Antimagic Total Labeling of Disjoint Union of Disconnected Graphs

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Contents

List of Tables

List of Figures

List of Symbols

C_n	=	Cycle
mC_n	=	Disjoint union of cycles
P_n	=	Path
P_n	=	Disjoint union of paths
$mP_n \bigcup \mu C_n$	=	Disjoint union of path and cycle
$K_{1,m} \bigcup K_{1,n}$	=	Disjoint union of two stars
$K_{1,m} \bigcup 2sK_{1,n}$	=	Disjoint union of more than two stars
$mK_{n,n,n}$	=	Disjoint union of m copies of tripartite graph
$mK\underbrace{n, n, \ldots, n}_{s}$	=	Disjoint union of m copies of complete s-partite graphs
$m(C_n \odot \overline{K}_s)$	=	Disjoint union of m copies of s-crown
$mS_{t_1,t_2,\ldots,t_n}$	=	Disjoint union of m copies of caterpillars
mL_n	=	Disjoint union of triangular ladder
$kP_{(n,2)}$	=	Disjoint union of generalized petersen graph
$mB_{(n,k)}$	=	Disjoint union of banana tree
$mF_{(n,k)}$	=	Disjoint union of firecracker
$W_0(3, j, 2)$	=	Generalized web with two pendant
$sW_0(3, j, 2)$	=	Disjoint union of s copies of generalized web
E_n	=	Graph E
mE_n	=	Disjoint union of m copies of graph E
$m\pounds_{(i,j,k)}$	=	Disjoint union of lobster
Dl_n	=	Diamond ladder
mDl_n	=	Disjoint union of diamond ladder
M_{2n}	=	Mountain graph
mM_{2n}	=	Disjoint union of mountain graph

Bt_n	=	Triangular book
mBt_n	=	Disjoint union of m copies triangular book
St_n	=	Stair graphs
mSt_n	=	Disjoint union of m copies of stair graphs
$C_n \cup C_{n+2} \cup C_{n+4}$	=	Disjoint union of non isomorphic cycles
$mC_n^k \cup (\mu - 1)C_n \cup \mu C_n^3 \cup mC_n$	=	Disjoint union of cycles with cycles with chord
Bat_n	=	Bat graph
$mBat_n$	=	Disjoint union of m copies bat graph
ϑ_n	=	Volcano graph
$m\vartheta_n$	=	Disjoint union of m copies volcano graph
TCL_n	=	Three-Circular Ladder graph
$mTCL_n$	=	Disjoint union of m copies Three-Circular Ladder graph
$mBtr_{i,4}$	=	Bermuda Triangle graph
$mmBtr_{i,4}$	=	Disjoint union of m copies Bermuda Triangle graph
$R_{m,n}$	=	Rocket graph
$sR_{m,n}$	=	Disjoint union of s copies rocket graph
$U_{m,n}$	=	UFO graph
$sU_{m,n}$	=	Disjoint union of s copies UFO graph
$CR_{n,m}$	=	Coconut Sprout graph
$sCR_{n,m}$	=	Disjoint union of s copies Coconut Sprout graph
S_n	=	Snail graph
mS_n	=	Disjoint union of s copies Snail graph

INTRODUCTION

In the early days of computer networks, interprocessor communication and scalability of applications was hampered by the high latency and the lack of bandwidth of the network. The IBM supercomputing project, which was begun in 1999, has proposed a new solution to the problem and built a new family of supercomputers optimizing the bandwidth, scalability and the ability to handle large amounts of transferring data. One of the world's fastest supercomputers was officially inaugurated at IBM's Zurich Research Laboratory (ZRL). The so-called BlueGene system, which is the IBM supercomputing project solution, has the same performance as the computer ranked 21st on the current list of the world's top 500 supercomputers (for more detail, see [46]). It will be used to address some of the most demanding problems faced by scientists regarding the future of information technology, such as, how computer chips can be made even smaller and more powerful. However, in massive parallel computers, the robustness of supercomputers is not the only factor. One of the most significant factors is the design of parallel processing systems circuits and, more precisely, the construction of their interconnection networks. Therefore, there has been a growing interest in the study of the design of large interconnection networks.

A communication network can be modelled as a *graph* or a *directed graph* (*digraph*, for short), where each processing element is represented by a *vertex* and the connection between two processing elements is represented by an *edge* (or, in the case of a digraph, by a directed *arc*). The number of vertices is called the *order* of the graph or digraph. The number of connections incident to a vertex is called the *degree* of the vertex. If the connections are one way only then we distinguish between in-coming and out-going connections and we speak of the *in-degree* and the *out-degree* of a vertex. The *distance* between two vertices is the length of the shortest path, measured by the number of edges or arcs that need to be traversed in order to reach one vertex from another vertex. In either case, the largest distance between any two vertices, called the *diameter* of the graph or digraph, represents the maximum data communication delay in a communication network.

In communication network design, Fiol and Lladó [38] identified several factors which should be considered. Some of these factors seem fundamental, for instance, there must always exist a path from any processing element to another. Also, the data communication delay during processing must be as short as possible. The complexity of the network will also increase dramatically if the number of elements (or computer) that are involved in the network increases, especially if the number of connections that are connected to a vertex is also getting larger, then the design of a network which admits a modularity, a good fault tolerance, a diameter vulnerability and a vertex-symmetric interconnection network properties will always be a major concern in network topology. One of the important efforts that can be done is to do labeling to the models of the network.

Graph labelings provide useful mathematical models for a wide range of applications, such as radar and communication network addressing systems and circuit design, bioinformatics, various coding theory problems, automata, x-ray crystallography and data security. More detailed discussions about applications of graph labelings can be found in Bloom and Golomb's papers [16] and [17].

Many studies in graph labeling refer to Rosa's research in 1967 [61] and Golomb's research in 1972 [42]. Rosa introduced a kind of labeling, called *β-valuation* and Golomb independently studied the same type of labeling and called this labeling *graceful labeling*. Surprisingly, in 1963 Sedláček [62] had already published a paper which introduced another type of graph labeling, namely, *magic labeling*. Stewart [66] called magic labeling *supermagic* if the set of edge labels consists of consecutive integers.

Motivated by Sedláček and Stewart's research, many other labelings of graphs have been studied since then, and many new results have been published. However, there still exist many interesting open problems and conjectures. No polynomial time bounded algorithm is known for determining whether or not the various types of graph labelings exist for particular classes of graphs. Therefore, the question of whether a specific family of graphs admits a property of a specific labeling is still widely open. In this book we present new results in super graph labeling for disjoint unions of multiple copies of special families of graphs. These results have been published in either proceeding conference or international journal. Since there are a lot of beginners and professional researchers searching a reference on graph labeling especially disjoint unions of disconnected graphs, we finally decide to write these collection of results in a book. Finally, I gratefully expect that this book will give a benefit to all readers.