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On super edge-antimagicness of connected generalized shackle of cycle with two chords

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Abstract. Let G be a simple graph of order p and size q. The graph G is called an (a, d)-*edge-antimagic total graph* if there exist a bijection $f : V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p + q\}$ such that the edge-weights, $w(uv) = f(u) + f(v) + f(uv), uv \in E(G)$, form an arithmetic sequence with first term a and common difference d. Such a graph is called *super* if the smallest possible labels appear on the vertices. In this paper we study a super edge-antimagicness of connected generalized shackle of cycle of order five with two chords, denoted by $gshack(C_5^2, v \in C_3, n)$. The result shows that the graph $gshack(C_5^2, v \in C_3, n)$ admits a super (a, d)-edge antimagic total labeling for some feasible $d \leq 2$.

Key Words *: Super (a, d)-edge antimagic total labeling, generalized shackle, cycle of order five with two chords.*

Introduction

Mathematics plays an important role in science and technology advancement. One of the interesting topic in mathematics is a graph theory. There are many research interests in graph theory, one of them is labeling of graph. Graph labelings provide useful mathematical models for a wide range of applications, such as radar and communication network addressing systems and circuit design, various coding theory problems, cryptography, automata, x-ray crystallography and data security. More detailed discussions about the applications of graph labelings can be found in Bloom and Golomb's papers (5).

Let G be a finite, simple and undirected graph, by a labeling of a graph G of order p and size q, we mean any mapping that sends some set of graph elements to a set of positive integers. If the domain is a vertex-set $V(G)$ or a edge-set $E(G)$, the labelings are called respectively vertex labelings or edge labelings. Moreover, if the domain is $V(G) \cup E(G)$ then the labelings are called *total* labelings. We define the *edge-weight* $w(uv)$ of an edge $uv \in E(G)$ under a total labeling as the sum of the vertex labels corresponding to vertices u, v and edge label corresponding to edge uv. If such a labeling exists then G is said to be an (a, d)-*edge-antimagic total graph*. Such a graph G is called *super* if we

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assign all the smallest possible labels on the vertices. Thus, a *super* (a, d)*-edge-antimagic total graph* is a graph that admits a super (a, d)-edge-antimagic total labeling.

These labelings, introduced by Simanjuntak *at al.* in (15), are natural extensions of the concept of magic valuation, studied by Kotzig and Rosa (12) (see also (2),(11)), and the concept of super edge-magic labeling, defined by Enomoto *et al.* in (10). Many other researchers expand their investigation for different forms of antimagic labeling either for connected or disconnected graphs. For example, see Bodendiek and Walther (6) and Bača in (1), and (3). Dafik *at al.* in (7) and (9) also found the results of super (a, d)-edge-antimagic total labelings of disjoint union of stars, s-crowns and tripartite graphs.

In this paper we are studying a super edge-antimagicness of connected generalized shackle of cycle of order five with two chords, denoted by $gshack(C_5^2, v \in C_3, n)$.

Some Useful Lemmas

In this section, we recall two known lemmas and one theorem that will be useful in the next section. The first lemma, see (16), is a necessary condition for a graph to be super (a,d)-edge antimagic total, providing a least upper bound for feasible value of d.

Lemma 1 *If a* (p, q)*-graph is super* (a, d)*-edge antimagic total then* $d \leq \frac{2p+q-5}{q-1}$.

Proof. Assume that a (p, q)-graph has a super (a, d)-edge antimagic total labeling $f : V(G) \cup E(G) \to \{1, 2, \ldots, p + q\}$ and the edge-weights $\{a, a + d, a + 2d, \ldots, a + (q - 1)d\}$. The minimum possible edge-weight in the labeling f is at least $1 + 2 + p + 1 = p + 4$. Thus, $a \geq p + 4$. On the other hand, the maximum possible edge-weight is at most $(p - 1) + p + (p + q) = 3p + q - 1$. So we obtain $a + (q - 1)d \leq 3p + q - 1$ which gives the desired upper bound for difference d. □

The second lemma obtained by Figueroa-Centeno et al (11), gives a necessary and sufficient condition for a graph to be super $(a, 0)$-edge-antimagic total.

Lemma 2 A (p, q)-graph G is super edge-magic if and only if there exists a bijective function $f : V(G) \to \{1, 2, \ldots, p\}$ such that the set $S = \{f(u) + f(v) : uv \in E(G)\}$ consists of q consecutive integers. In such a case, f extends to a super edge-magic labeling of G with magic constant $a = p + q + s$, where $s = min(S)$ and $S = \{a - (p + 1), a - (p + 2), \ldots, a - (p + q)\}$.

Previously, the lemma states that a (p, q)-graph G is super $(a, 0)$-edge-antimagic total if and only if there exists an $(a - p - q; 1)$-edge-antimagic vertex labeling.

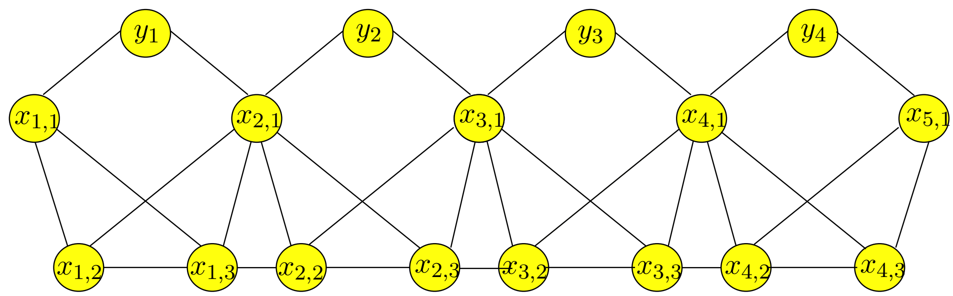

Figure 1: The graph $gshack(C_5^2, v \in C_3, 4)$

The Research Result

A generalized shackle of cycle of order five with two chords, denoted by $gshack$ $(C_5^2, v \in C_3, n)$, is a connected graph with vertex set $V = \{x_{ij}, y_i, 1 \le i \le n, 1 \le j \le 3\}$ and $E = \{x_{i1}x_{i2}, 1 \le i \le n\} \cup \{x_{i1}y_i, 1 \le i \le n\} \cup \{x_{i1}x_{i3}, 1 \le i \le n\} \cup \{x_{i2}x_{i3}, 1 \le i \le n\} \cup \{x_{i2}x_{(i+1)1}, 1 \le i \le n\} \cup \{x_{(i+1)1}y_i, 1 \le i \le n\} \cup \{x_{i3}x_{(i+1)1}, 1 \le i \le n\}$, see Figure 1. Thus $|V(gshack(C_5^2, v \in C_3, n))| = p = 4n + 1$ and $|E(gshack(C_5^2, v \in C_3, n))| = q = 8n - 1$.

If generalized shackle of cycle of order five with two chords has a super (a, d)-edge-antimagic total labeling then for $p = 4n + 1$ and $q = 8n - 1$, it follows from Lemma 1 the upper bound of d is $d \le 2$ or $d \in \{0, 1, 2\}$. Before describing the super antimagicness of total labeling of this graph, the study focuss on describing an $(a, 1)$-edge-antimagic vertex labeling. It can be found in Theorem 1.

◇ **Theorem 1** *The graph $gshack(C_5^2, v \in C_3, n)$ has an $(3, 1)$-edge-antimagic vertex labeling for $n \ge 1$*

Proof. Define the vertex labeling $f_1 : V(gshack(C_5^2, v \in C_3, n)) \rightarrow$

$\{1, 2, \ldots, 3n+2\}$ for $1 \le i \le n$ as follow:

$$f_1(x_{ij}) = \begin{cases} 4i + 3\frac{j+1}{2} - 6, & \text{for } 1 \le i \le n \ j = 1, 3 \\ 4i - 2, & \text{for } 1 \le i \le n \ j = 2 \end{cases}$$
$$f_1(y_i) = 4i - 1, \ 1 \le i \le n$$

The vertex labeling f_1 is a bijective function. The edge-weights of $gshack$ $(C_5^2, v \in C_3, n)$, for $1 \le i \le n$, under the labeling f_1, constitute the following sets:

$$\begin{aligned} w_{f_1}^1(x_{i1}x_{i2}) &= 8i - 5, \text{ for } 1 \le i \le n \\ w_{f_1}^2(x_{i1}y_i) &= 8i - 4, \text{ for } 1 \le i \le n \\ w_{f_1}^3(x_{i1}x_{i3}) &= 8i - 3, \text{ for } 1 \le i \le n \\ w_{f_1}^4(x_{i2}x_{i3}) &= 8i - 2, \text{ for } 1 \le i \le n \\ w_{f_1}^5(x_{i2}x_{(i+1)1}) &= 8i - 1, \text{ for } 1 \le i \le n \\ w_{f_1}^6(x_{(i+1)1}y_i) &= 8i, \text{ for } 1 \le i \le n \\ w_{f_1}^7(x_{i2}x_{(i+1)1}) &= 8i + 1, \text{ for } 1 \le i \le n \\ w_{f_1}^8(x_{i3}x_{(i+1)2}) &= 8i + 2, \text{ for } 1 \le i \le n - 1 \end{aligned}$$

The above formulas show that the set $\bigcup_{k=1}^{8} w_{f_1}^k$ $=\{3,, 4, 5 \ldots, 8n+1\}$ consists of consecutive integers. Hence α_1 is a $(3, 1)$-edge antimagic vertex labeling. It concludes the proof. □

With Theorem 1 in hand together with Lemma 2, we can establish the following theorem.

◇ **Theorem 2** *The graph $gshack(C_5^2, v \in C_3, n)$ admits a super $(12n+3, 0)$-edge-antimagic total labeling for $n \ge 1$.*

The next theorem deals with $d \in \{1, 2\}$. We start to state the theorem of $d = 2$.

◇ **Theorem 3** *The graph $gshack(C_5^2, v \in C_3, n)$ admits a super $(4n+5, 2)$-edge-antimagic total labeling for $n \ge 1$.*

Proof. Define the vertex labeling of the graph $gshack(C_5^2, v \in C_3, n)$: $f_2(x_{ij}) = f_1(x_{ij})$ and $f_2(y_i) = f_1(y_i)$ for $1 \le i \le n$, and also defined the edge labeling as follows:

$$\begin{aligned}
f_2(x_{i1}x_{i2}) &= 4n+8i-6, \text{ for } 1 \le i \le n\\
f_2(x_{i1}y_i) &= 4n+8i-5, \text{ for } 1 \le i \le n\\
f_2(x_{i1}x_{i3}) &= 4n+8i-4, \text{ for } 1 \le i \le n\\
f_2(x_{i2}x_{i3}) &= 4n+8i-3, \text{ for } 1 \le i \le n\\
f_2(x_{i2}x_{(i+1)1}) &= 4n+8i-2, \text{ for } 1 \le i \le n\\
f_2(x_{(i+1)1}y_i) &= 4n+8i-1, \text{ for } 1 \le i \le n\\
f_2(x_{i2}x_{(i+1)1}) &= 4n+8i, \text{ for } 1 \le i \le n\\
f_2(x_{i3}x_{(i+1)2}) &= 4n+8i+1, \text{ for } 1 \le i \le n-1
\end{aligned}$$

The total labeling f_2 is a bijective function from $V(gshack(C_5^2, v \in C_3, n)) \cup E(gshack(C_5^2, v \in C_3, n))$ to $\{1, 2, \ldots, p+q\}$. The edge-weights of $gshack(C_5^2, v \in C_3, n)$, can be given as follow:

$$\begin{aligned}
W^1_{f_2}(x_{i1}x_{i2}) &= w^1_{f_1}(x_{i1}x_{i2}) + f_2(x_{i1}x_{i2}) = 16i+4n-11, \text{ for } 1 \le i \le n\\
W^2_{f_2}(x_{i1}y_i) &= w^2_{f_1}(x_{i1}y_i) + f_2(x_{i1}y_i) = 16i+4n-9, \text{ for } 1 \le i \le n\\
W^3_{f_2}(x_{i1}x_{i3}) &= w^3_{f_1}(x_{i1}x_{i3}) + f_2(x_{i1}x_{i3}) = 16i+4n-7, \text{ for } 1 \le i \le n\\
W^4_{f_2}(x_{i2}x_{i3}) &= w^4_{f_1}(x_{i2}x_{i3}) + f_2(x_{i2}x_{i3}) = 16i+4n-5, \text{ for } 1 \le i \le n\\
W^5_{f_2}(x_{i2}x_{(i+1)1}) &= w^5_{f_1}(x_{i2}x_{(i+1)1}) + f_2(x_{i2}x_{(i+1)1}) = 16i+4n-3, \text{ for } 1 \le i \le n\\
W^6_{f_2}(x_{(i+1)1}y_i) &= w^6_{f_1}(x_{(i+1)1}y_i) + f_2(x_{(i+1)1}y_i) = 16i+4n-1, \text{ for } 1 \le i \le n\\
W^7_{f_2}(x_{i2}x_{(i+1)1}) &= w^7_{f_1}(x_{i2}x_{(i+1)1}) + f_2(x_{i2}x_{(i+1)1}) = 16i+4n+1, \text{ for } 1 \le i \le n\\
W^8_{f_2}(x_{i3}x_{(i+1)2}) &= w^8_{f_1}(x_{i3}x_{(i+1)2}) + f_2(x_{i3}x_{(i+1)2}) = 16i+4n+3, \text{ for } 1 \le i \le n-1
\end{aligned}$$

It is easy to understand that the set $\bigcup_{k=1}^{8} W^k_{f_2} = \{4n+5, \ldots, \}$. It implies that the graph $gshack(C_5^2, v \in C_3, n)$ admits a super $(4n+5, 2)$-edge-antimagic total labeling for $n \ge 1$. □

◇ **Theorem 4** *The graph* $gshack(C_5^2, v \in C_3, n)$ *admits a super* $(8n+4, 1)$*-edge-antimagic total labeling for* $n \ge 1$.

Proof. Define the vertex labeling of the graph $gshack(C_5^2, v \in C_3, n)$: $f_3(x_{ij}) = f_1(x_{ij})$ and $f_3(y_i) = f_1(y_i)$ for $1 \le i \le n$, and also defined the edge labeling as follows:

$$\begin{aligned}
f_3(x_{i1}x_{i2}) &= 8n-4i+5, \text{ for } 1 \le i \le n\\
f_3(x_{i1}y_i) &= 12n-4i+4, \text{ for } 1 \le i \le n\\
f_3(x_{i1}x_{i3}) &= 8n-4i+4, \text{ for } 1 \le i \le n\\
f_3(x_{i2}x_{i3}) &= 12n-4i+3, \text{ for } 1 \le i \le n\\
f_3(x_{i2}x_{(i+1)1}) &= 8n-4i+3, \text{ for } 1 \le i \le n\\
f_3(x_{(i+1)1}y_i) &= 12n-4i+2, \text{ for } 1 \le i \le n\\
f_3(x_{i2}x_{(i+1)1}) &= 8n-4i+2, \text{ for } 1 \le i \le n\\
f_3(x_{i3}x_{(i+1)2}) &= 12n-4i+1, \text{ for } 1 \le i \le n-1
\end{aligned}$$

The total labeling f_3 is a bijective function from $V(gshack(C_5^2, v \in C_3, n)) \cup E(gshack(C_5^2, v \in C_3, n))$ to $\{1, 2, \ldots, p+q\}$. The edge-weights of $gshack(C_5^2, v \in C_3, n)$, can be given as follow:

$$\begin{aligned}
W^1_{f_3}(x_{i1}x_{i2}) &= w^1_{f_1}(x_{i1}x_{i2}) + f_3(x_{i1}x_{i2}) = 8n+4i, \text{ for } 1 \le i \le n\\
W^2_{f_3}(x_{i1}y_i) &= w^2_{f_1}(x_{i1}y_i) + f_3(x_{i1}y_i) = 12n+4i, \text{ for } 1 \le i \le n\\
W^3_{f_3}(x_{i1}x_{i3}) &= w^3_{f_1}(x_{i1}x_{i3}) + f_3(x_{i1}x_{i3}) = 8n+4i+1, \text{ for } 1 \le i \le n\\
W^4_{f_3}(x_{i2}x_{i3}) &= w^4_{f_1}(x_{i2}x_{i3}) + f_3(x_{i2}x_{i3}) = 12n+4i+1, \text{ for } 1 \le i \le n\\
W^5_{f_3}(x_{i2}x_{(i+1)1}) &= w^5_{f_1}(x_{i2}x_{(i+1)1}) + f_3(x_{i2}x_{(i+1)1}) = 8n+4i+2, \text{ for } 1 \le i \le n\\
W^6_{f_3}(x_{(i+1)1}y_i) &= w^6_{f_1}(x_{(i+1)1}y_i) + f_3(x_{(i+1)1}y_i) = 12n+4i+2, \text{ for } 1 \le i \le n\\
W^7_{f_3}(x_{i2}x_{(i+1)1}) &= w^7_{f_1}(x_{i2}x_{(i+1)1}) + f_3(x_{i2}x_{(i+1)1}) = 8n+4i+3, \text{ for } 1 \le i \le n\\
W^8_{f_3}(x_{i3}x_{(i+1)2}) &= w^8_{f_1}(x_{i3}x_{(i+1)2}) + f_3(x_{i3}x_{(i+1)2}) = 12n+4i+3, \text{ for } 1 \le i \le n-1
\end{aligned}$$

It is easy to understand that the set $\bigcup_{k=1}^{8} W^k_{f_3}$ = $\{8n+4, \ldots, \}$. It implies that the graph $gshack(C_5^2, v \in C_3, n)$ admits a super $(8n+4, 1)$-edge-antimagic total labeling for $n \ge 1$. □

Conclusion

We have studied the existence of super antimagicness of the graph $gshack(C_5^2, v \in C_3, n)$. The result shows that the graph $gshack(C_5^2, v \in C_3, n)$ admits a super (a, d)-edge antimagic total labeling for some feasible $d \le 2$.

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