$. It obviously shows that every vertex in G is adjacent to $\{x_{1,j} \in D\}$, thus $\gamma(F_n \square P_m) \leq |D| = \lceil \frac{5m}{6} \rceil$. It completes the proof. \square

Theorem 6 *Let $n \geq 3$ and $r \geq 2$. The domination number of $Amal(F_n, v = A, r)$ is $\gamma Amal(F_n, v = A, r) = 1$.*

Proof. A composition of graphs $Amal(F_n, v = A, r)$ is a connected with vertex set $V(Amal(F_n, v = A, r)) = \{x_{i,j}; 1 \leq i \leq n, 1 \leq j \leq r\}$ and edge set $E(Amal(F_n, v = A, r)) = \{Ax_{i,j}; 1 \leq i \leq n, 1 \leq j \leq r\} \cup \{x_{i+1,j}; 1 \leq i \leq n, 1 \leq j \leq r\}$. Thus $|V(Amal(F_n, v = A, r))| = nr + 1$, $|E(Amal(F_n, v = A, r))| = 2nr - r$ and $\Delta(Amal(F_n, v = A, r)) = nr$. From Theorem 1, it follows that $\gamma Amal(F_n, v = A, r) \geq \lceil \frac{nr+1}{nr+1} \rceil$. We need to show that $\gamma Amal(F_n, v = A, r) \leq 1$, by choosing a dominating set $\{D = A\}$. It obviously shows that every vertex in G is adjacent to $\{A \in D\}$, thus $Amal(F_n, v = A, r) \leq |D| = 1$. It completes the proof. \square

Conclusions

We have studied the domination number of some families of special graphs of distance one, namely Flower graph Fl_n , Mountain graph $M_{2,n}$ and any regular graph. The results obtain that all the values of $\gamma(G)$ take a place in the interval

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