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## The construction of the Koch curve ( $n,c$ ) using L-system

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**Abstract.** The Koch curve is a fractal that has self-similarity. It is built from a straight line segment divided into three equal parts; then, the middle part is removed and transformed into a bottomless equilateral triangle. Fractal objects can be constructed by several methods, one of which is the L-system (Lindenmayer system). L-system is a form of notation of the system of repeating a symbol by building a simple part of the object using recursive rewriting rules. This paper aims to build the Koch ( $n, c$ ) curve generated by varying the middle segment, which is converted into a regular  $n$ -number, where  $n$  positive integers greater than or equal to 3. The value of  $c$  defines the length of the removed segment, where  $c$  is a real number ( $0 < c < 1$ ) so that the Koch ( $n, c$ ) curves do not overlap each other. From this rule, we obtained various Koch ( $n, c$ ) curves using the L-system by varying the value  $c$ , which approaches the lower and upper boundary of value  $c$ .

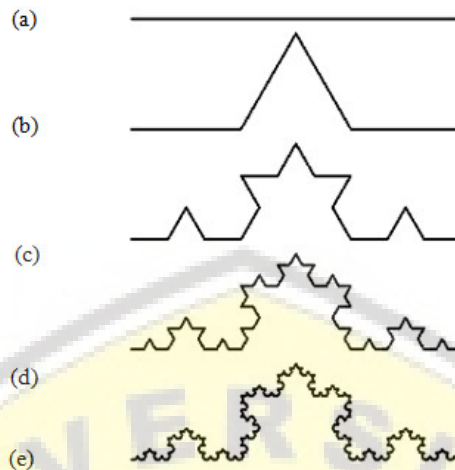
### INTRODUCTION

Fractals have the nature of self-similarity, which is part of them can be seen as a scale of the others. Fractals are also produced by repeating a pattern so that it has a similar shape or structure. Some examples of fractal objects are Sierpinski triangle, Sierpinski carpet, dragon curve, Koch curve, Hilbert curve, Koch snowflake, Mandelbrot set, Julia set, and Cantor set.

Fractal objects can be constructed with several methods, including L-System. L-System is a set of notations of a loop that form an object by replacing a part of a rule through a recursive mechanism. Research on L-System has been carried out by building fractal objects through different line segments and regulations [1].

The Koch curve is constructed from a straight line, which will be divided into three segments of equal length, and the middle part will be erased and converted into an equilateral triangle [2]. The Koch curve is generated by varying the middle segment that is omitted, where  $n$  defines the regular  $n$ -polygon with  $n \geq 3$  and the value of  $c$  establishes the length of the subinterval segments which are removed at the middle of a specified interval  $0 < c < 1$ , so that they do not overlap. Based on this description, the authors are interested in conducting researching on the Koch curve with its generation using L-system.

The formation of the Koch curve is denoted by  $K_i$  denotes the formation of the Koch curve while  $i$  describes an iteration in its generation. The steps for forming the Koch curve start from a straight line (Figure 1(a)). The first order Koch curve  $K_1$  is obtained from the path that divided into three sections and then the middle section is transformed into an equilateral triangle so that it forms four line segments (Figure 1(b)). The formation of the second-order Koch curve is created by dividing each line segment from the first-order Koch curve into three parts and the middle part being converted into an equal triangle (Figure 1(c)). The same way is done continuously to form the Koch curve with a higher-order from the previous Koch curve and so forth(Figure 1(d)-1(e))[4].



**Figure 1.** Generation of the Koch Curve

The construction of the Koch  $(n, c)$  curve is done by generating a line section that is removed from the middle segment and then replaced with a regular  $n$  with 3, while value  $c$  denotes the segment length omitted at intervals of  $0 < c < 1$ . The value of  $n$  is determined so that  $c$  has taken on the Koch  $(n, c)$  curve produces a non-overlapping curve. The rules for calculating the allowable  $c$  value intervals can be seen in Theorem 1.

### Theorem 1.

The Koch curve  $(n, c)$  will not overlap with the value of  $c$  that satisfies

$$c < \frac{\sin^2(\frac{\pi}{n})}{\cos^2(\frac{\pi}{n}) + 1} \quad (n \text{ even})$$

and

$$c < 1 - \cos(\frac{\pi}{n}) \quad (n \text{ odd}) [5].$$

L-System is a system of repetition or rewriting with the technique of building complex objects from repeating simple parts of objects using a recursive rewrite rule [6]. Interpretation of the L-System graphically describes a sequence that results from predetermined axioms and production rules. The symbols on the L-system can be stated as follows:

F: drawing forward as long as  $d$ ;

G: moving forward as long as  $d$  without drawing;

+: rotating counterclockwise with angle  $\theta$ ;

-: rotating clockwise with angle  $\theta$ .

## RESEARCH METHODS

This research would be done in several steps. The first step is to generate the Koch  $(n, c)$  curve by making a straight line segment which is transformed into three sections with the middle section length is  $c$ , and the length of other two sections are  $(1-c)/2$ . Then the center is changed with  $n$ -polygon with  $n \geq 3$ . This step will continue to be repeated based on straight line segments in the next iteration. The second step is drawing the Koch  $(n, c)$  curve using L-System. At this step, the value of  $n, c$ , the number of iterations, the rules of production used, the turning angle and length of the segment are determined. The third step is a program simulation consisting of input, process, and output. Information that is entering the value of  $n, c$ , and iteration. The process is the stage for describing the Koch  $(n, c)$  curve with predetermined production rules. The output is the result of a program simulation in the form of visualizing the Koch curve  $(n, c)$  using L-System.

## THE RESULTS

The visualization of each algorithm step will be carried out, namely determining the boundaries of values  $n$  and  $c$ , determining the production rules in the L-system, determining the angular rules on regular  $n$ -polygons,

determining the length of the Koch (n, c) curve. Determination of the boundaries of values n and c aims that the curves formed do not overlap. By the rules in Theorem 1, the boundaries can be seen in Table 1.

**TABLE 1.** Boundaries of value *n* and *c*

regular <i>n</i> -polygons	value of <i>c</i>
<i>n</i> =3	$0 < c < 0.5$
<i>n</i> =4	$0 < c < 0.33333$
<i>n</i> =5	$0 < c < 0.19098$
<i>n</i> =6	$0 < c < 0.14286$
<i>n</i> =7	$0 < c < 0.099031$
<i>n</i> =8	$0 < c < 0.079009$

The production rules on the L-system are the necessary steps for forming the Koch curve. The production rules carried out are five types of notations used, namely  $V = \{F, S1, S2, +, -\}$  where:

- F*: drawing forward with the previous segment length
- S1*: drawing forward with the reduced segment length
- S2*: drawing forward with the enlarged segment length
- +: rotating counter clockwise with an angle  $\alpha$
- : rotating clockwise with an angle  $\theta$

The production rules of each regular *n*-polygon are different because there are additions to each side of the constructed polygon. The following Table 2 is a form of L-system for regular *n*-polygon in iteration 1.

**TABLE 2.** Form of L-system for regular *n*-polygon in iteration 1

regular <i>n</i> -polygons	Form of L-system in iteration 1
<i>n</i> =3	$F + S1 - F + S2$
<i>n</i> =4	$F + S1 - F - F + S2$
<i>n</i> =5	$F + S1 - F - F - F + S2$
<i>n</i> =6	$F + S1 - F - F - F - F + S2$
<i>n</i> =7	$F + S1 - F - F - F - F - F + S2$
<i>n</i>	$F + S1 \underbrace{-F - F - F - F - F \dots -F}_{(n - 2) \text{ times}} + S2$

The production rule in the generation of the Koch curve (n, c) that uses L-system from the first iteration to the *i* iteration are interrelated. Suppose that in iteration 1 for *n*=3 is  $F+S1-F+S2$ ; then, for the second iteration insert the basic rule namely  $+ S1-F + S2$  based on the *n*-nodes used. Each symbol of the previous iteration is the first iteration, then the second iteration for *n* = 3 is  $F + S1-F + S2 + S1 + S1-F + S2-F + S1-F + S2 + S2 + S1-F + S2$ . For the *i* iteration production rule will always be related to the previous iteration by insert basic rules in the repetition. In this case, the production rules that we will use are as follows.

$$F \rightarrow F + S1 - F + S2$$

$$S1 \rightarrow S1 + S1 - F + S2$$

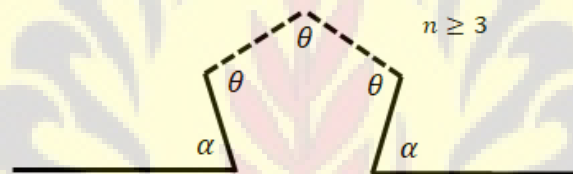
$$S2 \rightarrow S2 + S1 - F + S2$$

In the L-system, every turn in the production rule to draw a line has a certain angle that depends on the regular *n*-polygon to be formed. Therefore, it must be understood the angular magnitude of each regular *n*-polygon used. The size of the regular corner can be seen in Table 3.

**TABLE 3.** The regular  $n$ -polygon corner size

Regular $n$ -polygon	Total of angle size	Large size of each angle
Regular triangle	$180^\circ$	$1/3 \times 180^\circ = 60^\circ$
Regular rectangles	$2 \times 180^\circ = 360^\circ$	$2/4 \times 180^\circ = 90^\circ$
Regular pentagon	$3 \times 180^\circ = 540^\circ$	$3/5 \times 180^\circ = 108^\circ$
Regular hexagon	$4 \times 180^\circ = 720^\circ$	$4/6 \times 180^\circ = 120^\circ$
Regular 7-polygon	$5 \times 180^\circ = 900^\circ$	$5/7 \times 180^\circ = 128,57^\circ$
Regular 8-polygon	$6 \times 180^\circ = 1080^\circ$	$6/8 \times 180^\circ = 135^\circ$
Regular 9-polygon	$7 \times 180^\circ = 1260^\circ$	$7/9 \times 180^\circ = 140^\circ$
Regular 10-polygon	$8 \times 180^\circ = 1440^\circ$	$8/10 \times 180^\circ = 144^\circ$
Regular $n$ -polygon	$(n - 2) \times 180^\circ$	$((n - 2) / n) \times 180^\circ$

The formation of the corner of the Koch curve ( $n, c$ ) in the rules for each  $n$ -polygon is different. The size of each angle for regular  $n$ -polygon can be seen in Table 3. From the table, it is known that the size of the corner of a full circle is  $360^\circ$ . So that the illustration performed on the generation of normal  $n$ -polygon Koch curves as given in Figure 2.



**FIGURE 2.** Illustration of the angle of

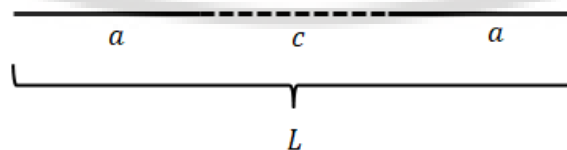
Every turn in L-system has a different name of edges depending on the value of  $n$ , where  $\alpha$  is a subtraction from the number of the half-circle angles, while  $\theta$  is the angle of a regular  $n$ -polygon. The equation of the description of the corner above can be written as below.

$$\theta = \frac{(n - 2)}{n} \times 180^\circ$$

and

$$\alpha = 180^\circ - \theta$$

The line length of the Koch curve is generally  $L$  or can be normalized to 1. The length of each part of the Koch curve ( $n, c$ ) will vary based on the specified value of  $c$ . If the length of the middle part to be replaced with the regular  $n$ -polygon is  $c$ , then the range of the other two parts is  $\frac{1-c}{2}$ . The equation that illustrates the length of each part can be seen in Figure 3.



**FIGURE 3.** Segment length of Koch ( $n,c$ ) curve

Each generation of the Koch curve ( $n, c$ ) has a value of  $c$  whose value changes. If the segment length of the Koch curve is generally 1, then the length of the other segments or the value of  $a$  that depends on  $c$  can be seen in the equation below.

$$a = \frac{L - c}{2}$$

Because it is known that  $L = 1$ , then

$$a = \frac{1 - c}{2}$$

The length  $a$  in the last iteration that will be used as the initial segment in the L-system, namely:

$$P = (a)^{\max \text{ iteration}}$$

$$P = \left(\frac{1 - c}{2}\right)^{\max \text{ iteration}}$$

The magnitude of the dilation scale to change  $a$  to  $c$  is as below.

$$m_{s1} = \frac{c}{a}$$

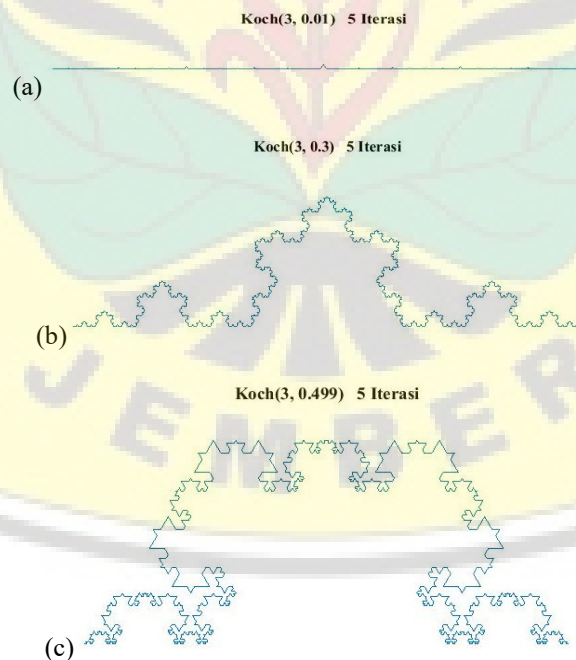
$$m_{s1} = \frac{2c}{1 - c}$$

The magnitude of the dilation scale to change  $c$  to  $a$  is as below.

$$m_{s2} = \frac{a}{c}$$

$$m_{s2} = \frac{1 - c}{2c}$$

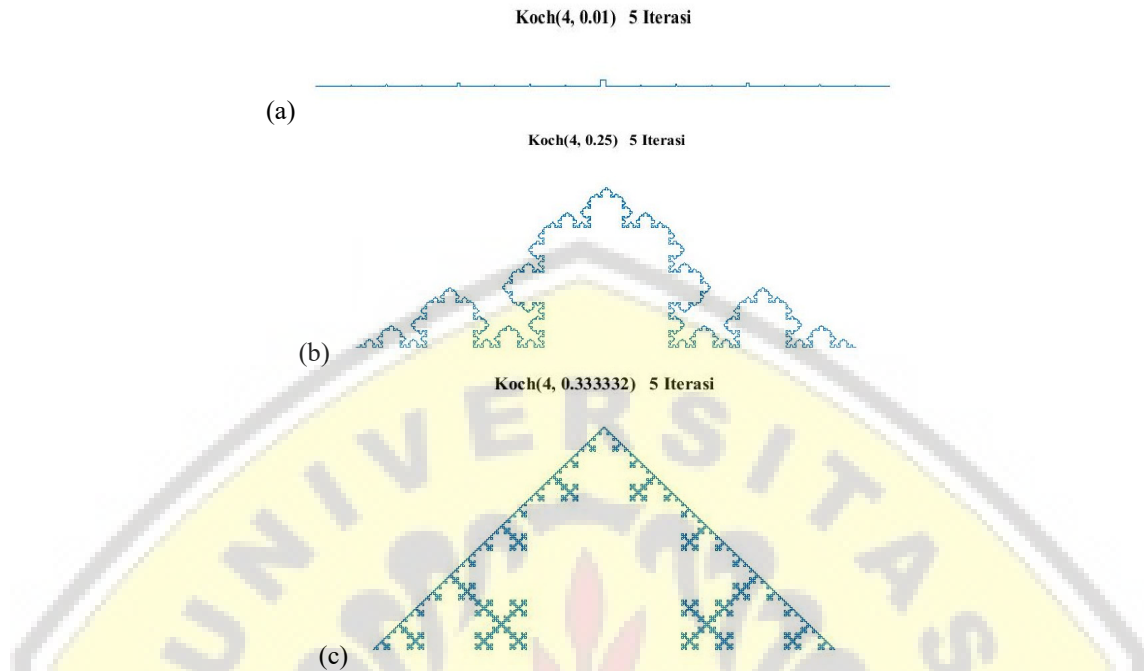
The results of the visualization carried out are six types of regular  $n$ -polygon generation, namely triangle, rectangle, pentagon, hexagon, heptagon, and octagon. The first generation has been generated by  $n = 3$  where the value of  $c$  is varied with the limit  $0 < c < 0.5$  using the 5th iteration. It is known that the generator is a triangle, and the results of the visualization in the curve will still form a triangle in accordance with Figure 4(c).



**FIGURE 4.** The Generation of Koch curve  $n = 3$  with the value of  $c$  varied

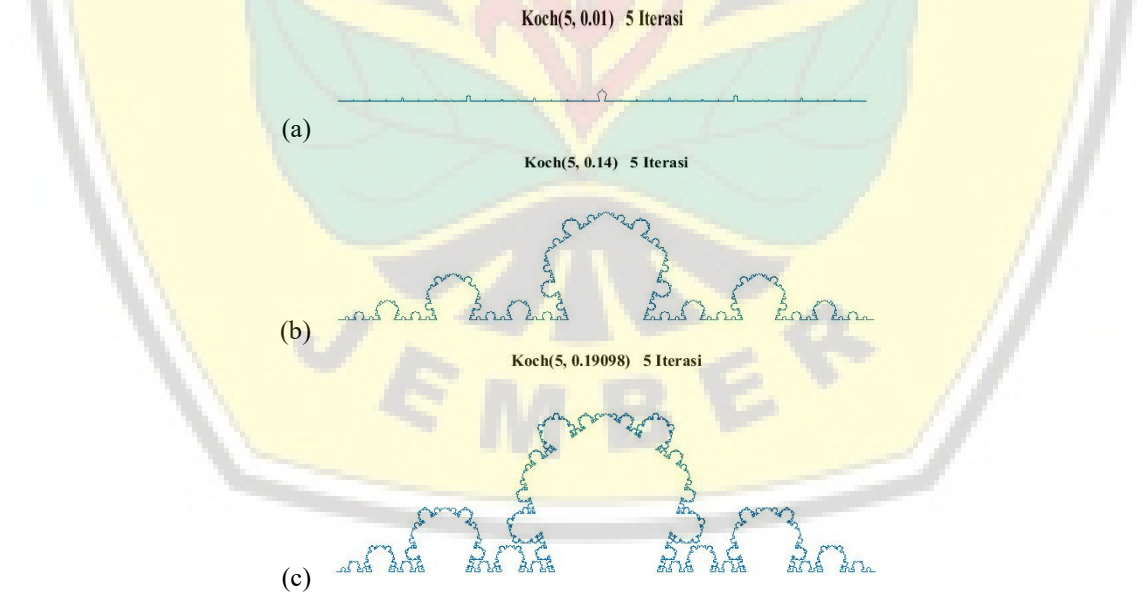
The second generation is Koch curve  $n = 4$  with a value of  $c$  varied within the limits of  $0 < c < 0.333333$  using the 5th iteration. In this case the generator is a quadrilateral so that the results of the visualization in the curve will still form quadrilateral according to Figure 5(c).





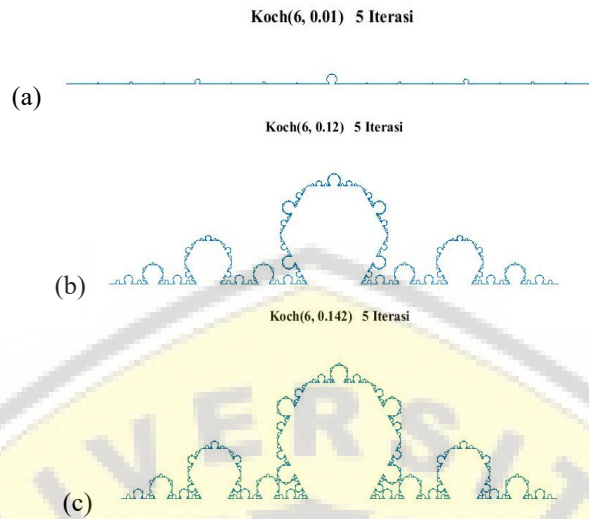
**FIGURE 5.** The Generation of Koch curven = 4 with the value of  $c$  varied

The third generation is Koch curven = 5 with  $c$  varied within the limits of  $0 < c < 0.19098$  using the 5th iteration and the generator is pentagonal. According to Figure 6(c), the results of the visualization in the curve will still form a pentagon..



**FIGURE 6.** The Generation of the Koch curve  $n=5$  with the value of  $c$  varied

The fourth generation is the Koch curve with  $n = 6$  where the value  $c$  varied within  $0 < c < 0.14286$  and the generator is hexagonal, so that the results will still form the hexagonal shape It can be seen as in Figure 7(c).



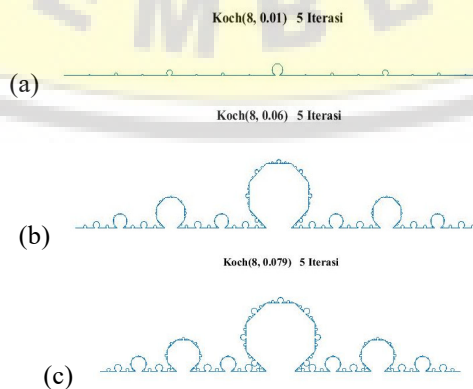
**FIGURE 7.** The Generation of the Koch curve  $n=6$  with the value of  $c$  varied

The fifth-generation is using  $n = 7$  where the value of  $c$  varied within the limits of  $0 < c < 0.099031$ . The results of the curve will still form a heptagon according to Figure 8(c).



**FIGURE 8.** The Generation of the Koch curve  $n=7$  with the value of  $c$  varied

The sixth-generation is Koch curve with  $n = 8$  and amount of  $c$  varied within  $0 < c < 0,079009$ . The results of the visualization in the curve will still form octagon, according to Figure 9(c).



**FIGURE 9.** The Generation of the Koch curve  $n=8$  with the value of  $c$  varied



Based on the description above from each result, it can be seen that each value of  $c$ , which approaches to zero will form the Koch curve, which is getting closer to a straight line while the amount of  $c$  that is close to the upper boundary will generate a Koch curve that varies based on the number of regular  $n$ -polygon used.

## CONCLUSIONS AND SUGGESTIONS

Based on the results and discussion, the Koch  $(n, c)$  curve can be generated using L-system with the following procedure:

- The axioms in the form of L-system in regular  $n$ -polygon for the first iteration is  $F + S1 \underbrace{-F - F - F - F - F \dots - F}_{(n-2) \text{ times}} + S2$ .
- The production rules used are five types of symbols, namely  $V = \{F, S1, S2, +, -\}$  which define symbols on L-system, that is:  
 $F \rightarrow F + S1 - F + S2$   
 $S1 \rightarrow S1 + S1 - F + S2$   
 $S2 \rightarrow S2 + S1 - F + S2$
- The angle rules of nodes are related to defining symbols  $+$ , which is rotating counterclockwise with an angle of  $\alpha = 180^\circ - \theta$ . The symbol  $-$ , which is rotating clockwise with an angle of  $\theta = \frac{(n-2)}{n} \times 180^\circ$ .
- The segment length of the Koch  $(n, c)$  curve is related to the rules for the symbol "S1" and "S2" where "S1" drawing forward an reduced segment length with the scale  $m_{S1} = \frac{2c}{1-c}$  and "S2" drawing forward an enlarged segment with the scale  $m_{S2} = \frac{1-c}{2c}$ .

We obtain other significant result. Each value of  $c$  which approaches to zero will form the Koch curve, which is getting closer to a straight line while the amount of  $c$  that is close to the upper limit will create a Koch curve that varies based on the number of regular  $n$ -polygon used.

In this study, the Koch  $(n, c)$  curve generation with L-system was carried out. Previous research has also been carried out using the IFS (Iterated Function System). It is hoped that further analysis can be developed by forming another Koch curve.

## ACKNOWLEDGMENT

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