



The Role of Mathematical Connections in Mathematical Problem Solving

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Abstract

Problem-solving and mathematical connections are two important things in learning mathematics, namely as the goal of learning mathematics. However, it is unfortunate that the ability of students' mathematical connections is very low so that it impacts on students' failure in solving mathematical problems. The writing of this paper aims to discuss the understanding of mathematical problems, mathematical problem solving, mathematical connections, and how they play a role in solving mathematical problems. The method used in writing this paper is a method of studying literature, which is reinforced by the example of a qualitative research result. The research subjects consisted of two eighth grade students of junior high school in Jember East Java, Indonesia, in 2017/2018. The research data consisted of written test results solving the mathematical problem as well as interview results. Data analysis uses descriptive qualitative analysis. From the results of literature studies and research results provide a conclusion that mathematical connections play an important role, namely as a tool for students to use in solving mathematical problems where students who have good mathematical connection skills succeed in solving mathematical problems well, while poor mathematical connection skills cause students to fail in solving mathematical problems.

Keywords: Mathematical Problems, Mathematical Problem Solving, Mathematical Connections

Abstrak

Pemecahan masalah dan koneksi matematis merupakan dua hal penting dalam pembelajaran matematika, yaitu sebagai tujuan dari pembelajaran matematika. Namun, sangat disayangkan kemampuan koneksi matematis siswa sangat rendah sehingga berdampak pada kegagalan siswa dalam aktivitas memecahkan masalah matematika. Penulisan paper ini bertujuan untuk membahas pengertian masalah matematika, pemecahan masalah matematika, koneksi matematis dan bagaimana peranannya dalam aktivitas memecahkan masalah matematika. Metode yang digunakan dalam penulisan paper ini adalah metode kajian literatur yang diperkuat dengan contoh sebuah hasil penelitian kualitatif. Subjek penelitian terdiri dari dua orang siswa kelas VIII Sekolah Menengah Pertama (SMP) di Jember Jawa Timur Indonesia tahun 2017/2018. Data penelitian terdiri dari hasil tes tertulis memecahkan masalah matematika serta hasil wawancara. Analisis data menggunakan analisis deskriptif kualitatif. Dari hasil kajian literatur maupun hasil penelitian memberikan kesimpulan bahwa koneksi matematis memegang peranan penting, yaitu sebagai alat untuk digunakan siswa dalam aktivitas memecahkan masalah matematika. Di mana, siswa yang memiliki kemampuan koneksi matematis yang baik berhasil memecahkan masalah matematika dengan baik, sedangkan kemampuan koneksi matematis yang kurang baik menyebabkan kegagalan siswa dalam memecahkan masalah matematika.

Kata kunci: Masalah Matematika, Pemecahan Masalah Matematika, Koneksi Matematis

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INTRODUCTION

In daily life, everyone always faces problems, both personal problems and work problems. Romberg (1994), Enrique (2016) states that a problem or problem is a situation that contains difficulties for someone and encourages that person to solve them. Enrique (2016) adds that a situation faced by someone will be a problem if the person is aware of the situation, and requires action to resolve it, but he cannot directly resolve it. Someone who is facing a problem certainly is

trying to overcome these problems by collecting information that is already known and relates it to the problem situation that he is facing. The more complete the information he relates, the faster he can overcome the problem. Of course, everyone wants to solve the problems they face well. But the fact is, not everyone can gather all the important information and relate all of the information to be used in solving the problems they face.

The above phenomenon also occurs in the world of mathematics education, where every student must have a problem in the process of learning mathematics in school. Now the question arises: which one can be called a mathematical problem and which one is not? Dossey (2006), Enrique (2016) said that a problem in mathematics is a problem if students can not answer the problem directly. If the problem can be answered directly by students, then the problem is called a routine problem and is not included in the mathematical problem. However, this understanding can be subjective, depending on the readiness and learning experience students have. There may be questions that are considered non-routine questions for a group of students, but these questions are considered as routine questions for other students. Pambudi (2018) gives an example if elementary school students and junior high school students are asked to answer questions (1) "*what is the area and circumference of a rectangle that is long and wide?*", then this question (1), not a problem but just a routine matter, for all of them; because students can directly answer using the method taught by the teacher, that is the area of length multiplied by width, the result is, and the circumference is twice the length plus twice the width, the result is. In contrast to the question (2) following, "*Mr. Budi has a rectangular piece of land with a length of twice its width and an area of, determine the circumference of Mr. Budi's land*". In this problem, elementary students cannot immediately answer because it is not known what the actual length and width is, but the student must try to find a way to find the length and width, then he can determine the circumference. As for junior high school students who are used to dealing with this problem, then this problem can be considered as a routine problem; but for junior high school students who are first facing this problem, then this problem will be a non-routine problem (problem) for these students.

In dealing with the problem (2), some junior high school students were able to solve it well, but many other junior high school students failed to answer properly. This is where students need the ability to associate the knowledge that students already have with the situation they face. This ability is called mathematical connection ability. Students who can associate the concept of a rectangle with the principle of the circumference of the rectangle and the area of the rectangle can certainly solve the problem easily. The first step he took was to make a sketch of Pak Budi's land in a rectangular shape, then to consider the length and width with the symbol $p = 2l$. Next, he made a mathematical model that contains the relationship between length, width, and area of the rectangle, namely: $(2l)(l) = 50 \text{ m}^2$ or $12 = 25 \text{ m}^2$. To get the width of these students use the concept of quadratic equations so that the value of $l = 5$ (why $l = -5$ does not meet?). Furthermore, students can associate the value of $l = 5$ with the equation $p = 2l$, so that the value of $p = 10 \text{ m}$ is obtained. Finally, students can

associate the values of p and l with the formula around the rectangle, which is $K = 2(p + l)$ so that it is obtained that the circumference of Mr. Budi's land is 30 m. If students are not able to associate various mathematical ideas that include concepts, principles/formulas, symbols, arithmetic operations such as the example, then students will experience failure in solving problems.

Every time students follow the process of learning mathematics in school; students are expected to be able to understand mathematical concepts, explain the interrelationships between concepts and apply these concepts flexibly, accurately, efficiently, and precisely in activities to solve mathematical problems. But the reality that happened was not by that desire. Where the facts show that the ability of students' mathematical connections in Indonesia is still very low. For example, the results of research conducted by Sugiman (2008) show that the mathematical connection ability of junior high school students only reaches an average of 53,8%, and is still relatively low. Saminanto & Kartono (2015) reported that students' mathematical connection skills in solving problems of daily life only reached an average score of 2%. Siregar & Surya (2017) also reported that the mathematical connection ability of junior high school students in associating ideas in mathematics only reached an average score of 51,11%, and the ability to associate mathematics with everyday life problems reached an average score of 17,78%.

In addition to the weak mathematical connection ability, Indonesian students are also still very weak in problem-solving abilities, as can be seen in the PISA (Programme for International Student Assessment) event. Where in 2012, Indonesian students only ranked 64 out of 65 participating countries with an average mathematical grade of 318, and for 2015 Indonesia's ranking was 62 out of 70 participating countries with an average mathematical grade of 335 (OECD, 2014; 2016). The results for the 2018 rank of Indonesia is 73 of 79 participating countries with an average mathematical grade of 379 (OECD, 2019). The score is also far from the average PISA score of around 500. There is a relationship between mathematical connections and problem-solving. Stacey & Tuner (2015), OECD (2016; 2019), Saputri & Zulkardi (2020) stated that PISA questions are designed to determine student literacy abilities based on various basic abilities such as reasoning, use of mathematical concepts, procedures, facts, and other basic skills for use as a tool in describing, explaining, and predicting an event. This shows that the ability of mathematical connections (using links between concepts, procedures, and facts) is related to mathematical problem-solving abilities because PISA questions are problem-solving problems that require literacy abilities, including mathematical connection abilities.

They are seeing the relationship between students' abilities in aspects of mathematical connections and mathematical problem solving, so the purpose of this study is to find out how the role of mathematical connections in mathematical problem-solving activities. This research is very important to do because both mathematical connections and problem-solving are very important to be taught in schools and become the skills needed in the life of modern society (Oktaviyanthi & Agus,

2019), as well as to give an idea to mathematics teachers how the role of students' mathematical connection abilities in solving mathematical problems. Every time students follow the process of learning mathematics in school; students are expected to be able to understand mathematical concepts, explain the interrelationships between concepts and apply these concepts flexibly, accurately, efficiently, and precisely in activities to solve mathematical problems. But the reality that happened was not by that desire. For example, the results of research conducted by Sugiman (2008) show that the mathematical connection ability of junior high school students only reaches an average of 53,8%, and is still relatively low. Saminanto & Kartono (2015) reported that students' mathematical connection skills in solving problems of daily life only reached an average score of 2%. Siregar & Surya (2017) also reported that the mathematical connection ability of junior high school students in associating ideas in mathematics only reached an average score of 51,11%, and the ability to associate mathematics with everyday life problems reached an average score of 17,78%.

METHODS

The method used in writing this paper is a method of studying literature, which is reinforced by the example of a qualitative research result. The literature has taken deals with the topic of mathematical problem solving and mathematical connections to describe how the role of mathematical connections as one of the factors that can influence student success in solving mathematical problems. The qualitative research that is used as an example is a case study of two subjects, namely eighth-grade students of junior high school in Jember, East Java, Indonesia, in 2017/2018. The research data consisted of written test results as well as the results of interviews of both subjects. Written test in the form of a story about the topic of geometry and social arithmetic with the context of land sales and the cost of making a fence around the land. Data analysis uses descriptive qualitative analysis with steps of data categorization, data reduction, data presentation, interpretation, and concluding (Moleong, 2013; Miles & Huberman, 2014). In the data categorization step, classification of data or information is carried out by the problem, followed by data reduction, which is removing data or information that is not needed with the problem being examined. Next, the data is presented in the form of verbal and picture and interpreted to conclude in answering the research problem.

RESULTS AND DISCUSSION

Position of Problem Solving and Mathematical Connection in the Mathematics Learning Process

In the mathematics curriculum in the United States, the Principles and Standards for School Mathematics National Council of Teachers of Mathematics (NCTM), strengthened by Sawyer (2008),

Jäder, Lithner & Sidenvall (2019) stated that five necessary mathematical abilities are standardized by the process learning mathematics in schools, namely (1) problem solving, (2) reasoning and proof, (3) communication, (4) connections, and (5) representation. The statement shows that problem solving and mathematical connections are two important things that need to be considered in the process of learning mathematics. Even in the USA, there has long been a large project called The Connected Mathematics Project (CMP) that emphasizes the importance of making mathematical connections in the process of learning mathematics in schools (Moyer, Robison, & Cai, 2018).

Many countries have followed in the USA's footsteps, such as Australia, Singapore, and Malaysia by including problem-solving and mathematical connections in their mathematics curriculum starting in 2002-2006 (The Australian Association of Mathematics Teachers, 2006, Curriculum Planning and Development Division of the Ministry of Education Singapore, 2006, Ministry of Education Malaysia, 2002). Likewise, the Government of Indonesia in the 2006 SBC and Curriculum 2013 have included mathematical connections and problem solving as a goal of learning mathematics in schools. This can be seen from the purpose of teaching mathematics in SMP/MTs, namely *"so that students understand mathematical concepts, explain the interrelationships between concepts and use concepts and algorithms flexibly, accurately, efficiently, and precisely in activities to solve mathematical problems"*. Thus the teacher's effort to improve mathematical connection skills in problem-solving activities is very important because it is the goal or heart of mathematics learning itself.

Romberg (1994) states that problem-solving is a process for accepting or dealing with problems and making efforts to solve these problems. Kirkley (2003), Enrique (2016) added that problem-solving is an embodiment of a mental activity consisting of various skills and cognitive actions intended to get the correct solution to the problem. Efforts to teach problem-solving to students are very important because, according to Pehkonen (1997), problems can develop cognitive skills in general, problem-solving fosters creativity, problem-solving is part of the mathematical application process, and problem-solving motivates students to learn mathematics itself. In line with this, Charles & O'Daffar (1997), Liljedahl, Santos-Trigo, Malaspina, & Brother (2016) stated that the purpose of teaching mathematical problem solving is to develop students' thinking skills, develop their ability to select and use ways of solving problems, develop attitudes and beliefs in problem-solving, develop students' ability to use the knowledge that is interconnected, develop students' ability to monitor and evaluate their thoughts from the results of their work while solving problems develop students' ability to solve problems in a cooperative atmosphere and develop students' ability to express correct answers to various problems.

Meanwhile, Liljedahl et al. (2016) suggested several reasons for the importance of problem-solving in teaching mathematics, namely: (a) students must learn to understand mathematics, develop models for standard problems, and reduce their fear of mathematical challenges. Understanding mathematical problems are part of solving mathematical problems, (b) students must develop a sense

of pride because, without pride and enthusiasm, students are not motivated to learn mathematics well. Students must continue to grow their mathematical abilities, among others through problem-solving, (c) students become more critical and analytical in dealing with problems, whereas developing critical and analytical thinking skills is an important part of educational goals, (d) Problem-solving ability is one of the goals in education, even is one of the indirect objects of study of mathematics.

Especially in the process of solving mathematical problems, two opinions support one another. First, the opinion of Polya (1973) which stated four important stages in solving problems, namely: (a) understanding the problem (understanding the problem), (b) devising a plan for solving the problem, (c) implementing the problem-solving plan (carrying out the plan), and (d) rechecking the results of problem-solving (looking back). Other opinions from Liljedahl, et al. (2016) also suggests four steps in solving problems, namely: (a) reading the problem, (b) choosing a strategy, (c) solving the problem, and (d) checking or looking back. From these two opinions, we can conclude that the process of solving a mathematical problem consists of four steps, namely understanding the problem, drawing up a plan or strategy, implementing a plan or strategy, and checking the answers.

The term "*mathematical connection*" comes from English, namely "*mathematical connection*". This term has long appeared in mathematics education, for example, by Brownell around 1935 (Bergeson, 2000), but is still limited to connections between concepts in arithmetic. The term connection became popular after NCTM stated that "... *mathematics is not a set of isolated topics but rather a web of closely connected ideas*". Although mathematics consists of various objects or ideas, such as facts, concepts, operations, and principles, all of these ideas need to be linked to be able to study more difficult material (Thompson, 2008). Baki, Çatlıoğlu, Costu, & Birgin, (2009), added that to be able to solve mathematical problems, students should understand problems and make mathematical connections between various ideas or objects in mathematics, which include facts, concepts/principles, procedures, representations in the form of verbal, numerical, symbols, formulas, equations, tables, pictures, and graphs. Connections made must be used effectively to solve mathematical problems properly.

Eli, Schroeder & Lee (2013) define mathematical connections as follows "*mathematical connection is a part of a network structured like a spider's web; the junctures, or nodes, can be thought of as pieces of represented information, and threads between them as the connections or relationships*". From this, it can be interpreted that the mathematical connection as part of a mental network that is structured like a cobweb. Points or nodes can be considered as pieces of information, and the thread between them is the connection. It was added that all nodes on the network are always connected, so that spider travel is always smooth without obstacles by following established connections. Some nodes are connected directly, and some nodes are not related. The shape of the net that occurs can be very simple or may be very complex, with many connections originating from each node. The simplicity or complexity of the connections formed is related to the level of difficulty of the problem at hand.

Regarding the types of mathematical connections, there are several opinions. For example, the NCTM states that *"mathematics instruction should enable students to: recognize and use connections between mathematical ideas, and recognize and apply mathematics in contexts outside of mathematics"*. This means that in the mathematics learning process, teachers should guide students to recognize and use two types of mathematical connections, namely internal and external connections. Internal connections are links between ideas in mathematics itself, while external connections are links between mathematical ideas with the outside world, both with other fields or subjects and with the application of mathematics in everyday life. Next, it's explained that *"mathematical connections are"* tools *"for problems Solving ..."*, which means mathematical connections made by students become tools for use in mathematical problem-solving activities. Sugiman (2008) divides mathematical connections into four types, namely (1) inter-topic connections in mathematics, namely the association of ideas in one topic, (2) connections between one topic with other topics in mathematics, such as the relation of mathematical ideas in the topic of quadratic equations with the topic of searching for area and circumference of rectangles, (3) connections between mathematics and other fields of study, such as ideas in mathematics with physics, engineering, and other sciences, and (4) mathematical connections with the real world or everyday life, i.e., mathematical ideas that are associated with real daily problems.

Anthony & Walshaw (2009), Sumarmo (2010) states that mathematics teachers need to guide students to be skilled in making mathematical connections through activities that include: looking for connections or relationships between various representations of concepts and procedures, understanding links or contacts between mathematical topics, looking for connections or other procedures in equivalent representation, using connections between mathematical topics and between topics with other topics, and using mathematics in other fields of study or everyday life.

As for some of the results of research that are relevant to this study are mathematical connections are tools used to solve mathematical problems (Eli et al., 2013), the ability of students to make mathematical connections affect the success of students in solving problems of everyday life (Altay, Yalvaç, & Yeltekin, 2017), the ability of mathematical connections and mathematical communication contribute to improving students' problem-solving abilities (Suharto & Widada, 2018). Good mathematical connection ability (strong) causes students to succeed in solving mathematical problems, and conversely, poor mathematical connection ability (weak) causes students to fail in solving mathematical problems (Michigan State University, 2012; Pambudi, 2018).

Results of Research that has been Done regarding Mathematical Connections of Middle School Students in Mathematical Problem Solving Activities

To show how the role of mathematical connections in mathematical problem-solving activities, the following is given an example of a qualitative study conducted by Pambudi (2018) relating to

examples of mathematical connections made by two eighth grade students in Jember East Java Indonesia in problem-solving activities. Both students were coded with the initials PMN, and CFA.

Both students were given the same Problem Solving Task (TPM), as shown in Figure 1.

Mr. Amir has a rectangular piece of land measuring 21 meters x 8 meters. A year later this land was hit by a road-widening project, so Mr. Amir sold part of his land which was located in a corner in the form of a right triangle with sides of the elbows measuring 6 meters and 8 meters at Rp100.000 per m^2 . Pak Amir used the proceeds from the sale of the land to make a fence around his current land. The cost of making the fence Rp50.000. per meter, while for the door made of the 4-meter long iron door, at a cost of Rp1.000.000. How many funds do you still need to add, Mr. Amir?

Figure 1. Problems given to research subjects in TPM

The results of the answers from PMN students are presented in Figure 2.

Diketahui : - P. Amir mempunyai sebidang tanah berbentuk persegi panjang dengan ukuran 21 m x 8 m
 - P. Amir menjual sebagian tanah yang terletak di pojok dgn bentuk segitiga siku-siku yg sisi siku-sikunya berukuran 6m dan 8m dengan harga Rp 100.000/ m^2
 - Hasil penjualan tanah digunakan untuk membuat pagar dengan biaya Rp 50.000/m dan untuk membuat pintu besi dengan biaya Rp 1.000.000
 Ditanya : Berapakah dana yang masih perlu ditambah Pak Amir?
 Jawab :

1
2

3

4
5
6

C. tanah yang dijual = $\frac{6m \times 8m}{2}$
 $= 24 m^2$
 Hasil penjualan tanah = Rp 100.000/ m^2 x $24 m^2$
 $= Rp 2.400.000$
 $BD^2 = BC^2 + CD^2$
 $BD^2 = 6^2 + 8^2$
 $BD^2 = 36 + 64$
 $BD^2 = 100$
 $BD = \sqrt{100} = 10$
 K. tanah sekarang = $15m + 8m + 21m + 10m = 54 m$
 pagar = $54 m - 4m$
 $= 50 m$
 Biaya pembuatan pagar = Rp 50.000/m x 50 m
 $= Rp 2.500.000$
 Biaya pintu besi = Rp 1.000.000
 Total biaya = Rp 2.500.000 + Rp 1.000.000
 $= Rp 3.500.000$
 Dana yang perlu ditambah = Rp 3.500.000 - Rp 2.400.000
 $= Rp 1.100.000$
 Jadi, dana yang masih perlu ditambah Pak Amir adalah Rp 1.100.000

Information:

1. Written what is known, written explicitly, and clearly.
2. Some facts/data are correct and equipped with units and symbols.
3. Write what is asked, and students understand what to look for / answer.
4. There are clear and systematic plans and procedures as well as steps for solving problems.
5. Some concepts and principles are connected, and the amount is sufficient and the mathematical ideas that are connected are all right (true).
6. The connection of mathematical ideas is used effectively so students succeed in answering problems correctly

Figure 2. Results of PMN student answers

After students finish answering the problem, interviews are conducted to find out how students' understanding of the answers. The following are excerpts from the interview (R = Researcher, and PMN = Student).

R : Try to explain what do you mean by "Known", and "Asked" on your answer sheet

PMN : That's the information that I understood from the problem and what I was looking for.

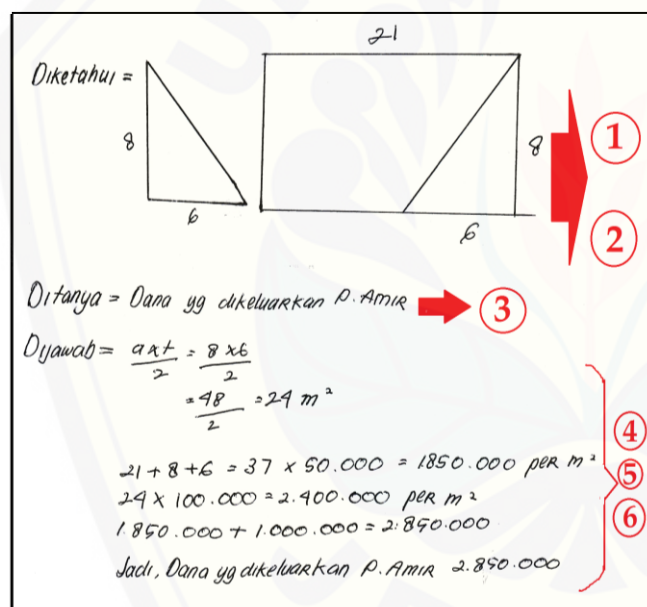
- R : *Then what is your purpose in making this picture and what is this calculation to find?*
- PMN : *The picture is the land owned by Pak Amir in the shape of a rectangle, and the corner of the land being sold is in the form of a right triangle. That calculation ... I calculated the area of land for sale, ... I used the formula area of a triangle, base times height divided by two.*
- R : *Then how do you make money from the sale of Mr. Amir's land?*
- PMN : *I calculated the selling price of the land per square meter and then multiplied the area of the land, so I got two million four hundred thousand rupiahs.*
- R : *Then what do you do next?*
- PMN : *I'm looking for the long side of a right triangle.*
- R : *What for?*
- PMN : *To calculate the circumference of Mr. Amir's land which will be made of a trapezoid-shaped fence, so I add all the sides, ... so that it can be 50 meters.*
- R : *Why 54 m – 4 m?*
- PMN : *Because of the 4 meters made iron door.*
- R : *Oh yeah, you were looking for a right triangle with what is this formula?*
- PMN : *That's the Pythagorean formula*
- R : *Alright, now what are you looking for this?*
- PMN : *I'm looking for costs to build a fence.*
- R : *How to do it?*
- PMN : *I multiply the cost of the fence per meter by the circumference of the fence which has been reduced by the length of the door by the price of making the fence per meter, so I get two million and five hundred thousand rupiahs.*
- R : *Then what does it mean for the cost of the iron door and this total cost?*
- PMN : *I know the cost of the iron door, about one million rupiahs. So the total cost required by Mr. Amir to make a fence and an iron door ... I add, so he can get three million five hundred thousand rupiahs.*
- R : *Then what's the next one?*
- PMN : *That's I counting the funds that Pak Amir needs to add, so Rp3.500.000 – Rp2.400.000 = Rp1.100.000.*
- R : *Where's the Rp2.400.000 of this from?*
- PMN : *That's money from the sale of land.*

From the results of answers and interviews with PMN students, it can be seen that students understand the problem by writing down what is known in the form of facts or data or information that they understand from the problem explicitly and clearly and writing down what the answers to the given questions will be sought. It shows that PMN students have understood the connection between mathematical ideas in the form of facts/data with the context of everyday life, which is related to the sale of a plot of land, and the cost of making fences and gates. PMN students sketch the ground completed with clear data, then write down each step or plan taken to answer the question. PMN students carry out plans consistently and systematically from each step. In the clear answer, PMN students connect various concepts to look for the land area sold by Mr. Amir along with the money from the sale of land, look for the slanted side with the Pythagorean formula, look for the circumference of the land of Pak Amir after the sale and purchase, find the length of the fence and the cost of making the fence, look for how much funds does Mr. Amir need to build fences and gates and finally look for how much funds Mr. Amir still needs. PMN students have used a lot of facts/data following the correct formula and continued to do the counting operations correctly. All mathematical

ideas that include facts/data with concepts, principles, representations, and mathematical procedures that are connected by PMN students are all right, the number of ideas that are connected quite a lot and according to the needs and connections of mathematical ideas are used effectively so that PMN students managed to solve the problem correctly.

From this, it can be said that the mathematical connection ability of PMN students is very good and used as an effective tool in solving mathematical problems, so students succeed in solving these problems properly. This is consistent with existing theories, namely mathematical connections are tools used to solve mathematical problems (Eli et al., 2013), good 8th-grade student skills in making mathematical connections can deliver students' success in solving problems of everyday life (Altay, Yalvaç, & Yeltekin, 2017), and the ability of mathematical connections and mathematical communication contribute to improving students' problem-solving abilities (Suharto & Widada, 2018).

The results of the answers from the second subject, namely CFA students are presented in Figure 3.



Information:

1. Written what is known, but not explicit (not clear) students' understanding of the problem/problem, because only the picture without verbal explanation
2. There are facts/data, but not complete with units and symbols
3. Written what is asked, but students do not understand what to look for / answer (misunderstand questions)
4. Unclear plans and procedures carried out, no problem-solving steps
5. Some concepts and principles are connected/used, such as formulas, but not enough in number and the mathematical ideas that are connected are not all right (something is wrong)
6. Connection of these ideas is less effective to answer the problem

Figure 3. Results of CFA student answers

The following are a few snippets of interviews with CFA students (R = Researcher, and CFA = Student) shortly after students finish answering written tests.

- R : Try to explain what is meant by "Unknown" and this picture?
 CFA : That was the land that Pak Amir sold and all the land before it was sold.
 R : Then what does it mean to you write "Asked?"
 CFA : I have to look for how much money should Mr. Amir spends.
 R : Try to explain what this calculation means? (researchers point to the text *ax*)
 CFA : I am looking for the land area for sale ... the formula for the area of a triangle, base times height divided by two ... so it can be 24 m^2 .

- R : *What is this then? (the researcher pointed to the students' writing $21 + 8 + 6 = 37 \times 50.000 = \dots$)*
- CFA : *This calculation is looking for costs to make a fence, ... which is $21 + 8 + 6 = 37 \times 50.000 = 1.850.000$ per m^2 .*
- R : *What are you looking for on this one? (researchers point to 24×100.000 student writings = $2.400.000$ per m^2)*
- CFA : *Looking for money earned by Mr. Amir to sell land*
- R : *What does it mean per m^2 ?*
- CFA : *That means all the money is $2.400.000$*
- R : *Then what did you look for last?*
- CFA : *I am looking for funds issued by Mr. Amir.*
- R : *Where did you get this number? (researchers appoint $1.850.000 + 1.000.000 = 2.850.000$)*
- CFA : *The $1.850.000$ is money to make fences, ... and the $1.000.000$ is money to make iron doors ... so all of them are $2.850.000$.*

From the results of written answers and interviews with CFA students, it can be said that CFA students do not understand the problem correctly, because when making a connection between mathematical ideas in the form of facts/data with the context of daily life is not right, namely, students write down to find the funds spent Pak Amir, even though the truth is to find how much funds Mr. Amir needs to add to make fences and gates. Furthermore, CFA students carry out problem-solving plans by making connections between mathematical ideas in the form of facts/data with mathematical concepts, principles, representations, and procedures. Unfortunately, the mathematical ideas connected by CFA students are not all correct (there are wrong ideas), and the number of ideas connected is less than needed. For example, CFA students did not associate the length of the right-angled triangle by using the Pythagorean formula to find the circumference of the land of Amir, CFA students also did not look for the length of the side of the land affected by the sale, ie ($21 - 6 = 15$ meters). CFA students' mathematical connections regarding the concept of traveling and the concept of long wholes are not good so that it results in incorrect calculations on the circumference of the land of Mr. Amir after the sale and purchase ($21 + 8 + 6$). CFA students also miscalculated $21 + 8 + 6$ should have 35 written 37, then CFA students when calculating the cost of making a fence made a mistake not to reduce the length of the gate (as long as 4 meters) and write the square meter (m^2) symbol at $1.850.000$ per m^2 , which should be rupiah (Rp). The final answer of CFA students is looking for funds issued by Mr. Amir, which is $2.850.000$, there is an error here, not using the symbol Rp. and CFA students do not deduct from the money Pak Amir obtained from the sale of part of his land. From the students' answers, it is clear that CFA students have poor or poor mathematical connection skills that fail CFA students to solve problems.

From this, it can be said that the CFA student has poor mathematical connection skills, so he fails in solving mathematical problems. The mathematical connections made by CFA students are not effective in solving mathematical problems, because the connections of mathematical ideas made are insufficient, and the association of mathematical ideas made by students is not right. This is consistent

with the theory which states that weak mathematical connections cannot be used as effective tools in solving mathematical problems (Eli et al. 2013), the 8th-grade students' skills that are not good at making mathematical connections cause students to fail in solving problems of daily life days (Altay, Yalvaç, & Yeltekin, 2017).

The results of this study support the theory and the consequences of previous studies which state that mathematical connections are tools to solve mathematical problems. Effective mathematical connections to solve mathematical problems are the links of mathematical ideas made by students are enough, and the links are right. Conversely, ineffective mathematical connections are lacking in the number of connections, and the association of mathematical ideas made by students is not right (Eli et al., 2013). Strong mathematical connection skills cause students to succeed in solving mathematical problems, and conversely, weak mathematical connection abilities cause students to fail in solving mathematical problems (Michigan State University, 2012; Pambudi, 2018).

Several factors can cause students' mathematical connection abilities to be weak. Pambudi (2018), Budayasa, and Lukito (2018) stated that students who are passively involved in learning mathematics in class, lack adequate learning experience, causing students to have weak mathematical connection abilities. This causes students to fail in solving mathematical problems, also, Arthur, Y. D, Owusu, EK, Addo, S. A, & Arhin, AK (2018) stated that learning mathematics in schools that lack connecting between mathematical concepts and problems in daily life makes students not interested in learning mathematics. Therefore, mathematics teachers in schools need to guide students to practice making mathematical connections in mathematical problem-solving activities. Mathematical connection skills that need to be practiced include connections between one concept and other mathematical concepts as well as connections between mathematical concepts and problems in everyday life. Through intensive training, it is expected to improve students' mathematical connection skills and ability to solve mathematical problems.

CONCLUSION

From the results of the study, it can be concluded that mathematical connections have a very important role, namely as a tool that students can use to solve mathematical problems. Where, the mathematical connections of various mathematical ideas, whether in the form of facts/data, concepts, principles, representations, and mathematical procedures in sufficient numbers and precise connections become effective tools for achieving student success in solving mathematical problems. On the other hand, the mathematical connections of various mathematical ideas that are lacking in number and inaccurate links become less effective tools for achieving success, so students experience failure in solving the mathematical problems they face. In other words, students who have good mathematical connection skills tend to succeed in solving mathematical problems, while students with poor mathematical connection skills tend to fail in solving mathematical problems.

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