



**PROCEEDINGS OF THE 7TH SEAMS UGM
INTERNATIONAL CONFERENCE ON MATHEMATICS AND
ITS APPLICATIONS 2015: Enhancing the Role of
Mathematics in Interdisciplinary Research**

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PROCEEDINGS OF THE 7TH SEAMS UGM INTERNATIONAL CONFERENCE ON MATHEMATICS AND ITS APPLICATIONS 2015: Enhancing the Role of Mathematics in Interdisciplinary Research



Conference date: 18–21 August 2015

Location: Yogyakarta, Indonesia

ISBN: 978-0-7354-1354-2

Editors: Fajar Adi Kusumo, Indah Emilia Wijayanti, Irwan Endrayanto Aluicius and Yeni Susanti

Volume number: 1707

Published: Feb 11, 2016

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No Access • Published Online: 11 February 2016

Preface: The 7th SEAMS UGM International Conference on Mathematics and its Applications 2015

AIP Conference Proceedings **1707**, 010001 (2016); <https://doi.org/10.1063/1.4940800>

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TOOLS

PREFACE

First of all, we would like to thank to all participants of “The 7th SEAMS UGM 2015 International Conference on Mathematics and Its Applications” which was held on August 18-21, 2015. The conference is very important as a communication forum of the mathematicians, not only in Indonesia, but also in Southeast Asia and surrounding areas. We also thank to the steering committee members and all of the reviewers for all supports during the conference and the preparation of this proceedings.

As the scientific documentation of the conference, we provide two-types of proceedings. The first one is the AIP Proceedings which contains the high quality paper selected by blind review process. The second one is the regular proceedings, which contain the selected papers which are not published in the AIP Proceedings and the paper of our invited speakers.


We would like to say thanks to all authors who have submitted the paper to our proceedings. During the review process, we found that almost all papers has good quality. However due to the limitation number of the paper which can be published in our proceedings, we should select the submitted paper based on the reviewer recommendation and score. So, there are some papers which are not accepted to publish in this proceedings, we apologize to the authors about this inconvenience.

Lastly, we would like to say thanks for all partners and all sponsors in supporting our conference.




Warm regards,

Dr. Fajar Adi Kusumo
Editor in Chief.




ALGEBRA AND COMBINATORICS Contributed Papers

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On Ramsey $(3K_2, K_3)$ –minimal graphs

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Abstract. The Ramsey graph theory has many interesting applications, such as in the fields of communications, information retrieval, and decision making. One of growing topics in Ramsey theory is Ramsey minimal graph. For any given graphs G and H , find graphs F such that any red-blue coloring of all edges of F contains either a red copy of G or a blue copy of H . If this condition is not satisfied by the graph $F - e$, then we call the graph F as a Ramsey (G, H) –minimal. In this paper, we derive the properties of $(3K_2, K_3)$ –minimal graphs. We, then, characterize all Ramsey $(3K_2, K_3)$ –minimal graphs.

Keywords: Ramsey minimal graph, edge coloring, matching, complete graph.

PACS: 02.10.Ox

INTRODUCTION

All graphs in this paper are simple, undirected, and finite. For any given graphs G and H , any red-blue coloring on all edges of F contains a red copy of G or a blue copy of H , denoted by $F \rightarrow (G, H)$. A (G, H) –coloring is a red-blue coloring on the edges of F such that F does not contains a red G and blue H . If a graph F has a (G, H) –coloring, then we write $F \not\rightarrow (G, H)$. A graph F is called a Ramsey (G, H) –minimal if $F \rightarrow (G, H)$ but for each $e \in E(F)$, $(F - e)$ has a (G, H) –coloring. The set of all Ramsey (G, H) –minimal graphs will be denoted by $\mathcal{R}(G, H)$. The set $\mathcal{R}(G, H)$ can be finite or infinite, depends on the pair of graphs G and H . Here we will focus on Ramsey-finite.

Burr *et al.* [3] proved that if G is a matching, $G = mK_2$, then $\mathcal{R}(mK_2, H)$ is finite for any graph H . A complete determination graphs Ramsey (G, H) –minimal is a challenging but difficult problem, even for some small-order graphs G and H . Some researchers have characterized graphs in $\mathcal{R}(mK_2, H)$, for some classes graph H and positive integer $m > 1$. For H a triangle, $\mathcal{R}(2K_2, K_3) = \{K_5, 2K_3, A\}$, where A is the graph in Figure 1, was given by Burr *et al.* [3]. For H a star $K_{1,n}$, the set $\mathcal{R}(2K_2, K_{1,n})$ for $n \leq 3$, was determined by Mengersen and Oeckermann [4]. Later on, the set of Ramsey minimal graphs for a pair matching and path was given by some researchers, such as $\mathcal{R}(2K_2, P_4)$ and $\mathcal{R}(2K_2, P_5)$ by Baskoro and Yulianti [2], $\mathcal{R}(3K_2, P_3)$ by Mushi and Baskoro [5], and $\mathcal{R}(4K_2, P_3)$ by Wijaya *et al.* [7]. Meanwhile, for H a cycle, the set $\mathcal{R}(2K_2, C_4)$ was given by Wijaya *et al.* [8]. For H a complete graph, $\mathcal{R}(2K_2, K_4)$

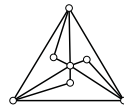


FIGURE 1. The graph $A \in \mathcal{R}(2K_2, K_3)$.

was determined by, Burr *et al.* [3], Baskoro and Wijaya [1], and Wijaya *et al.* [7]. For every connected graph H and integer $m > 1$, all disconnected graphs in $\mathcal{R}(mK_2, H)$ was given by Wijaya *et al.* [9]. In the same paper, Wijaya *et al.* [9] gave the relationship between graphs in $\mathcal{R}(mK_2, H)$ and in $\mathcal{R}((m - 1)K_2, H)$ and gave a corollary as follows.

Corollary 1. [9] Let H be a graph and m be a positive integer. If $F \in \mathcal{R}(mK_2, H)$, then for any $v \in V(F)$ and any $K_3 \subseteq F$, both graphs $F - \{v\}$ and $F - E(K_3)$ contain a Ramsey $((m - 1)K_2, H)$ -minimal graph. \square

In this paper, we characterize all connected graphs in $\mathcal{R}(3K_2, K_3)$. We do not consider disconnected graphs in $\mathcal{R}(3K_2, K_3)$, since by [3, 9], we have obtained them, namely $3K_3$, $K_5 \cup K_3$, and $A \cup K_3$, where A is the graph in Figure 1. We also derive the properties of graphs in $\mathcal{R}(3K_2, K_3)$. Note that the results related to the properties of graphs in $\mathcal{R}(3K_2, K_3)$ has been published in Proceeding of International Seminar on Mathematics Education and Graph Theory, Islamic University of Malang (UNISMA), Indonesia, [6], which is considered to be unofficial publication. Therefore, for the reader's convenience, we provide the results in [6] which are used in determining graphs in $\mathcal{R}(3K_2, K_3)$.

MAIN RESULTS

In this section, we will characterize all connected graphs in $\mathcal{R}(3K_2, K_3)$. Before doing that, we derive the necessary and sufficient condition of graphs belonging to $\mathcal{R}(3K_2, H)$ in the following lemmas.

Lemma 1. [6] Let H be a graph. $F \rightarrow (3K_2, H)$ holds if and only if the following four cases are satisfied:

- (i) for every $u, v \in V(F)$, $F - \{u, v\} \supseteq H$,
- (ii) for every $u \in V(F)$ and triangle K_3 in F , $F - \{u\} - E(K_3) \supseteq H$,
- (iii) for every two triangles $2K_3$ in F , $F - E(2K_3) \supseteq H$,
- (iv) for every induced subgraph with 5 vertices S in F , $F - E(S) \supseteq H$.

Proof. (\Rightarrow) Let H be a graph and $F \rightarrow (3K_2, H)$. Then, every red-blue coloring on the edges of F containing no red $3K_2$ will contains a blue H . It means that the subgraph induced by red edges forms one of four following subgraphs, namely two stars, a star and triangle, two triangles, or induced subgraph with 5 vertices. The four cases above are satisfied by the removing all the red edges of F . The removing red edges can be done by delete some vertices or edges in F . So, the remaining edges, namely the subgraph induced by blue edges contains a graph H . We obtain the cases (i) - (iv).

(\Leftarrow) Let four cases (i) - (iv) be satisfied. Suppose, on the contrary, that there is a $(3K_2, H)$ -coloring on F . So, the removing all red edges on F causes there is no graph H in the remaining edges, contradict all cases. \square

Next lemma is a negation of Lemma 1. Therefore, it can be proved in the same fashion.

Lemma 2. [6] Let e be an edge of a graph F . $F - e \not\rightarrow (3K_2, H)$ holds if and only if at least one of the following cases is satisfied.

- (i) There are two vertices $u, v \in V(F)$, such that $(F - e) - \{u, v\} \not\supseteq H$.
- (ii) There are a vertex $u \in V(F)$ and a triangle K_3 in F , such that $(F - e) - \{u\} - E(K_3) \not\supseteq H$.
- (iii) There are two triangles $2K_3$ in F , such that $(F - e) - E(2K_3) \not\supseteq H$.
- (iv) There is an induced subgraph with 5 vertices S in F , such that $(F - e) - E(S) \not\supseteq H$. \square

Lemma 3. If $F \in \mathcal{R}(3K_2, H)$, then a $(3K_2, H)$ -coloring of $F - e$ contains exactly a red $2K_2$, for every $e \in E(F)$.

Proof. Let $e \in E(F)$ and φ_1 be a $(3K_2, H)$ -coloring of $F - e$. For a contradiction, suppose that the red subgraph of $F - e$ contains at most a K_2 . We define φ as coloring on all edges of F such that $\varphi(x) = \varphi_1(x)$ for $x \in E(F - e)$ and $\varphi(e) = \text{red}$. Thus, under coloring φ , F contains at most a red $2K_2$. Hence, φ is a $(3K_2, H)$ -coloring of F , a contradiction. \square

For $H = K_3$, by the previous lemmas, we have the following observation.

Observation 1. Let $F \in \mathcal{R}(3K_2, K_3)$. Then, the following cases are satisfied:

- the minimum degree of F , $\delta(F) \geq 2$,
- F is not a tree,
- every edge $e \in E(F)$ is contained in some K_3 in F ,
- every vertex $v \in V(F)$ is contained in some K_3 in F . \square

Lemma 4. Let $F \in \mathcal{R}(3K_2, K_3)$. Then, $|V(F)| \geq 7$.

Proof. Let F be a graph in $\mathcal{R}(3K_2, K_3)$. By Lemma 1 (iv), $F - E(K_5)$ must contain a triangle. To form a triangle, we must involve two vertices other than vertices in K_5 . Hence, $|V(F)| \geq 7$. \square

We, now, give a class of graphs in $\mathcal{R}(3K_2, K_n)$ for any positive integer $n \geq 3$.

Theorem 1. Let $n \geq 3$ be a positive integer. The only connected graph of order $n + 4$ in $\mathcal{R}(3K_2, K_n)$ is K_{n+4} .

Proof. It is easily noticed that the complete graph K_{n+4} satisfies four conditions in Lemma 1. Thus, $K_{n+4} \rightarrow (3K_2, K_n)$. Next, let e be an edge in K_{n+4} . Then, $K_{n+4} - e = K_{n+2} + \overline{K_2}$. So, there exists a K_5 in $K_{n+4} - e$, such that $(K_{n+4} - e) - E(K_5) = K_{n-3} + \overline{K_5} + \overline{K_2}$ does not contain a graph K_n . Therefore, for every $e \in E(K_{n+4})$, $K_{n+4} - e \not\rightarrow (3K_2, K_n)$. Since K_{n+4} is the graph with the maximum number of edges, K_{n+4} is the only graph of order $n + 4$ in $\mathcal{R}(3K_2, K_n)$. \square

By Theorem 1, we have the following corollary.

Corollary 2. The only connected graph of order 7 in $\mathcal{R}(3K_2, K_3)$ is K_7 . \square

Next, we construct all graphs of order greater than 7, $F \in \mathcal{R}(3K_2, K_3)$. Clearly, F contains a triangle K_3 . By Corollary 1, $F - E(K_3)$ must contain a $2K_3, K_5$, or A . There are 8 possibilities, namely F contains the graph B_1, B_2, \dots, B_7 , or B_8 as depicted in Figure 2.

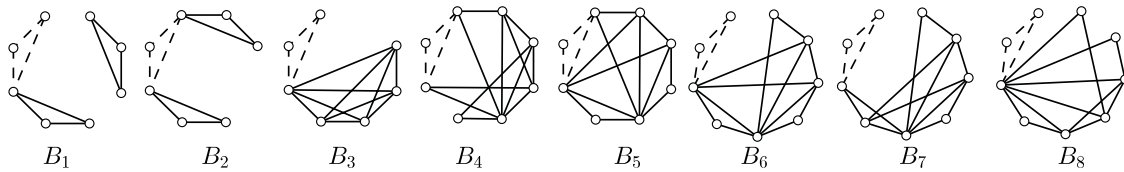


FIGURE 2. The possibilities of $F - E(K_3)$ must contain a $2K_3, K_5$, or A .

First, we will construct a graph of order 8 in $\mathcal{R}(3K_2, K_3)$. We can see that to construct a graph of order 8 in $\mathcal{R}(3K_2, K_3)$ is enough to consider F containing B_1, B_2, B_3, B_4 , or B_5 . Next theorem, we consider the graphs $C_8(1, 2)$, F_1 , and F_2 in Figure 3. The graph $C_8(1, 2)$ is a circulant graph with the vertex set $\{v_0, v_1, \dots, v_7\}$ in which the i^{th} vertex is adjacent to the $(i + j)^{th}$ and $(i - j)^{th}$ vertices for each $i \in [0, 7]$ and $j \in \{1, 2\}$.

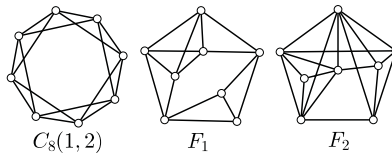


FIGURE 3. The graphs of order 8 in $\mathcal{R}(3K_2, K_3)$.

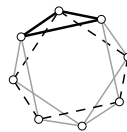


FIGURE 4. A red-blue coloring on edges of $C_8(1, 2)$

Theorem 2. The graphs $C_8(1, 2)$, F_1 , and F_2 are the only connected graphs of order 8 in $\mathcal{R}(3K_2, K_3)$.

Proof. We can show easily that the graphs $C_8(1, 2)$, F_1 , and F_2 satisfy Lemma 1. The proof of the minimality of $C_8(1, 2)$, F_1 , and F_2 is described as follows. We color all edges of $C_8(1, 2)$ by red and blue so that all red edges are depicted by dash lines. Observe that such a coloring induces a red $2K_2$ and blue K_3 (drawn in bold line). Thus, removing an arbitrary bold line edge e in K_3 results in a $(3K_2, K_3)$ -coloring of $C_8(1, 2) - e$. Since the circulant graph $C_8(1, 2)$ is a regular graph of degree 4, for any $e \in E(C_8(1, 2))$, a $(3K_2, K_3)$ -coloring of $C_8(1, 2) - e$ can be represented

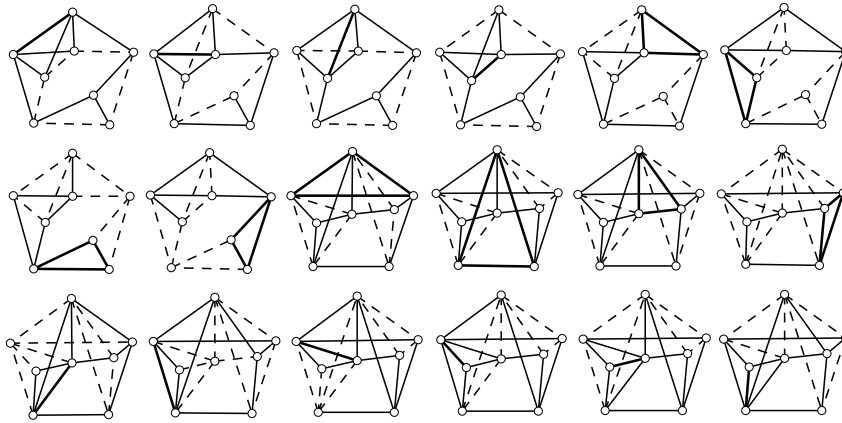


FIGURE 5. Some red-blue coloring on edges of F_1 and F_2 , where the dash line represents the red color.

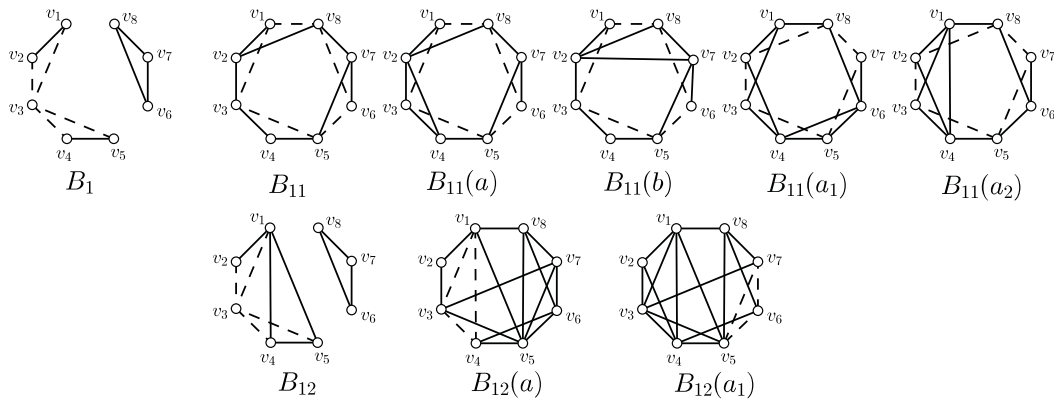


FIGURE 6. The construction of graph F of order 8 in $\mathcal{R}(3K_2, K_3)$ by applying Corollary 1 and Lemma 1 when $F \supseteq B_1$.

in Figure 4. In the same fashion it can be done for the other edges of $C_8(1, 2)$. Meanwhile, some red-blue coloring of F_1 and F_2 are depicted in Figure 5, where the bold lines represent the removed edge.

Let $F \in \mathcal{R}(3K_2, K_3)$ be a connected graph of order 8. Then, F contains the graph B_1, B_2, B_3, B_4 , or B_5 , in Figure 2. First, we consider $F \supseteq B_1$, where $V(B_1) = \{v_1, v_2, \dots, v_8\}$ as depicted in Figure 6. By Corollary 1, $F - \{v_3\}$ must contain a $2K_3, K_5$, or A but it does not contain a graph B_3, B_4 , or B_5 . So, there are two possibilities, namely the graph B_{11} or B_{12} as depicted in Figure 6. Moreover, we consider $F \supseteq B_{11}$. Since for every $v \in V(B_{11})$ and triangle in B_{11} , both graphs $F - \{v\}$ and $F - E(K_3)$ contain a $2K_3 \in \mathcal{R}(2K_2, K_3)$, then we apply Lemma 1. By Lemma 1, $F - E(S)$ must contain a triangle K_3 , for $V(S) = \{v_1, v_3, v_5, v_6, v_8\}$. So, up to isomorphism, F must contain a triangle with the set $\{v_2, v_3, v_4\}$ or $\{v_2, v_7, v_8\}$ as depicted by the graph $B_{11}(a)$ or $B_{11}(b)$, respectively, in Figure 6. For $F \supseteq B_{11}(a)$, by Lemma 1, $F - E(S)$ must contain a triangle K_3 , for $V(S) = \{v_2, v_3, v_5, v_7, v_8\}$. Up to isomorphism, the triangle K_3 in $F - E(S)$ is formed by the vertex set $\{v_4, v_5, v_6\}$ or $\{v_1, v_3, v_4\}$ as depicted by the graph $B_{11}(a_1)$ or $B_{11}(a_2)$, respectively, in Figure 6. Otherwise, F is not minimal. The graph $B_{11}(a_2)$ is isomorphic to the graph F_1 . Now, we consider $F \supseteq B_{11}(a_1)$. By Lemma 1, $F - E(S)$ must contain a triangle K_3 , for $V(S) = \{v_2, v_3, v_5, v_6, v_8\}$. By the minimality, up to isomorphism, the triangle K_3 in F is only formed by the vertex set $\{v_1, v_7, v_8\}$ which is isomorphic to $C_8(1, 2)$. For $F \supseteq B_{11}(b)$, it can be done in the same fashion.

Furthermore, we consider F contains the graph B_{12} . By Corollary 1, for a triangle K_3 with the set $\{v_1, v_3, v_4\}$, $F - E(K_3)$ must contain a $2K_3, K_5$, or A , but it does not contain the graph B_3, B_4, B_5 , or B_{11} . Hence, $F - E(K_3)$ contains the graph A as depicted by the graph $B_{12}(a)$ in Figure 6. Next, by Corollary 1, for a triangle K_3 with the set

$V(K_3) = \{v_5, v_6, v_7\}$, $F - E(K_3)$ must contain a $2K_3, K_5$, or A . By the minimality, $F - E(K_3)$ contains a $2K_3$ as depicted by the graph $B_{12}(a_1)$, in Figure 6. The graph $B_{12}(a_1)$ is isomorphic to F_2 . In the same argument, it can be done for F containing B_2, B_3, B_4 , or B_5 . Thus, the graphs of order 8 in $\mathcal{R}(3K_2, K_3)$ are $C_8(1, 2), F_1$, and F_2 . \square

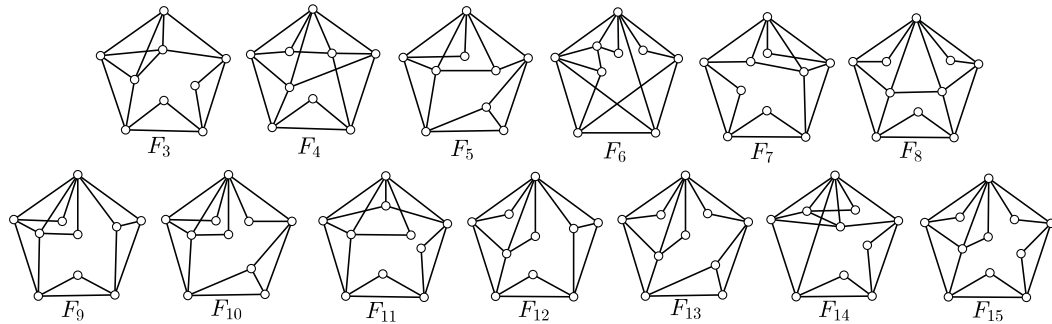


FIGURE 7. The graphs of order at least 9 in $\mathcal{R}(3K_2, K_3)$.

Next theorem, we consider all graphs in Figure 7, namely F_3, F_4, \dots, F_{15} .

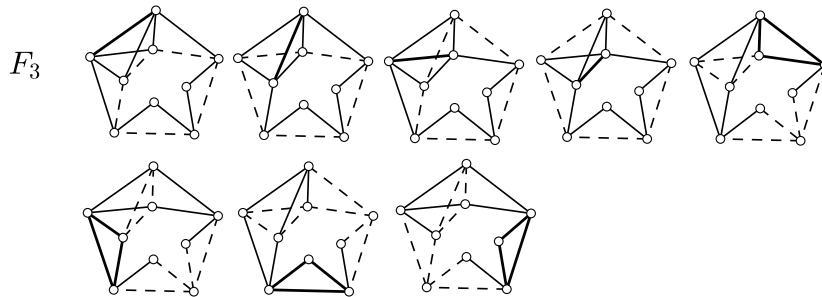


FIGURE 8. Some red-blue coloring on edges of F_3 , where the dash line represents the red color.

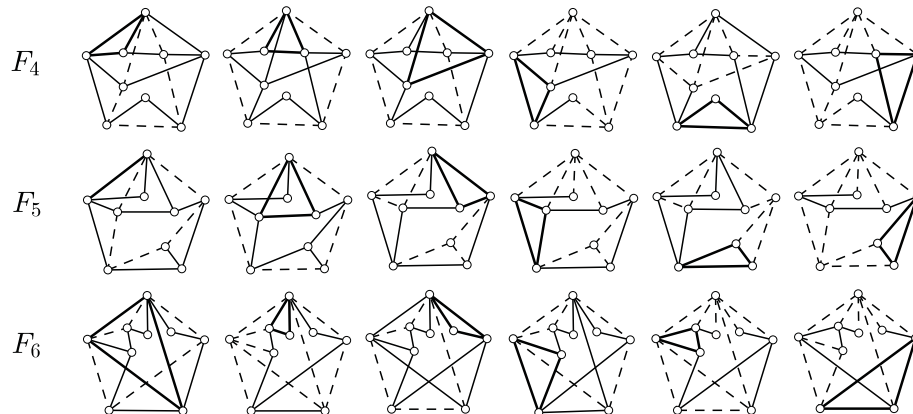


FIGURE 9. Some red-blue coloring on edges of F_4, F_5 , and F_6 , where the dash line represents the red color.

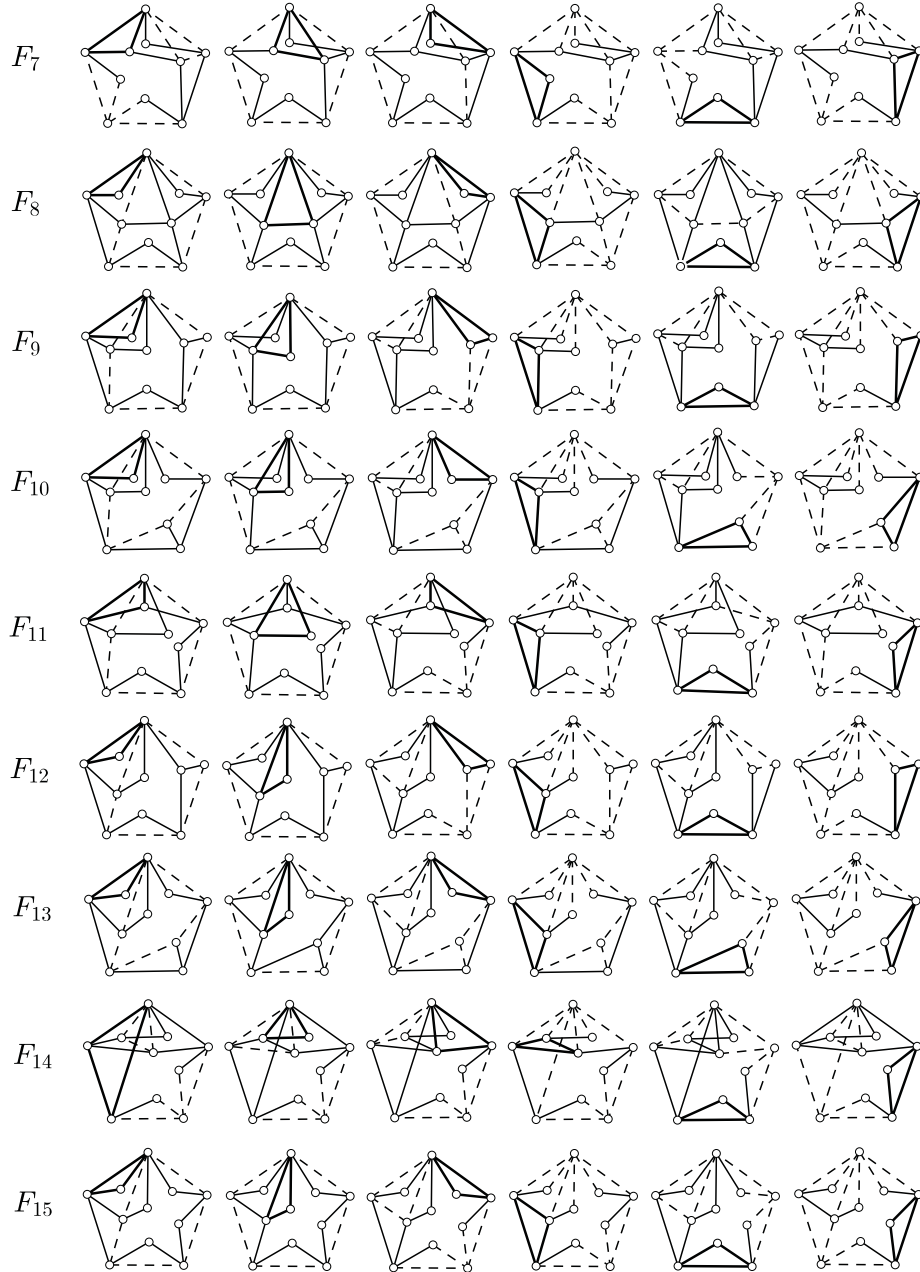


FIGURE 10. Some red-blue coloring on edges of F_7, F_8, \dots, F_{15} , where the dash line represents the red color.

Theorem 3. *The graphs F_3, F_4, \dots, F_{15} are the only connected graphs of order at least 9 in $\mathcal{R}(3K_2, K_3)$.*

Proof. It is easily noticed that the graphs F_3, F_4, \dots, F_{14} , and F_{15} satisfy Lemma 1. The proof of the minimality of the graphs F_3, F_4, \dots, F_{14} , and F_{15} is depicted in Figure 8, 9, and 10. Furthermore, let F be a graph of order at least 9 in $\mathcal{R}(3K_2, K_3)$. Then, F contains B_i for $i \in [1, 8]$ (see Figure 2). Next, we apply Corollary 1 and Lemma 1 such that four conditions in Lemma 1 are satisfied. These can be done in the same fashion as the graphs of order 8 in $\mathcal{R}(3K_2, K_3)$. \square

ACKNOWLEDGMENTS

This research was supported by Research Grant "Program Hibah Riset Unggulan ITB-DIKTI", Ministry of Research, Technology and Higher Education, Indonesia.

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