AIP Conference Proceedings

scitation.org/journal/apc

Volume 2326

The Third International Conference on Mathematics
Education, Theory and Application

Surakarta, Indonesia • 20 October 2020

Editors • Diari Indriati, Tri Atmojo Kusmayadi, Sutrima Sutrima, Dewi Retno Sari Saputro and Putranto Hadi Utomo

AIP Publishing

Preface: The 3rd International Conference on Mathematics: Education, Theory, and Application (ICMETA)

Cite as: AIP Conference Proceedings **2326**, 010001 (2021); https://doi.org/10.1063/12.0003103
Published Online: 08 February 2021

Preface: The 3rd International Conference on Mathematics: Education, Theory, and Application (ICMETA)

We are pleased to introduce conference proceedings of the 3rd. International Conference on Mathematics: Education, Theory, and Application which was held on October 20, 2020 in Surakarta, Central of Java, Indonesia. Due to the covid-19 pandemic, the conference was held online by zoom application. The main objective of this conference is to bring together academic scientists, researchers, practitioners, teachers and students to exchange and share their experiences and research results on most aspects of science and mathematics, as well as discuss practical challenges faced and the solutions adopted. The theme of the 3rd International Conference on Mathematics: Education, Theory and Application is "Empowering of Research and Learning Mathematics towards 4.0 Industrial Revolution Era".

At this conference, there are research papers of several researchers from institutions and several universities, which was presented in the form of a parallel oral presentation class. Besides paper presentation in the parallel class, the committee also invited five keynote speakers, namely Prof. Andrea Semanicova-Fenovcikova Ph.D. from the Technical University, Kosice, Slovakia, Prof. Shompong Jitman, Ph.D. from Silpakorn University, Nakhon Pathom, Thailand, Prof. Khalil Ezzinbi from Cadi Ayyat University, Marrakesh, Prof. Dr. Marsudi W. Kisworo from Perbanas Institute and PT. TELKOM, Indonesia, and Dr. Putranto Hadi Utomo from the Department of Mathematics, Sebelas Maret University, Surakarta, Indonesia.

We would like to thank you for the advice and support given by Prof. Martin Baca from Technical University, Kosice, Slovakia, Dr. S. Lavanya from the Department of Mathematics, Bharati Woman College, Chennai, Tamil Nadu, India, and Professor Widodo from Gadjah Mada University, Yogyakarta, Indonesia, for the success of the conference. We would also like to thanks to the organization staff, program committee members, and reviewers. They have worked very hard on reviewing papers and providing valuable suggestions for authors to improve their work. We would also like to thank the external reviewers, who provided extra assistance in the review process, and the authors who contributed their research to the conference.

The program, which is held every two years, provides the opportunity for all participants to interact with each other, can broaden scientific insight and expand communication networks among researchers and institutions. We look forward to the participant's experience in the 3rd. ICMETA 2020 is paying off and it lasts a long time.

We also hope to see all of you at the same conference in the next 2 years.

The Third International Conference on Mathematics
AIP Conf. Proc. 2326, 010001-1–010001-1; https://doi.org/10.1063/12.0003103
Published by AIP Publishing. 978-0-7354-4070-8/$30.00

Table of Contents

‹PREV NEXT›

THE THIRD INTERNATIONAL CONFERENCE ON MATHEMATICS: Education, Theory and Application

Conference date: 20 October 2020
Location: Surakarta, Indonesia
ISBN: 978-0-7354-4070-8
Editors: Diari Indriati, Tri Atmojo Kusmayadi, Sutrima Sutrima, Dewi Retno Sari Saputro and Putranto Hadi Utomo
Volume number: 2326
Published: Feb 8, 2021

DISPLAY : 20 50 100 all

PRELIMINARY

PDF E-READER ADD TO FAVORITES SHARE EXPORT CITATION

ICMETA PROCEEDING

Free . February 2021

Graceful labeling on a multiple-fan graph with pendants

A. Akerina and K. A. Sugeng

AIP Conference Proceedings **2326**, 020001 (2021); https://doi.org/10.1063/5.0039411

SHOW ABSTRACT | PDF | E-READER ADD TO FAVORITES SHARE EXPORT CITATION

Free . February 2021

The effectiveness of outpatient installation of Rumah Sakit Jiwa Daerah (RSJD) Surakarta through queueing analysis

G. Angela, I. Slamet, R. Gupta and S. Sugiyanto

AIP Conference Proceedings **2326**, 020002 (2021); https://doi.org/10.1063/5.0039284

SHOW ABSTRACT | PDF | E-READER ADD TO FAVORITES SHARE EXPORT CITATION

Free . February 2021

Super (a,d)-H-antimagic total labeling of generalized fan corona product with path and generalized fan corona product with cycle

Doni Ardiyanto, Titin Sri Martini and Santoso Budi Wiyono

AIP Conference Proceedings **2326**, 020003 (2021); https://doi.org/10.1063/5.0040078

SHOW ABSTRACT | PDF | E-READER ADD TO FAVORITES SHARE EXPORT CITATION

Free . February 2021

The total profit model of a manufacturer – Two retailers through online and offline sales media

Dian Dwi Aryani, Ririn Setiyowati and Santoso Budi Wiyono

AIP Conference Proceedings **2326**, 020004 (2021); https://doi.org/10.1063/5.0039549

SHOW ABSTRACT | PDF | E-READER ADD TO FAVORITES SHARE EXPORT CITATION

Free . February 2021

The diameter of annihilator graph of non-commutative semirings

Muhammad Rofi' Ashidiqi, Vika Yugi Kurniawan and Putranto Hadi Utomo

AIP Conference Proceedings **2326**, 020005 (2021); https://doi.org/10.1063/5.0042278

SHOW ABSTRACT | PDF | E-READER ADD TO FAVORITES SHARE EXPORT CITATION

Free . February 2021

Edge irregular reflexive labeling on tadpole graphs $T_{m,1}$ and $T_{m,2}$

Nadia Indarwati Setia Budi, Diari Indriati and Bowo Winarno

AIP Conference Proceedings **2326**, 020006 (2021); https://doi.org/10.1063/5.0039337

SHOW ABSTRACT | PDF | E-READER | ADD TO FAVORITES | SHARE | EXPORT CITATION

Free . February 2021

Solving capacitated vehicle routing problem using saving matrix, sequential insertion, and nearest neighbor of product 'X' in Grobogan district

Nur Anisa Fitriani, Rama Aditya Pratama, Siti Zahro, Putranto Hadi Utomo and Titin Sri Martini

AIP Conference Proceedings **2326**, 020007 (2021); https://doi.org/10.1063/5.0039295

SHOW ABSTRACT | PDF | E-READER | ADD TO FAVORITES | SHARE | EXPORT CITATION

Free . February 2021

Schemes of network coding on fan graph $F_{2,4}$ and closed helm graph CH_4

Himawanti Rizqi Khasanah, Putranto Hadi Utomo and Bowo Winarno

AIP Conference Proceedings **2326**, 020008 (2021); https://doi.org/10.1063/5.0039774

SHOW ABSTRACT | PDF | E-READER | ADD TO FAVORITES | SHARE | EXPORT CITATION

Free . February 2021

The eccentric digraph of caterpillar graph and generalized Jahangir graph

Tri Atmojo Kusmayadi and Nugroho Arif Sudibyo

AIP Conference Proceedings **2326**, 020009 (2021); https://doi.org/10.1063/5.0040222

SHOW ABSTRACT | PDF | E-READER | ADD TO FAVORITES | SHARE | EXPORT CITATION

Free . February 2021

Final size distribution of stochastic susceptible infected recovered (SIR) epidemic model

Jane Lola Mahasmara, Respatiwulan and Yuliana Susanti

AIP Conference Proceedings **2326**, 020010 (2021); https://doi.org/10.1063/5.0039277

SHOW ABSTRACT | PDF | E-READER | ADD TO FAVORITES | SHARE | EXPORT CITATION

Free . February 2021

Conflict driven clause learning approach for satisfiability modulo theory

Vie'an Huzair Majalawa, Putranto Hadi Utomo, Tri Atmojo Kusmayadi and Diari Indriati

AIP Conference Proceedings **2326**, 020011 (2021); https://doi.org/10.1063/5.0039296

SHOW ABSTRACT | PDF E-READER ADD TO FAVORITES SHARE EXPORT CITATION

Free . February 2021

Zero annihilator graph of semiring of matrices over Boolean semiring

Haniiam Mariiaa, Vika Yugi Kurniawan and Sutrima

AIP Conference Proceedings **2326**, 020012 (2021); https://doi.org/10.1063/5.0039557

SHOW ABSTRACT | PDF E-READER ADD TO FAVORITES SHARE EXPORT CITATION

Free . February 2021

On H supermagic labeling and super antimagic labeling of $C_n \diamond W_n$

Titin Sri Martini

AIP Conference Proceedings **2326**, 020013 (2021); https://doi.org/10.1063/5.0039312

SHOW ABSTRACT | PDF E-READER ADD TO FAVORITES SHARE EXPORT CITATION

Free . February 2021

Forecasting cash outflow and inflow in Jember with ARIMAX calendar variation effect

Marieta Monica and Agus Suharsono

AIP Conference Proceedings **2326**, 020014 (2021); https://doi.org/10.1063/5.0039283

SHOW ABSTRACT | PDF E-READER ADD TO FAVORITES SHARE EXPORT CITATION

Free . February 2021

The application of Black-Litterman Bayesian model for the portfolio optimization on the liquid index 45 (LQ45) with information ratio assessment

Uswatul Auliya Murtadina, Dewi Retno Sari Saputro and Putranto Hadi Utomo

AIP Conference Proceedings **2326**, 020015 (2021); https://doi.org/10.1063/5.0039684

SHOW ABSTRACT | PDF E-READER ADD TO FAVORITES SHARE EXPORT CITATION

Free . February 2021

Edge irregular reflexive labeling on banana tree graphs $B_{2,n}$ and $B_{3,n}$

Jihan Almaas Novelia and Diari Indriati

AIP Conference Proceedings **2326**, 020016 (2021); https://doi.org/10.1063/5.0039316

SHOW ABSTRACT | PDF | E-READER | ADD TO FAVORITES | SHARE | EXPORT CITATION

Free . February 2021

Rasch analysis on item response theory: Review of model suitability

Triwik Jatu Parmaningsih and Dewi Retno Sari Saputro

AIP Conference Proceedings **2326**, 020017 (2021); https://doi.org/10.1063/5.0040305

SHOW ABSTRACT | PDF | E-READER | ADD TO FAVORITES | SHARE | EXPORT CITATION

Free . February 2021

On the local metric dimension of dipyramidal graph and king graph

Javas Alfreda Belva Yoga Pratama and Tri Atmojo Kusmayadi

AIP Conference Proceedings **2326**, 020018 (2021); https://doi.org/10.1063/5.0039319

SHOW ABSTRACT | PDF | E-READER | ADD TO FAVORITES | SHARE | EXPORT CITATION

Free . February 2021

Odd harmonious labeling of two graphs containing star

Diah Ayu Pujiwati, Ikhsanul Halikin and Kristiana Wijaya

AIP Conference Proceedings **2326**, 020019 (2021); https://doi.org/10.1063/5.0039644

SHOW ABSTRACT | PDF | E-READER | ADD TO FAVORITES | SHARE | EXPORT CITATION

Free . February 2021

The diameter of annihilator ideal graph of $\mathbb{Z}_n$

Ami Rahmawati, Vika Yugi Kurniawan and Supriyadi Wibowo

AIP Conference Proceedings **2326**, 020020 (2021); https://doi.org/10.1063/5.0039847

SHOW ABSTRACT | PDF | E-READER | ADD TO FAVORITES | SHARE | EXPORT CITATION

Free . February 2021

Edge irregular reflexive labeling on umbrella graphs $U_{3,n}$ and $U_{4,n}$

Nabilla Ayu Rahmawati and Diari Indriati

AIP Conference Proceedings **2326**, 020021 (2021); https://doi.org/10.1063/5.0039336

SHOW ABSTRACT | PDF | E-READER | ADD TO FAVORITES | SHARE | EXPORT CITATION

Free . February 2021

Numerical solution of the one dimentional shallow water wave equations using finite difference method : Lax-Friedrichs scheme

V. A. Riestiana, R. Setiyowati and V. Y. Kurniawan

AIP Conference Proceedings **2326**, 020022 (2021); https://doi.org/10.1063/5.0039545

SHOW ABSTRACT | PDF | E-READER | ADD TO FAVORITES | SHARE | EXPORT CITATION

Free . February 2021

Clustering human development index data with gravitational search algorithm-fuzzy 4-means (GSA-F4M)

Dewi Syifaur Rohmah, S. Dewi Retno Sari and K. Vika Yugi

AIP Conference Proceedings **2326**, 020023 (2021); https://doi.org/10.1063/5.0039661

SHOW ABSTRACT | PDF | E-READER | ADD TO FAVORITES | SHARE | EXPORT CITATION

Free . February 2021

On totally irregular total labeling of caterpillars having even number of internal vertices with degree three

Isnaini Rosyida, Mulyono and Diari Indriati

AIP Conference Proceedings **2326**, 020024 (2021); https://doi.org/10.1063/5.0039314

SHOW ABSTRACT | PDF | E-READER | ADD TO FAVORITES | SHARE | EXPORT CITATION

Free . February 2021

The modeling of human development index (HDI) in Papua–Indonesia using geographically weighted ridge regression (GWRR)

Dewi Retno Sari Saputro, Rina Dwi Hastutik and Purnami Widyaningsih

AIP Conference Proceedings **2326**, 020025 (2021); https://doi.org/10.1063/5.0040329

SHOW ABSTRACT | PDF | E-READER | ADD TO FAVORITES | SHARE | EXPORT CITATION


Scilight
Sign up for free weekly alerts!
AIP Publishing

Odd harmonious labeling of two graphs containing star

Cite as: AIP Conference Proceedings **2326**, 020019 (2021); https://doi.org/10.1063/5.0039644
Published Online: 08 February 2021

Diah Ayu Pujiwati, Ikhsanul Halikin, and Kristiana Wijaya

Odd Harmonious Labeling of Two Graphs Containing Star

Diah Ayu Pujiwati,[a)] Ikhsanul Halikin,[b)] and Kristiana Wijaya[c)]

Graph Combinatorics and Algebra Research Group,
Department of Mathematics-Faculty of Mathematics and Natural Sciences,
Universitas Jember (UNEJ),
Jl. Kalimantan 37 Jember 68121 Indonesia.

a) diahayupujiwati@gmail.com
b) ikhsan.fmipa@unej.ac.id
c)Corresponding author: kristiana.fmipa@unej.ac.id

Abstract. An odd harmonious labeling of a graph G is an injective function $f : V(G) \rightarrow \{0,1,2,\ldots,2|E(G)|-1\}$ such that the induced function $f^* : E(G) \rightarrow \{1,3,\ldots,2|E(G)|-1\}$ defined by $f^*(xy) = f(x) + f(y)$ is a bijection. A graph that admits odd harmonious labeling is called an odd harmonious graph. The concept of odd harmonious labeling was initiated by Liang and Bai in 2009. By the result of Liang and Bai, a star is an odd harmonious graph. Motivated by a result, we prove that two graphs containing star are still odd harmonious. In this case, we prove that a double stars is an odd harmonious graph. The remaining we prove that an even cycle and a star which is sharing a common vertex is also an odd harmonious graph.

INTRODUCTION

Throughout here, all graphs are simple, undirected, and connected. The graph labeling is an assignment of integers to the set of vertices or edges or both, subject to certain conditions. Graph labeling was first introduced in the mid1960s. There are many various of graphs labeling. In the intervening 50 years over 200 graph labelings techniques have been studied in over 2800 papers. An exhaustive survey of graph labeling can be seen in Gallian [1].

A harmonious labeling was introduced by Graham and Sloane [2] in 1980. A harmonious labeling of a graph G is an injective function $f : V(G) \longrightarrow \{0,1,\ldots,|E(G)|-1\}$ such that the induced function $f^* : E(G) \longrightarrow \{0,1,\ldots,|E(G)|-1\}$, defined by $f^*(xy) = f(x) + f(y)$ for each edge $xy \in E(G)$, is a bijection. Two various problems of harmonious la-beling are odd harmonious labeling introduced by Liang and Bai [3] in 2009 and even harmonious labeling introduced by Sarasija and Binthiya [4] in 2011. In this paper, we focus on odd harmonious labeling of a graph.

A graph G is an odd harmonious if there exists an injective function $f : V(G) \longrightarrow \{0,1,\ldots,2|E(G)|-1\}$ such that the induced function $f^*(xy) = f(x) + f(y)$ from the edges of G to the odd integers from 1 to $2|E(G)|-1$ is a bijection. Liang and Bai [3] have obtained the necessary conditions for the existence of odd harmonious labeling of graph, by theorem below.

Theorem 1. [3] *Let G be a graph.*
(i) If G is odd harmonious, then G is a bipartite.
(ii) If G is odd harmonious, then $2\sqrt{|E(G)|} \leq |V(G)| \leq 2|E(G)|-1$.

Liang and Bai [3] also proved that a cycle C_n is odd harmonious if and only if $n \equiv 0 \bmod 4$, K_2 is the only one of complete graph which is odd harmonious, a complete k-partite graph $K_{n_1,n_2,\ldots,n_k}$ is odd harmonious if and only if $k = 2$. Vaidya and Shah [5] proved that the shadow and the splitting graphs of path P_n and star $K_{1,n}$ are odd harmonious graphs. A star $K_{1,n}$ is a connected graph with one center vertex of degree n and n vertices of degree one. Many researcher have showed many classes of graph admitted odd harmonious labeling, see [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16].

In this paper, we investigate whether two graphs containing stars are odd harmonious or not. The first graph is a double star $S_{m,n}$, that is a graph obtained from two stars $K_{1,m}$ and $K_{1,n}$, where its center vertex is adjacent. So, a double star $S_{m,n}$ has $m+n+2$ vertices and $m+n+1$ edges. We investigate whether a double stars $S_{m,n}$ is an odd harmonious graph for every positive integers m and n or not. We also investigate whether the graphs obtained by a identification

The Third International Conference on Mathematics
AIP Conf. Proc. 2326, 020019-1–020019-5; https://doi.org/10.1063/5.0039644
Published by AIP Publishing. 978-0-7354-4070-8/$30.00

operation of two graphs, a cycle and star, are odd harmonious or not. The definition of identification of two graphs, we follow Wijaya and Baskoro [17]. Let G and H be graphs, $x \in V(G)$ and $y \in V(H)$. The graph $G \odot_{x,y} H$ is a graph obtained from two disjoint graphs G and H by identifying vertices $x \in V(G)$ and $y \in V(H)$. So, the graph $G \odot_{x,y} H$ has $(|V(G)| + |V(H)| - 1)$ vertices and $(|E(G)| + |E(H)|)$ edges.

ODD HARMONIOUS LABELING OF DOUBLE STARS

Let $V(S_{m,n}) = \{c_i | 1 \le i \le 2\} \cup \{x_i \cup y_j | 1 \le i \le m,\ 1 \le j \le n\}$ be the vertex-set of the double stars $S_{m,n}$, where $d(c_1) = m+1$, $d(c_2) = n+1$, and $d(x_i) = d(y_j) = 1$, for each $i \in [1,m]$ and $j \in [1,n]$. Let again $E(S_{m,n}) = \{c_1c_2, c_1x_i, c_2y_j | 1 \le i \le m, 1 \le j \le n\}$ be the edge-set of the double star $S_{m,n}$. We prove that the double stars $S_{m,n}$ is an odd harmonious graph for each m and n natural numbers.

Theorem 2. *Let m and n be natural numbers. Then a double star $S_{m,n}$ is odd harmonious.*

Proof. First, we consider the case where $m = n$. An injective function $f : V(S_{m,n}) \to \{0, 1, 2, \ldots, 2m+2n+1\}$ is defined by

$$f(c_k) = k-1, \quad \text{for } 1 \le k \le 2,$$
$$f(x_i) = 4i-1, \quad \text{for } 1 \le i \le m,$$
$$f(y_j) = 4j, \quad \text{for } 1 \le j \le n.$$

We get the induced edge labels as follows.

$$f^*(c_1c_2) = 1,$$
$$f^*(c_1x_i) = 4i-1, \quad \text{for } 1 \le i \le m,$$
$$f^*(c_2y_j) = 4j+1, \quad \text{for } 1 \le j \le n.$$

The induced label of edges are $\{1, 3, \ldots, 2m+2n+1\}$. It is bijective function $f^* : E(S_{m,n}) \longrightarrow \{1, 3, \ldots, 2m+2n+1\}$ defined by $f^*(xy) = f(x) + f(y)$.

Next, for $m < n$, we now define an injective function $f : V(S_{m,n}) \to \{0, 1, \ldots, 2m+2n+1\}$ by

$$f(c_k) = k-1, \quad \text{for } 1 \le k \le 2,$$
$$f(x_i) = 4i+1, \quad \text{for } 1 \le i \le m,$$
$$f(y_j) = \begin{cases} 4j-2, & \text{for } 1 \le j \le m+1, \\ 2m+2j, & \text{for } m+2 \le j \le n. \end{cases}$$

So, we obtain the induced function $f^* : E(S_{m,n}) \to \{1, 3, \ldots, 2m+2n+1\}$ as follows.

$$f^*(c_1c_2) = 1,$$
$$f^*(c_1x_i) = 4i+1, \quad \text{for } 1 \le i \le m,$$
$$f^*(c_2y_j) = \begin{cases} 4j-1, & \text{for } 1 \le j \le m+1, \\ 2m+2j+1, & \text{for } m+2 \le j \le n. \end{cases}$$

It is easily seen that f^* is bijection. Hence, $S_{m,n}$ is odd harmonious for any natural numbers m and n. □

For example, an odd harmonious labeling of a double star $S_{4,4}$ is depicted in Figure 1.

ODD HARMONIOUS LABELING OF $C_m \odot_{x,y} K_{1,n}$

There are two non-isomorphic graphs of $C_m \odot_{x,y} K_{1,n}$ for each $x \in V(C_m)$ and $y \in V(K_{1,n})$. Since the cycle C_m is a regular graph, these differences is determined by the identification vertex of a star $K_{1,n}$, namely when y is either the center or a pendant vertex of $K_{1,n}$. Two next theorem prove that two non-isomorphic graphs of $C_m \odot_{x,y} K_{1,n}$ for each $x \in V(C_m)$ and $y \in V(K_{1,n})$ are odd harmonious only if m is even. Before we discuss these theorem, we prove that for m odd, the graph $C_m \odot_{x,y} K_{1,n}$ is not odd harmonious for each $x \in V(C_m)$ and $y \in V(K_{1,n})$ by the following lemma.

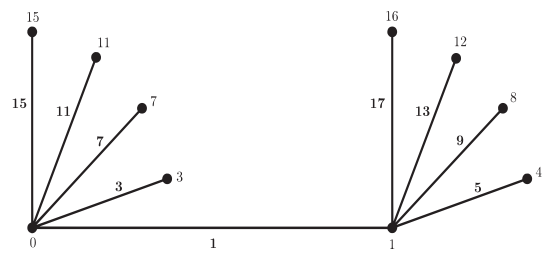

FIGURE 1. An odd harmonious labeling of $S_{4,4}$.

Lemma 3. *If m is odd then $C_m \odot_{x,y} K_{1,n}$ is not odd harmonious for each $x \in V(C_m)$ and $y \in V(K_{1,n})$.*

Proof. Suppose m is odd. Then $C_m \odot_{x,y} K_{1,n}$ is not bipartite graph for each $x \in V(C_m)$ and $y \in V(K_{1,n})$. According to the contrapositive of Theorem 1(i), $C_m \odot_{x,y_0} K_{1,n}$ is not odd harmonious. □

Theorem 4. *Let y_0 be the center of $K_{1,n}$. Then for each $x \in V(C_m)$, the graph $C_m \odot_{x,y_0} K_{1,n}$ is odd harmonious if and only if m is even.*

Proof. First we prove that if $C_m \odot_{x,y_0} K_{1,n}$ is odd harmonious then m is even. Suppose to the contrary that m is odd. By Lemma 3, $C_m \odot_{x,y_0} K_{1,n}$ is not odd harmonious, a contradiction. Hence, it should be m even.

Next, we prove that for m even, $C_m \odot_{x,y_0} K_{1,n}$ is odd harmonious, by given odd harmonious labeling of $C_m \odot_{x,y_0} K_{1,n}$. Let $V(C_m \odot_{x,y_0} K_{1,n}) = \{z\} \cup \{x_i | 1 \le i \le m-1\} \cup \{y_j | 1 \le j \le n\}$ be the vertex-set of $C_m \odot_{x,y_0} K_{1,n}$, where $d(z) = n+2$, $d(x_i) = 2$, and $d(y_j) = 1$ for $i \in [1, m-1]$ and $j \in [1, n]$. In this case, z is the identified vertex by $x_m \in V(C_m)$ and $y_0 \in V(K_{1,n})$. Let again $E(C_m \odot_{x,y_0} K_{1,n}) = \{zx_1, zx_{m-1}\} \cup \{x_i x_{i+1} | 1 \le i \le m-2\} \cup \{zy_j | 1 \le j \le n\}$ be the edge-set of $C_m \odot_{x,y_0} K_{1,n}$.

We break up the proof into two cases.

(i) For $m \equiv 0 \bmod 4$. Let $f : V(C_m \odot_{x,y_0} K_{1,n}) \to \{0, 1, \ldots, 2m+2n-1\}$ be an injective function defined by

$$f(z) = 0,$$

$$f(x_i) = \begin{cases} i, & \text{for } i \text{ odd and } 1 \le i \le \frac{m}{2} - 1, \\ i+2, & \text{for } i \text{ odd and } \frac{m}{2} + 1 \le i \le m-1, \\ i, & \text{for } i \text{ even.} \end{cases}$$

$$f(y_j) = 2m + 2j - 1, \quad \text{for} \quad 1 \le j \le n.$$

Therefore for each $xy \in E(C_m \odot_{x,y_0} K_{1,n})$, we obtain $f^*(xy) = f(x) + f(y)$ as follows.

$$f^*(zx_i) = \begin{cases} 1, & \text{for } i = 1, \\ m+1, & \text{for } i = m-1, \end{cases}$$

$$f^*(x_i x_{i+1}) = \begin{cases} 2i+1, & \text{for } 1 \le i \le \frac{m}{2} - 1, \\ 2i+3, & \text{for } \frac{m}{2} \le i \le m-2, \end{cases}$$

$$f^*(zy_j) = 2m + 2j - 1, \quad \text{for } 1 \le j \le n.$$

It is easy to see that f^* is bijective from the edge-set $E(C_m \odot_{x,y_0} K_{1,n})$ to the set of odd integers $\{1, 3, \ldots, 2m + 2n - 1\}$.

(ii) For $m \equiv 2 \bmod 4$, let an injective function $f : V(C_m \odot_{x,y_0} K_{1,n}) \to \{0, 1, \ldots, 2m+2n-1\}$ be defined by

$$f(z) = 0,$$

$$f(x_i) = \begin{cases} i, & \text{for } i \text{ odd and } 1 \le i \le m-3, \\ m+1, & \text{for } i \text{ odd and } i = m-1, \\ i, & \text{for } i \text{ even and } 2 \le i \le \frac{m}{2} - 1, \\ i+2, & \text{for } i \text{ even and } \frac{m}{2} + 1 \le i \le m-2, \end{cases}$$

$$f(y_j) = \begin{cases} 2m-1, & \text{for } j = 1, \\ 2m + 2j - 1, & \text{for } 2 \le j \le n. \end{cases}$$

Then, by $f^*(xy) = f(x) + f(y)$, we obtain $f^* : E(C_m \odot_{x,y_0} K_{1,n}) \to \{1,3,\ldots,2m+2n-1\}$ as follows.

$$f^*(zx_i) = \begin{cases} 1, & \text{for } i = 1, \\ m+1, & \text{for } i = m-1, \end{cases}$$

$$f^*(x_ix_{i+1}) = \begin{cases} 2i+1, & \text{for } 1 \le i \le \frac{m}{2} - 1, \\ 2i+3, & \text{for } \frac{m}{2} \le i \le m-3, \\ 2m+1, & \text{for } i = m-2, \end{cases}$$

$$f^*(zy_j) = \begin{cases} 2m-1, & \text{for } j = 1, \\ 2m+2j-1, & \text{for } 2 \le j \le n. \end{cases}$$

We obtain $R_{f^*} = \{1, m+1\} \cup \{3,5,\ldots,m-1\} \cup \{m+3, m+5, \ldots, 2m-3, 2m+1\} \cup \{2m-1, 2m+3, 2m+5, \ldots, 2m+2n-1\} = \{1,3,\ldots,2m+2n-1\}$ which is the same as codomain of f^*. Since $|R_{f^*}| = |D_{f^*}| = m+n$, we conclude that f^* is bijective. □

As an illustration, Figure 2 presents an odd harmonious labeling on $C_8 \odot_{x,y_0} K_{1,6}$.

Theorem 5. *Let y be the vertex of degree 1 of $K_{1,n}$. Then for each $x \in V(C_m)$, the graph $C_m \odot_{x,y} K_{1,n}$ is an odd harmonious if and only if m is even.*

Proof. By Lemma 3, we enough prove that for m even, $C_m \odot_{x,y} K_{1,n}$ is odd harmonious, by given odd harmonious labeling of $C_m \odot_{x,y} K_{1,n}$. Let $V(C_m \odot_{x,y} K_{1,n}) = \{z\} \cup \{x_i | 1 \le i \le m-1\} \cup \{y_0\} \cup \{y_j | 1 \le j \le n-1\}$ be the vertex-set of $C_m \odot_{x,y} K_{1,n}$, where x_i has degree 2, y_0 has degree n, y_j has degree 1, and z has degree 3, where z is the identified vertex by $x \in V(C_m)$ and $y \in V(K_{1,n})$. Suppose now $E(C_m \odot_{x,y} K_{1,n}) = \{zx_1\} \cup \{zx_{m-1}\} \cup \{x_ix_{i+1} | 1 \le i \le m-2\} \cup \{zy_0\} \cup \{y_0y_j | 1 \le j \le n-1\}$ is the edge-set of $C_m \odot_{x,y} K_{1,n}$.

Suppose an injective function $f : V(C_m \odot_{x,y} K_{1,n}) \to \{0,1,\ldots,2m+2n-1\}$ is defined by

$$f(z) = m+1,$$

$$f(y_j) = \begin{cases} m, & \text{for } j = 0, \\ m+2j+1, & \text{for } 1 \le j \le n-1. \end{cases}$$

The label of the vertices x_i, we break up into two cases.
For $m \equiv 0 \bmod 4$

$$f(x_i) = \begin{cases} i-1, & \text{for } i \text{ odd and } 1 \le i \le m-1, \\ i-1, & \text{for } i \text{ even and } 2 \le i \le \frac{m}{2}, \\ i+1, & \text{for } i \text{ even and } \frac{m}{2}+2 \le i \le m-2. \end{cases}$$

For $m \equiv 2 \bmod 4$

$$f(x_i) = \begin{cases} i-1, & \text{for } i \text{ odd and } 1 \le i \le \frac{m}{2}, \\ i-1, & \text{for } i \text{ even and } 2 \le i \le m-2, \\ i+1, & \text{for } i \text{ odd and } \frac{m}{2}+2 \le i \le m-1. \end{cases}$$

We get $f^* : E(C_m \odot_{x,y} K_{1,n}) \to \{1,3,\ldots,2m+2n-1\}$ as follows.

$$f^*(zx_i) = \begin{cases} m+1, & \text{for } i = 1, \\ 2m-1, & \text{for } i = m-1, \end{cases}$$

$$f^*(x_ix_{i+1}) = \begin{cases} 2i-1, & \text{for } 1 \le i \le \frac{m}{2}, \\ 2i+1, & \text{for } \frac{m}{2}+1 \le i \le m-2, \end{cases}$$

$$f^*(zy_0) = 2m+1,$$

$$f^*(y_0y_j) = 2m+2j+1, \quad \text{for } 1 \le j \le n-1.$$

It is easily seen that f^* a bijection. We conclude that $C_m \odot_{x,y} K_{1,n}$ is odd harmonious. □

For example, an odd harmonious labeling of $C_8 \odot_{x,y} K_{1,7}$ can be seen in Figure 2.

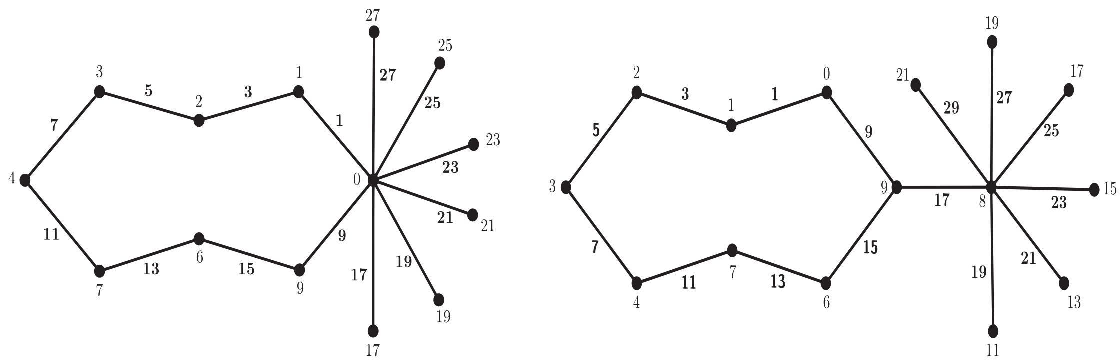

FIGURE 2. An odd harmonious labeling of $C_8 \odot_{x,y_0} K_{1,6}$ and $C_8 \odot_{x,y} K_{1,7}$.

ACKNOWLEDGMENTS

This research has been supported by "Hibah KeRis Batch 1, Universitas Jember, Tahun Anggaran 2020".

REFERENCES

1. J. A. Gallian, "A dynamic survey of graph labeling," The Electronic Journal of Combinatorics. **DS6** (2019).
2. R. L. Graham and N. J. A. Sloane, "On additive bases and harmonious graphs," SIAM J. Alg. Disc. Meth. **1**, 382–404 (1980).
3. Z. Liang and Z. Bai, "On the odd harmonious graphs with applications," J. Appl. Math. Comput. **29**, 105–116 (2009).
4. P. B. Sarasija and R. Binthiya, "Even harmonious graphs with application," International Journal of Computer Science and Information Security (2011).
5. S. K. Vaidya and N. H. Shah, "Some new odd harmonious graph," International Journal of Mathematics and Soft Computing **1**, 9–16 (2011).
6. S. K. Vaidya and N. H. Shah, "Odd harmonious labeling of some graphs," International J.Math. Combin. **3**, 105–112 (2012).
7. G. A. Saputri, K. A. Sugeng, and D. Froncek, "The odd harmonious labeling of dumbbell and generalized prims graphs," AKCE Int, J. Graphs Combin. **10**, 221–228 (2013).
8. F. Alyani, F. Firmansah, W. Giyarti, and K. A. Sugeng, "The odd harmonious labeling of kC_n- snake graphs for spesific values of n, that is, for $n = 4$ and $n = 8$," Proceeding of IICMA 2013 , 225–230 (2013).
9. P. Jeyanthi, S. Philo, and K. A. Sugeng, "Odd harmonious labeling of some new families of graphs," SUT Journal of Mathematics **51**, 181–193 (2015).
10. P. Jeyanthi and S. Philo, "Odd harmonious labeling of some new families of graphs," Electronic Notes in Discrete Mathematics **48**, 165–168 (2015).
11. P. Jeyanthi and S. Philo, "Odd harmonious labeling of some cycle related graphs," Proyecciones Journal of Mathematics **35**, 85–98 (2016).
12. P. Jeyanthi and S. Philo, "Odd harmonious labeling of plus graphs," Bulletin of the International mathematical Virtual Institute **7**, 515–526 (2017).
13. P. Jeyanthi, S. Philo, and M. K. Siddiqui, "Odd harmonious labeling of super subdivision graphs," Proyecciones Journal of Mathematics **38**, 1–11 (2019).
14. P. Jeyanthi and S. Philo, "Odd harmonious labeling of certain graphs," Journal of Applied Science and Computations **06**, 1224–1232 (2019).
15. P. Jeyanthi, S. Philo, and M. Youssef, "Odd harmonious labeling of grid graph," Proyecciones Journal of Mathematics **38**, 411–428 (2019).
16. K. A. Sugeng, S. Surip, and R. Rismayati, "On odd harmonious labeling of m -shadow of cycle, gear with pendant and shuriken graphs," AIP Conference Proceedings **2192**, 040015 (2019).
17. K. Wijaya and E. T. Baskoro, "On ramsey $(2K_2, 2H)$-minimal graphs," Springer Proc. Math. Stat. **186**, 219–225 (2016).