Journal of Physics: Conference Series

PAPER • OPEN ACCESS

The 1st International Conference of Combinatorics, Graph Theory, and Network Topology

To cite this article: 2018 *J. Phys.: Conf. Ser.* **1008** 011001

IOP Conf. Series: Journal of Physics: Conf. Series **1008** (2018) 011001 doi:10.1088/1742-6596/1008/1/011001

The First International Conference on Combinatorics, Graph Theory and Network Topology (ICCGANT)

Dafik

Editor in Chief of ICCGANTs Publication, University of Jember, Jember, Indonesia

E-mail: d.dafik@unej.ac.id

Preface

It is with my great pleasure and honor to organize the First International Conference on Combinatorics, Graph Theory and Network Topology which is held from 25-26 November 2017 in the University of Jember, East Java, Indonesia and present a conference proceeding index by Scopus. It is the first international conference organized by CGANT Research Group University of Jember in cooperation with Indonesian Combinatorics Society (INACOBMS). The conference is held to welcome participants from many countries, with broad and diverse research interests of mathematics especially combinatorical study. The mission is to become an annual international forum in the future, where, civil society organization and representative, research students, academics and researchers, scholars, scientist, teachers and practitioners from all over the world could meet in and exchange an idea to share and to discuss theoretical and practical knowledge about mathematics and its applications. The aim of the first conference is to present and discuss the latest research that contributes to the sharing of new theoretical, methodological and empirical knowledge and a better understanding in the area mathematics, application of mathematics as well as mathematics education.

The themes of this conference are as follows: (1) Connection of distance to other graph properties, (2) Degree/diameter problem, (3) Distance-transitive and distance-regular graphs, (4) Metric dimension and related parameters, (5) Cages and eccentric graphs, (6) Cycles and factors in graphs, (7) Large graphs and digraphs, (8) Spectral Techniques in graph theory, (9) Ramsey numbers, (10) Dimensions of graphs, (11) Communication networks, (12) Coding theory, (13) Cryptography, (14) Rainbow connection, (15) Graph labelings and coloring, (16). Applications of graph theory

The topics are not limited to the above themes but they also include the mathematical application research of interest in general including mathematics education, such as:(1) Applied Mathematics and Modelling, (2) Applied Physics: Mathematical Physics, Biological Physics, Chemistry Physics,(3) Applied Engineering: Mathematical Engineering, Mechanical engineering, Informatics Engineering, Civil Engineering,(4) Statistics and Its Application,(5) Pure Mathematics (Analysis, Algebra and Geometry),(6) Mathematics Education, (7) Literacy of Mathematics,(8) The Use of ICT Based Media In Mathematics Teaching and Learning,(9) Technological, Pedagogical, Content Knowledge for Teaching Mathematics, (10) Students Higher Order Thinking Skill of Mathematics, (11) Contextual Teaching and Realistic Mathematics, (12) Science, Technology, Engineering, and Mathematics Approach, (13) Local Wisdom Based

IOP Conf. Series: Journal of Physics: Conf. Series **1008** (2018) 011001 doi:10.1088/1742-6596/1008/1/011001

Education: Ethnomathematics, (14) Showcase of Teaching and Learning of Mathematics, (16) The 21st Century Skills: The Integration of 4C Skill in Teaching Math.

The participants of this ICCGANT 2017 conference were 200 people consisting research students, academics and researchers, scholars, scientist, teachers and practitioners from many countries. The selected papers to be publish of Journal of Physics: Conference Series are 80 papers. On behalf of the organizing committee, finally we gratefully acknowledge the support from the University of Jember of this conference. We would also like to extend our thanks to all lovely participants who are joining this unforgettable and valuable event.

Prof. Drs. Dafik, M.Sc., Ph.D.

PAPER • **OPEN ACCESS**

The Committees of The First International Conference on Combinatorics, Graph Theory and Network Topology (ICCGANT)

To cite this article: 2018 *J. Phys.: Conf. Ser.* **1008** 011002

View the article online for updates and enhancements.

Related content

- List of committees
- List of Committees
- Committees

IOP Conf. Series: Journal of Physics: Conf. Series **1008** (2018) 011002 doi:10.1088/1742-6596/1008/1/011002

The Committees of The First International Conference on Combinatorics, Graph Theory and Network Topology (ICCGANT)

Dafik

Editor in Chief of ICCGANTs Publication, University of Jember, Jember, Indonesia
Professor of Combinatorics and Graph Theory

E-mail: d.dafik@unej.ac.id

Advisory Committee

Moch. Hasan	Rector of the University of Jember
Zulfikar	Vice Rector of the University of Jember
Slamin	President of Indonesian Combinatorial Society

Organizing Committee

Dafik	Chairperson
Ika Hesti Agustin	Secretary

Advisory Editorial Board

Surahmat	University of Islam Malang, Indonesia
Syafrizal Sy	University of Andalas, Indonesia

Editorial Board

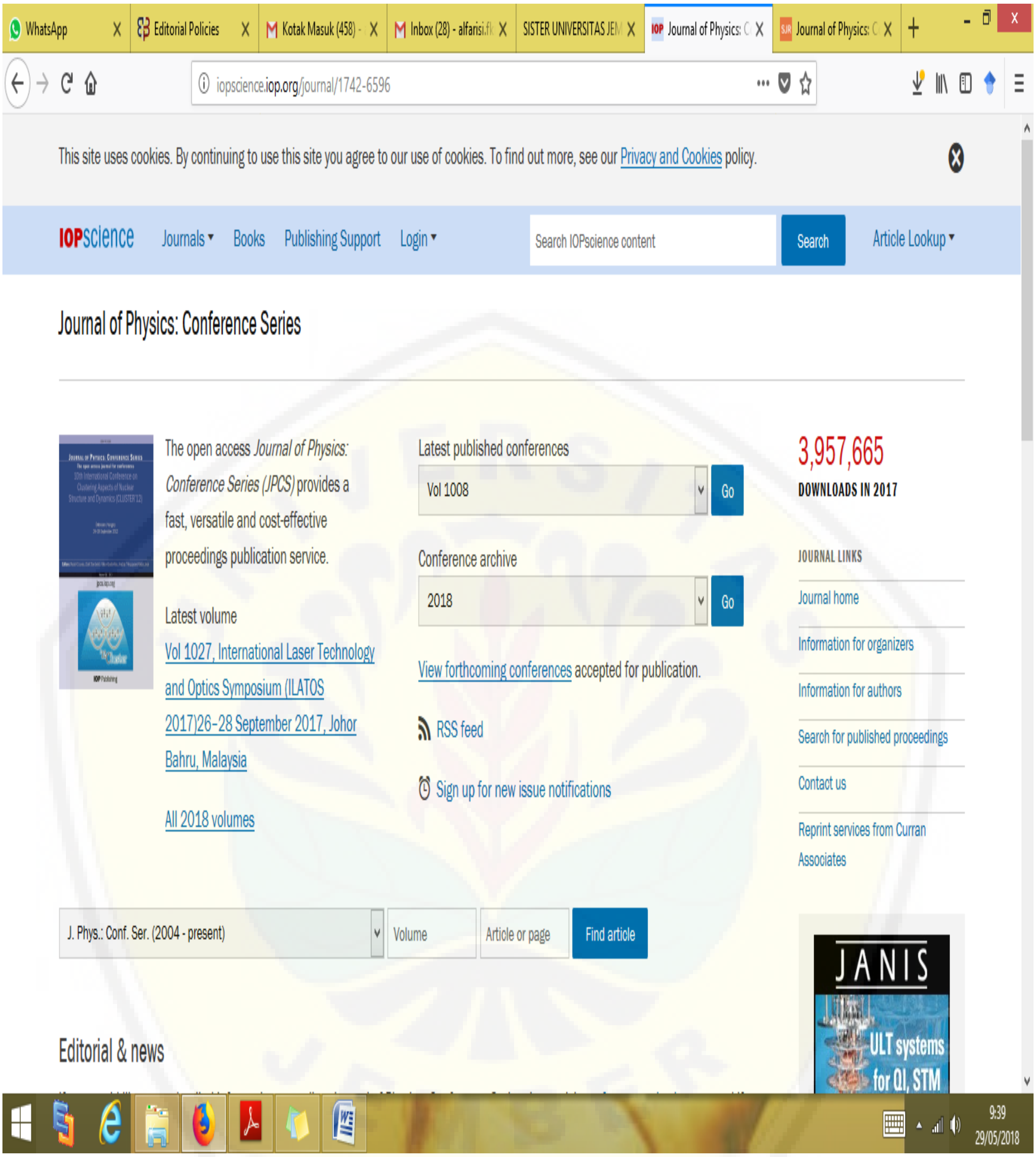
Arika Indah Kristiana	University of Jember, Indonesia
Abduh Riski	University of Jember, Indonesia
Ikhsanul Halikin	University of Jember, Indonesia
Ridho Alfarisi	University of Jember, Indonesia
Rafiantika Megahnia Prihandini	University of Jember, Indonesia
Kusbudiono	University of Jember, Indonesia
Ermita Rizky Albirri	University of Jember, Indonesia
Robiatul Adawiyah	University of Jember, Indonesia
Dwi Agustin Retno Wardani	IKIP PGRI Jember, Indonesia

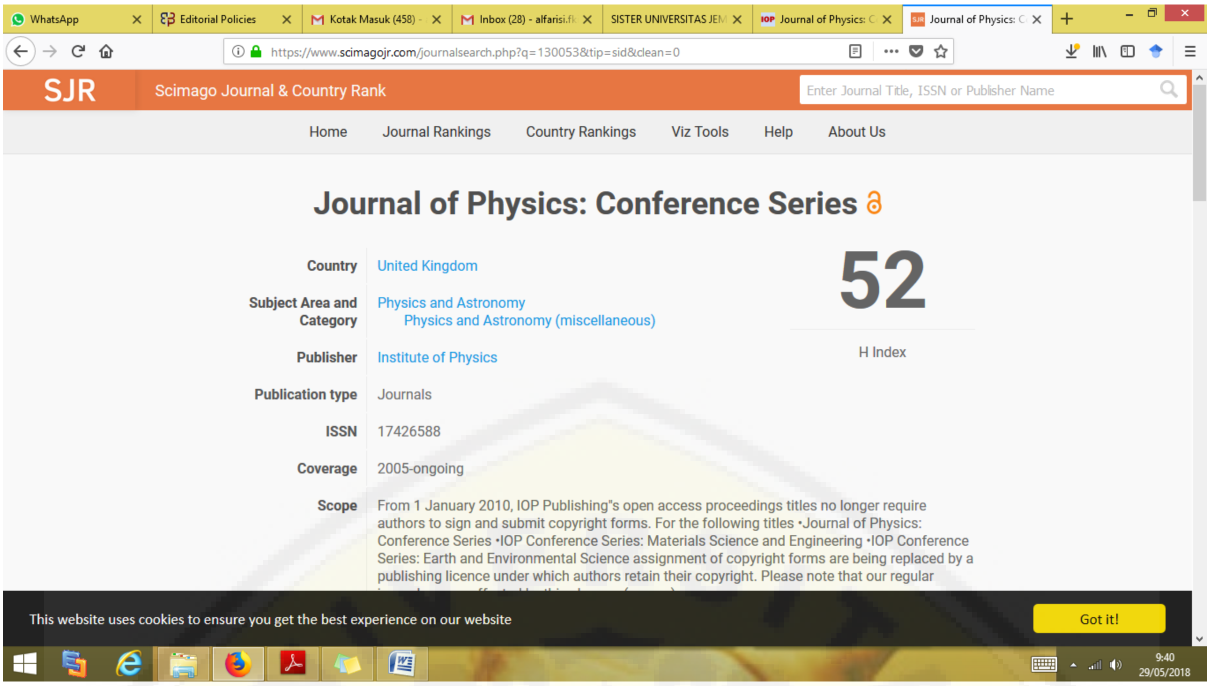
IOP Conf. Series: Journal of Physics: Conf. Series **1008** (2018) 011002 doi:10.1088/1742-6596/1008/1/011002

Scientific Committee and Reviewers

Joe Ryan	University of Newcastle, Australia
Kinkar Chandra Das	Sungkyunkwan University, Republic of Korea
Octavio Paulo Vera Villagran	University of Bio-Bio, Chile
Ali Ahmad	Jazan University, Saudi Arabia
Roslan Hasni	Universiti Malaysia Terengganu, Malaysia
Kiki A. Sugeng	University of Indonesia, Indonesia
Rinovia Simajuntak	Institut Teknologi Bandung, Indonesia
Hilda Assiyatun	Institut Teknologi Bandung, Indonesia
Liliek Susilowati	Universitas Airlangga, Indonesia
Diary Indriati	Universitas Sebelas Maret, Indonesia
Syaiful Bukhori	University of Jember, Indonesia
Antonius Cahya Prihandoko	University of Jember, Indonesia
Bambang Sujanarko	University of Jember, Indonesia
Khairul Anam	University of Jember, Indonesia

The committees of the First International Conference on Combinatorics, Graph Theory and Network Topology would like to express gratitude to all Committees for the volunteering support and contribution in the editing and reviewing process.


WhatsApp
Editorial Policies
Kotak Masuk (458)
Inbox (28)
SISTER UNIVERSITAS JEM
Journal of Physics: C
Journal of Physics: C
iopscience.iop.org/journal/1742-6596
This site uses cookies. By continuing to use this site you agree to our use of cookies. To find out more, see our Privacy and Cookies policy.
IOPscience
Journals
Books
Publishing Support
Login
Search IOPscience content
Search
Article Lookup
Journal of Physics: Conference Series
The open access Journal of Physics: Conference Series (JPCS) provides a fast, versatile and cost-effective proceedings publication service.
Latest volume
Vol 1027, International Laser Technology and Optics Symposium (ILATOS 2017)26-28 September 2017, Johor Bahru, Malaysia
All 2018 volumes
Latest published conferences
Vol 1008
Go
Conference archive
2018
Go
View forthcoming conferences accepted for publication.
RSS feed
Sign up for new issue notifications
3,957,665
DOWNLOADS IN 2017
JOURNAL LINKS
Journal home
Information for organizers
Information for authors
Search for published proceedings
Contact us
Reprint services from Curran Associates
J. Phys.: Conf. Ser. (2004 - present)
Volume
Article or page
Find article
Editorial & news
JANIS
ULT systems for QI, STM
9:39
29/05/2018


WhatsApp
Editorial Policies
Kotak Masuk (458)
Inbox (28) - alfarisi.fk
SISTER UNIVERSITAS JEM
Journal of Physics: C
Journal of Physics: C
https://www.scimagojr.com/journalsearch.php?q=130053&tip=sid&clean=0
SJR
Scimago Journal & Country Rank
Enter Journal Title, ISSN or Publisher Name
Home
Journal Rankings
Country Rankings
Viz Tools
Help
About Us
Journal of Physics: Conference Series
Country
United Kingdom
Subject Area and Category
Physics and Astronomy
Physics and Astronomy (miscellaneous)
Publisher
Institute of Physics
Publication type
Journals
ISSN
17426588
Coverage
2005-ongoing
Scope
From 1 January 2010, IOP Publishing's open access proceedings titles no longer require authors to sign and submit copyright forms. For the following titles •Journal of Physics: Conference Series •IOP Conference Series: Materials Science and Engineering •IOP Conference Series: Earth and Environmental Science assignment of copyright forms are being replaced by a publishing licence under which authors retain their copyright. Please note that our regular
52
H Index
This website uses cookies to ensure you get the best experience on our website
Got it!
9:40
29/05/2018

Journal of Physics: Conference Series

PAPER • **OPEN ACCESS**

The non-isolated resolving number of k-corona product of graphs

To cite this article: Ridho Alfarisi *et al* 2018 *J. Phys.: Conf. Ser.* **1008** 012040

View the article online for updates and enhancements.

ICCGANT IOP Publishing
IOP Conf. Series: Journal of Physics: Conf. Series **1008** (2018) 012040 doi:10.1088/1742-6596/1008/1/012040

The non-isolated resolving number of k-corona product of graphs

Ridho Alfarisi[1,4], Dafik[1,2], Slamin[1,5], I. H. Agustin[1,3], A. I. Kristiana[1,2]
[1]CGANT-University of Jember, Jember, Indonesia
[2]Department of Mathematics Education, University of Jember, Jember, Indonesia
[3]Department of Mathematics, University of Jember, Jember, Indonesia
[4]Department of Elementary School Teacher Education, University of Jember, Jember, Indonesia
[5]Department of Information System, University of Jember, Jember, Indonesia

E-mail: alfarisi.fkip@unej.ac.id, d.dafik@unej.ac.id, s.slamin@unej.ac.id

Abstract. Let all graphs be a connected and simple graph. A set $W = \{w_1, w_2, w_3, \ldots, w_k\}$ of veretx set of G, the $k-$vector ordered $r(v|W) = (d(x, w_1), d(x, w_2), \ldots, d(x, w_k))$ of is a representation of v with respect to W, for $d(x, w)$ is the distance between the vertices x and w. The set W is called a resolving set for G if different vertices of G have distinct representation. The metric dimension is the minimum cardinality of resolving set W, denoted by $dim(G)$. Through analogue, the resolving set W of G is called non-isolated resolving set if there is no $\forall v \in W$ induced by non-isolated vertex. The non-isolated resolving number is the minimum cardinality of non-isolated resolving set W, denoted by $nr(G)$. In our paper, we determine the non isolated resolving number of k-corona product graph.

1. Introduction

In this paper, All graphs G is a nontrivial and connected graph, for more detail definition of graph see [1, 2]. The concept of metric dimension was independently introduced by Slater [3] and Harrary and Melter [4]. In his paper, Slater called this concept as a locating set. Let u, v be two vertices in G. The distance $d(u, v)$ is the length of a shortest path between two vertices u and v in connected graph G. For an ordered set $W = \{w_1, w_2, ..., w_k\}$ subset of vertex set $V(G)$. The representation $r(v|W)$ of v with respect to W is the ordered k-tuple $r(v|W) = (d(v, w_1), d(v, w_2), ..., d(v, w_k))$. The set W is called resolving set of G if every vertices of G have distinct representation respect to W, let u and v be two any vertices in G if $r(u|W) = r(v|W)$ implies that $u = v$. Hence if W is a resolving set of cardinality k for a graph G, then the representation set $r(v|W), v \in V(G)$ consists of $|V(G)|$ distinct k-vector. A resolving set of minimum cardinality for a graph G is called a minimum resolving set for G and this cardinality is the metric dimension of G, denoted by $dim(G)$. Saenpholphat and Zhang [7] introduced the concept of connected resolving set. A resolving set W of graph G is connected if each subgraph $\langle W \rangle$ induced by W has no isolated vertices in G. The minimum cardinality of a non-isolated set in a graph G is the non-isolated resolving number, denoted by $nr(G)$. For more detail notation of $nr(G)$ please see in Chitra and Arumugam [5].

Until today, Chartrand et.al.[6] determined the bounds of the metric dimensions for any connected graphs and determined the metric dimensions of some well known families of graphs

IOP Conf. Series: Journal of Physics: Conf. Series **1008** (2018) 012040 doi:10.1088/1742-6596/1008/1/012040

such as tree, path, and complete graph. Citra and Arumugam [5] studied resolving set without isolated vertices of some family graphs. Furthermore, Baca, *et.al* [8] showed the metric dimension of regular bipartite graphs. In recent years, this metric dimension has been studied widely (see [7], [9], [10], [11], [12], [13]). In the following, we present some known results and Yunika [14] studied the metric dimension with non-isolated resolving number of corona product of graphs. Alfarisi et al [16] studied non-isolated resolving number of graphs with pendant edges

The known results on metric dimension and local metric dimension of some particular classes of graphs and graph operation have been discovered Chitra and Arumugam [5], Dafik [15], Yunika [14] as follows.

Proposition 1.1 *(Chitra and Arumugam [5]) Let G be a connected graph of order $n \geq 2$*

- *For P_n, $n \geq 2$, $nr(P_n) = 2$.*
- *For K_n, $n \geq 3$, $nr(K_n) = n - 1$.*
- *For $K_{m,n}$, $m, n \geq 2$, $nr(K_{m,n}) = m + n - 2$.*
- *For friendship G with k-triangles, $k \geq 2$, $nr(G) = k + 1$.*
- *For $P_n + K_1$, $n \geq 2$, $nr(P_n + K_1) = \lfloor n/2 \rfloor$.*

Proposition 1.2 *(Dafik [15]) Let G and H be a connected graph, then the metric dimensio with non-ioslated resolving number of $G + H$ is $\lfloor \frac{|V(G)|}{2} \rfloor + \lfloor \frac{|V(H)|}{2} \rfloor \leq nr(G + H) \leq \lceil \frac{|V(G)|}{2} \rceil + \lceil \frac{|V(H)|}{2} \rceil + max\{nr(G), nr(H)\}$*

Proposition 1.3 *(Yunika [14]) Let G be a connected graph of order $n \geq 2$*

- *For $G \cong K_1 \odot P_n$, $n \geq 2$, $nr(G) = \lfloor \frac{n}{2} \rfloor + 1$.*
- *For $G \cong K_1 \odot K_n$, $n \geq 2$, $nr(G) = n$.*
- *For $G \cong K_1 \odot C_n$, $n \geq 2$,*

$$nr(G) = \begin{cases} 3, & \text{if } 3 \leq n \leq 4 \\ \lfloor \frac{n}{2} \rfloor + 1, & \text{if } n \geq 5,\ n \text{ is odd} \\ \lfloor \frac{n}{2} \rfloor, & \text{if } n \geq 5,\ n \text{ is even} \end{cases}$$

Fruct and Harary [17] was introduced a type corona product of grpah. Let G be a connected graph of order n and H (not necessarily connected) be a graph of order m. A graph G corona product H, $G \odot H$, is defined as a graph obtained by taking one copy of G and $|V(G)|$ copies of graph $H_1, H_2, ..., H_n$ of H and connecting i-th vertex of G to the vertices of H_i, $1 \leq i \leq n$. By definition of corona product, we can say that

$$V(G \odot H) = V(G) \cup \bigcup_{i \in V(G)} V(H_i),$$

$$E(G \odot H) = E(G) \cup \bigcup_{i \in V(G)} (E(H_i) \cup \{iu_i | u_i \in V(H_i)\}),$$

For any integer $k \geq 2$, we define the corona product of graph $G \odot_k H$ recursively of $G \odot H$ as $G \odot_k H = (G \odot_{k-1} H) \odot H$. The graph $G \odot_k H$ is named as $k-$corona product or multicorona product of graph G and H for more detail definiton can be seen in [18]. Figure 1 is an ilustration of k-corona product graphs.

IOP Conf. Series: Journal of Physics: Conf. Series **1008** (2018) 012040 doi:10.1088/1742-6596/1008/1/012040

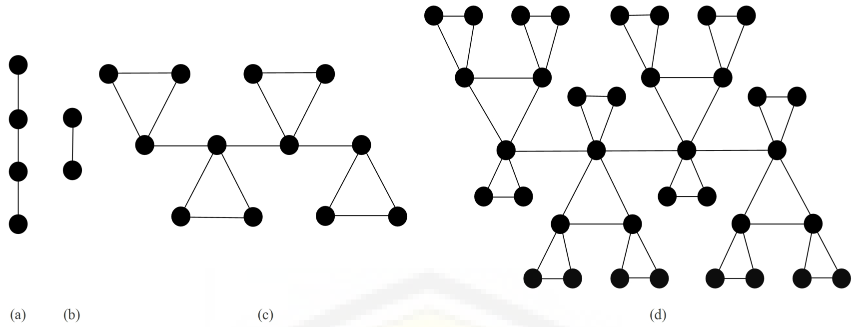


Figure 1. Example of $k-$corona product: (a) P_4; (b) P_2; (c) $P_4 \odot P_2$; (d) $P_4 \odot_2 P_2$

2. Main Results

In this section, we find the metric dimension with non-isolated resolving set of k-corona product graphs. We determine the sharp lower bound and the exact value of $P_n \odot_2 P_m$, $P_n \odot_2 K_m$, $K_m \odot_2 P_n$ and $K_n \odot_2 K_m$

Lemma 2.1 *Let G be a connected graph of order $|V(G)| \geq 2$ and H be a graph of order $|V(H)| \geq 2$, then non-isolated resolving number of $G \odot_k H$ is $nr(G \odot_k H) \geq |V(G \odot_{k-1} H)|nr(K_1 + H)$.*

Proof: The k-corona product of graph $G \odot_k H$ recursively of $G \odot H$ as $G \odot_k H = (G \odot_{k-1} H) \odot H$. Given a resolving set W' of $G \odot_k H$ with $W' \subset V(H) \neq \emptyset$ and resolving set W" of $G \odot_k H$ with $W' \subset V(G \odot_{k-1} H) \neq \emptyset$. It can be shown with use Lemma 2.6.2 (i). Based on definition of non-isolated resolving set that we get $W_i = W'_i \cup W"_i$ for every $i \in V(G \odot_{k-1} H)$ with $W"_i = \{u_i\}$ so that $W_i = W'_i \cup \{u_i\}$. In this section, we can assume —$W'_i \cup \{u_i\}| = nr(K_1 + H)$. Thus, we get resolving set $W = W_1 \cup W_2 \cup W_3 \cup \ldots \cup W_{|V(G \odot_{k-1} H)|}$ or $W = \bigcup_{i=1}^{|V(G \odot_{k-1} H)|} W_i$, then we obtain the lower bound non-isolated resolving set W of $G \odot_{k-1} H$ is

$$
\begin{aligned}
|W| &\leq |\bigcup_{i=1}^{|V(G \odot_{k-1} H)|} (W_i)| \\
&= \Sigma_{i=1}^{|V(G \odot_{k-1} H)|} |W_i| \\
&= |W_1| + |W_2| + \ldots + |W_{V(G \odot_{k-1} H)|}| \\
&= \underbrace{nr(K_1 + H) + nr(K_1 + H) + \ldots + nr(K_1 + H)}_{|V(G \odot_{k-1} H)| times} \\
&= V(G \odot_{k-1} H)|nr(K_1 + H)
\end{aligned}
$$

Hence, it is clearly that the lower bound non-isolated resolving number of $G \odot_k H$ is $nr(G \odot_k H) \geq |V(G \odot_{k-1} H)|nr(K_1 + H)$. □

Theorem 2.1 *Let P_n and P_m be a connected graph of order $n, m \geq 2$, then non-isolated resolving number of $P_n \odot_2 P_m$ is $nr(P_n \odot_2 P_m) = (nm + n)(\lfloor \frac{m}{2} \rfloor + 1)$.*

Proof: Let $P_n \odot_2 P_m$ be a corona product of path P_n and P_m with vertex set $V(P_n \odot_2 P_m) = \{x_i; 1 \leq i \leq n\} \cup \{x_{i,j_1}; 1 \leq i \leq n, 1 \leq j_1 \leq m\} \cup \{x_{i,j_1,j_2}; 1 \leq i \leq n, 1 \leq j_1 \leq m, 1 \leq j_2 \leq m\} \cup \{x_{i,j_2}; 1 \leq i \leq n, 1 \leq j_2 \leq m\}$ and edge set $E(P_n \odot_2 P_m) = \{x_i x_{i+1}; 1 \leq i \leq n-1\} \cup \{x_i x_{i,j_1}, x_{i,j_1} x_{i,j_1,j_2}, x_i x_{i,j_2}; 1 \leq i \leq n, 1 \leq j_1 \leq m, 1 \leq j_2 \leq m\} \cup \{x_{i,j_1} x_{i,j_1+1}, x_{i,j_1,j_2} x_{i,j_1,j_2+1}, x_{i,j_2} x_{i,j_2+1}; 1 \leq i \leq n, 1 \leq j_1 \leq m-1, 1 \leq j_2 \leq m-1\}$

IOP Conf. Series: Journal of Physics: Conf. Series **1008** (2018) 012040 doi:10.1088/1742-6596/1008/1/012040

with the cardinality of vertex set $|V(P_n \odot_2 P_m)| = nm^2 + 2nm + n$ and the cardinality of edge set $|E(P_n \odot_2 P_m)| = 2nm^2 + 3mn - n - 1$.

For $n, m \geq 2$, based on Lemma 2.1 and Proposition 1.3 then we have $nr(P_n \odot_2 P_m) \geq |V(P_n \odot P_m)|nr(K_1 + P_m) = (nm+n)(\lfloor \frac{m}{2} \rfloor + 1)$. However, we can attain the sharpest lower bound. Furthermore, we prove that $nr(P_n \odot_2 P_m) \leq (nm+n)(\lfloor \frac{m}{2} \rfloor + 1)$. Choosing $W \subset V(P_n \odot_2 P_m)$ with $W = \{x_i, x_{i,j_1}; 1 \leq i \leq n, 1 \leq j_1 \leq m\} \cup \{x_{i,j_1,j_2}, x_{i,j_2}; 1 \leq i \leq n, 1 \leq j_1 \leq m-1, 1 \leq j_2 \leq m-1, j_1 \text{ and } j_2 \text{ is odd}\}$ is a non-isolated resolving set of $P_n \odot_2 P_m$ and the cardinality of non-isolated resolving set is $|W| = |\{x_i, x_{i,j_1}; 1 \leq i \leq n, 1 \leq j_1 \leq m\}| + |\{x_{i,j_1,j_2}, x_{i,j_2}; 1 \leq i \leq n, 1 \leq j_1 \leq m-1, 1 \leq j_2 \leq m-1, j_1 \text{ and } j_2 \text{ is odd}\}| = (nm+n)(\lfloor \frac{m}{2} \rfloor + 1)$. Thus, the representation of vertices $v \in V(P_n \odot_2 P_m) - W$ respecting to W are as follows.
The vertex representation of x_{i,j_1,j_2} for m even respect to W is

$$r(x_{i,j_1,j_2}|W) = (a, t_{j_1-1}, \underbrace{t'_{j_1-1}, \ldots, t'_{j_1-1}}_{\lfloor \frac{m}{2} \rfloor}, \ldots, t_1, \underbrace{t'_1, \ldots, t'_1}_{\lfloor \frac{m}{2} \rfloor}, 1, \underbrace{2, \ldots, 2}_{\frac{j_2}{2}-1}, 1, 1, \underbrace{2, \ldots, 2}_{\lfloor \frac{m}{2} \rfloor - \frac{j_2}{2} -}, t_1, \underbrace{t'_1, \ldots, t'_1}_{\lfloor \frac{m}{2} \rfloor}, \ldots,$$
$$t_{m-j_1}, \underbrace{t'_{m-j_1}, \ldots, t'_{m-j_1}}_{\lfloor \frac{m}{2} \rfloor} 2, \underbrace{3, \ldots, 3}_{\lfloor \frac{m}{2} \rfloor}, c);\ 1 \leq i \leq n,\ 1 \leq j_1 \leq m\ 1 \leq j_2 \leq m-1 \text{ and } j \text{ is even.}$$

with $t_l = l+1$, $1 \leq l \leq m - j_1$ and $t'_l = l+2$, $1 \leq l \leq m - j_1$, $1 \leq j_1 \leq m$; $t_k = k+1$, $1 \leq k \leq j_1 - 1$ and $t'_k = k+2$, $1 \leq k \leq j_1 - 1$, $1 \leq j_1 \leq m$.
The vertex representation of $x_{i,j_1,m}$ for m even respect to W is

$$r(x_{i,j_1,m}|W) = (a, t_{j_1-1}, \underbrace{t'_{j_1-1}, \ldots, t'_{j_1-1}}_{\lfloor \frac{m}{2} \rfloor}, \ldots, t_1, \underbrace{t'_1, \ldots, t'_1}_{\lfloor \frac{m}{2} \rfloor}, 1, \underbrace{2, \ldots, 2}_{\frac{m}{2}-1}, 1, t_1, \underbrace{t'_1, \ldots, t'_1}_{\lfloor \frac{m}{2} \rfloor}, \ldots, t_{m-j_1},$$
$$\underbrace{t'_{m-j_1}, \ldots, t'_{m-j_1}}_{\lfloor \frac{m}{2} \rfloor} 2, \underbrace{3, \ldots, 3}_{\lfloor \frac{m}{2} \rfloor}, c);\ 1 \leq i \leq n,\ 1 \leq j_1 \leq m,\ 1 \leq j_2 \leq m-1.$$

with $t_l = l+1$, $1 \leq l \leq m - j_1$ and $t'_l = l+2$, $1 \leq l \leq m - j_1$, $1 \leq j_1 \leq m$; $t_k = k+1$, $1 \leq k \leq j_1 - 1$ and $t'_k = k+2$, $1 \leq k \leq j_1 - 1$, $1 \leq j_1 \leq m$.
The vertex representation of x_{i,j_1,j_2} for m odd respect to W is

$$r(x_{i,j_1,j_2}|W) = (a, t_{j_1-1}, \underbrace{t'_{j_1-1}, \ldots, t'_{j_1-1}}_{\lfloor \frac{m}{2} \rfloor}, \ldots, t_1, \underbrace{t'_1, \ldots, t'_1}_{\lfloor \frac{m}{2} \rfloor}, 1, \underbrace{2, \ldots, 2}_{\frac{j_2}{2}-1}, 1, 1, \underbrace{2, \ldots, 2}_{\lfloor \frac{m}{2} \rfloor - \frac{j_2}{2} -}, t_1, \underbrace{t'_1, \ldots, t'_1}_{\lfloor \frac{m}{2} \rfloor}, \ldots,$$
$$t_{m-j_1}, \underbrace{t'_{m-j_1}, \ldots, t'_{m-j_1}}_{\lfloor \frac{m}{2} \rfloor} 2, \underbrace{3, \ldots, 3}_{\lfloor \frac{m}{2} \rfloor}, c);\ 1 \leq i \leq n,\ 1 \leq j_1 \leq m\ 1 \leq j_2 \leq m-2 \text{ and } j \text{ is even.}$$

with $t_l = l+1$, $1 \leq l \leq m - j_1$ and $t'_l = l+2$, $1 \leq l \leq m - j_1$, $1 \leq j_1 \leq m$; $t_k = k+1$, $1 \leq k \leq j_1 - 1$ and $t'_k = k+2$, $1 \leq k \leq j_1 - 1$, $1 \leq j_1 \leq m$.
The vertex representation of $x_{i,j_1,m-1}$ and $x_{i,j_1,m}$ for m odd respect to W is

$$r(x_{i,j_1,m-1}|W) = (a, t_{j_1-1}, \underbrace{t'_{j_1-1}, \ldots, t'_{j_1-1}}_{\lfloor \frac{m}{2} \rfloor}, \ldots, t_1, \underbrace{t'_1, \ldots, t'_1}_{\lfloor \frac{m}{2} \rfloor}, 1, \underbrace{2, \ldots, 2}_{\lfloor \frac{m}{2} \rfloor - 1}, 1, t_1, \underbrace{t'_1, \ldots, t'_1}_{\lfloor \frac{m}{2} \rfloor}, \ldots, t_{m-j_1},$$
$$\underbrace{t'_{m-j_1}, \ldots, t'_{m-j_1}}_{\lfloor \frac{m}{2} \rfloor} 2, \underbrace{3, \ldots, 3}_{\lfloor \frac{m}{2} \rfloor}, c);\ 1 \leq i \leq n,\ 1 \leq j_1 \leq m,\ 1 \leq j_2 \leq m-1.$$

$$r(x_{i,j_1,m}|W) = (a, t_{j_1-1}, \underbrace{t'_{j_1-1}, \ldots, t'_{j_1-1}}_{\lfloor \frac{m}{2} \rfloor}, \ldots, t_1, \underbrace{t'_1, \ldots, t'_1}_{\lfloor \frac{m}{2} \rfloor}, 1, \underbrace{2, \ldots, 2}_{\lfloor \frac{m}{2} \rfloor}, t_1, \underbrace{t'_1, \ldots, t'_1}_{\lfloor \frac{m}{2} \rfloor}, \ldots, t_{m-j_1},$$
$$\underbrace{t'_{m-j_1}, \ldots, t'_{m-j_1}}_{\lfloor \frac{m}{2} \rfloor} 2, \underbrace{3, \ldots, 3}_{\lfloor \frac{m}{2} \rfloor}, c);\ 1 \leq i \leq n,\ 1 \leq j_1 \leq m,\ 1 \leq j_2 \leq m-1.$$

with $t_l = l+1$, $1 \leq l \leq m - j_1$ and $t'_l = l+2$, $1 \leq l \leq m - j_1$, $1 \leq j_1 \leq m$; $t_k = k+1$, $1 \leq k \leq j_1 - 1$ and $t'_k = k+2$, $1 \leq k \leq j_1 - 1$, $1 \leq j_1 \leq m$.
where

IOP Conf. Series: Journal of Physics: Conf. Series **1008** (2018) 012040 doi:10.1088/1742-6596/1008/1/012040

$a = (t^1_{i-1}, \underbrace{t^2_{i-1}, \dots, t^2_{i-1}}_{\lfloor \frac{m}{2} \rfloor}, \underbrace{t^3_{i-1}, \dots, t^3_{i-1}}_{m}, \underbrace{t^4_{i-1}, \dots, t^4_{i-1}}_{m\lfloor \frac{m}{2} \rfloor}, \dots, t^1_1, \underbrace{t^2_1, \dots, t^2_1}_{\lfloor \frac{m}{2} \rfloor}, \underbrace{t^3_1, \dots, t^3_1}_{m}, \underbrace{t^4_1, \dots, t^4_1}_{m\lfloor \frac{m}{2} \rfloor})$
$c = (t^1_1, \underbrace{t^2_1, \dots, t^2_1}_{\lfloor \frac{m}{2} \rfloor}, \underbrace{t^3_1, \dots, t^3_1}_{m}, \underbrace{t^4_1, \dots, t^4_1}_{m\lfloor \frac{m}{2} \rfloor}, \dots, t^1_{n-i}, \underbrace{t^2_{n-i}, \dots, t^2_{n-i}}_{\lfloor \frac{m}{2} \rfloor}, \underbrace{t^3_{n-i}, \dots, t^3_{n-i}}_{m}, \underbrace{t^4_{n-i}, \dots, t^4_{n-i}}_{m\lfloor \frac{m}{2} \rfloor})$
with $t^1_l = l+2,\ 1 \le l \le n-i$; $t^2_l = l+3,\ 1 \le l \le n-i$; $t^3_l = l+3,\ 1 \le l \le n-i$; $t^4_l = l+4,\ 1 \le l \le n-i$; $t^1_k = k+2,\ 1 \le k \le i-1$; $t^2_k = k+3,\ 1 \le k \le i-1$; $t^3_k = k+3,\ 1 \le k \le i-1$; $t^4_k = k+4,\ 1 \le k \le i-1$

The vertex representation of x_{i,j_2} for m even respect to W is
$r(x_{i,j_2}|W) = (a, 1, \underbrace{2, \dots, 2}_{\frac{j_2}{2}-1}, 1, 1, \underbrace{2, \dots, 2}_{\lfloor \frac{m}{2} \rfloor - \frac{j_2}{2} -}, \underbrace{2, \dots, 2}_{m}, \underbrace{3, \dots, 3}_{m(\lfloor \frac{m}{2} \rfloor)}, c)$; $1 \le i \le n$, $1 \le j_2 \le m-1$ and j is even.
The vertex representation of $x_{i,m}$ for m even respect to W is
$r(x_{i,m}|W) = (a, 1, \underbrace{2, \dots, 2}_{\frac{m}{2}}, 1, c)$; $1 \le i \le n$, $1 \le j_1 \le m$, $1 \le j_2 \le m-1$.

The vertex representation of x_{i,j_2} for m odd respect to W is
$r(x_{i,j_2}|W) = (a, 1, \underbrace{2, \dots, 2}_{\frac{j_2}{2}-1}, 1, 1, \underbrace{2, \dots, 2}_{\lfloor \frac{m}{2} \rfloor - \frac{j_2}{2} -}, \underbrace{2, \dots, 2}_{m}, \underbrace{3, \dots, 3}_{m(\lfloor \frac{m}{2} \rfloor)}, c)$; $1 \le i \le n$, $1 \le j_2 \le m-2$ and j is even.
The vertex representation of $x_{i,m-1}$ and $x_{i,m}$ for m odd respect to W is
$r(x_{i,m-1}|W) = (a, 1, \underbrace{2, \dots, 2}_{\lfloor \frac{m}{2} \rfloor - 1}, 1, \underbrace{2, \dots, 2}_{m}, \underbrace{3, \dots, 3}_{m(\lfloor \frac{m}{2} \rfloor)}, c)$; $1 \le i \le n$.
$r(x_{i,m}|W) = (a, 1, \underbrace{2, \dots, 2}_{\lfloor \frac{m}{2} \rfloor}, \underbrace{2, \dots, 2}_{m}, \underbrace{3, \dots, 3}_{m(\lfloor \frac{m}{2} \rfloor)}, c)$; $1 \le i \le n$.
where
$a = (t^1_{i-1}, \underbrace{t^2_{i-1}, \dots, t^2_{i-1}}_{\lfloor \frac{m}{2} \rfloor}, \underbrace{t^3_{i-1}, \dots, t^3_{i-1}}_{m}, \underbrace{t^4_{i-1}, \dots, t^4_{i-1}}_{m\lfloor \frac{m}{2} \rfloor}, \dots, t^1_1, \underbrace{t^2_1, \dots, t^2_1}_{\lfloor \frac{m}{2} \rfloor}, \underbrace{t^3_1, \dots, t^3_1}_{m}, \underbrace{t^4_1, \dots, t^4_1}_{m\lfloor \frac{m}{2} \rfloor})$
$c = (t^1_1, \underbrace{t^2_1, \dots, t^2_1}_{\lfloor \frac{m}{2} \rfloor}, \underbrace{t^3_1, \dots, t^3_1}_{m}, \underbrace{t^4_1, \dots, t^4_1}_{m\lfloor \frac{m}{2} \rfloor}, \dots, t^1_{n-i}, \underbrace{t^2_{n-i}, \dots, t^2_{n-i}}_{\lfloor \frac{m}{2} \rfloor}, \underbrace{t^3_{n-i}, \dots, t^3_{n-i}}_{m}, \underbrace{t^4_{n-i}, \dots, t^4_{n-i}}_{m\lfloor \frac{m}{2} \rfloor})$
with $t^1_l = l+1,\ 1 \le l \le n-i$; $t^2_l = l+2,\ 1 \le l \le n-i$; $t^3_l = l+2,\ 1 \le l \le n-i$; $t^4_l = l+3,\ 1 \le l \le n-i$; $t^1_k = k+1,\ 1 \le k \le i-1$; $t^2_k = k+2,\ 1 \le k \le i-1$; $t^3_k = k+2,\ 1 \le k \le i-1$; $t^4_k = k+3,\ 1 \le k \le i-1$

It is clearly that every vertices $v \in V(P_n \odot_2 P_m) - W$ has the distinct representation respect to W. Furthermore, we need to shown that all vertices in non-isolated resolving set W without isolated vertex. All vertices in vertex set $W = \{x_i, x_{i,j_1}; 1 \le i \le n, 1 \le j_1 \le m\} \cup \{x_{i,j_1,j_2}, x_{i,j_2}; 1 \le i \le n, 1 \le j_1 \le m-1, 1 \le j_2 \le m-1, j_1 \text{ and } j_2 \text{ is odd}\}$ without isolated vertex by the edge set $\{x_i x_{i,j_1}, x_{i,j_1} x_{i,j_1,j_2}, x_i x_{i,j_2}; 1 \le i \le n, 1 \le j_1 \le m, 1 \le j_2 \le m\}$ which all vertices in W induces subgraph $P_n \odot_2 P_m$ with pendant edges. Hence, $\langle W \rangle$ has no isolated vertices. So, the upper bound non-isolated resolving number of $P_n \odot_2 P_m$ is $nr(P_n \odot_2 P_m) \le (nm+n)(\lfloor \frac{m}{2} \rfloor + 1)$.

Hence, the lower bound non-isolated resolving number of $P_n \odot_2 P_m$ is $nr(P_n \odot_2 P_m) \ge (nm+n)(\lfloor \frac{m}{2} \rfloor + 1)$. It concludes that $nr(P_n \odot_2 P_m) = (nm+n)(\lfloor \frac{m}{2} \rfloor + 1)$. $\square$

Theorem 2.2 *Let P_n and K_m be a connected graph of order $n \ge 2$ and $m \ge 3$, then non-isolated resolving number of $P_n \odot_2 K_m$ is $nr(P_n \odot_2 K_m) = nm^2 + nm$.*

IOP Conf. Series: Journal of Physics: Conf. Series **1008** (2018) 012040 doi:10.1088/1742-6596/1008/1/012040

Proof: Let $P_n \odot_2 K_m$ be a be a corona product of path P_n and complete graph K_m with vertex set $V(P_n \odot_2 K_m) = \{x_i; 1 \le i \le n\} \cup \{x_{i,j_1}; 1 \le i \le n, 1 \le j_1 \le m\} \cup \{x_{i,j_1,j_2}; 1 \le i \le n, 1 \le j_1 \le m, 1 \le j_2 \le m\} \cup \{x_{i,j_2}; 1 \le i \le n, 1 \le j_2 \le m\}$ and edge set $E(P_n \odot_2 K_m) = \{x_i x_{i+1}; 1 \le i \le n-1\} \cup \{x_i x_{i,j_1}, x_{i,j_1} x_{i,j_1,j_2}, x_i x_{i,j_2}; 1 \le i \le n, 1 \le j_1 \le m, 1 \le j_2 \le m\} \cup \{x_{i,j_1} x_{i,j_1+r_1}, x_{i,j_1,j_2} x_{i,j_1,j_2+r_2}, x_{i,j_2} x_{i,j_2+r_2}; 1 \le i \le n, 1 \le j_1 \le m, 1 \le j_2 \le m, 1 \le r_1 \le m-j_1$ and $1 \le r_2 \le m-j_2\}$ with the cardinality of vertex set $|V(P_n \odot_2 K_m)| = nm^2 + 2nm + n$ and the cardinality of edge set $|E(P_n \odot_2 K_m)| = 2nm^2 + mn + n + mn(\frac{m^2-m}{2}) - 1$.

For $n \ge 2$ and $m \ge 3$, based on Lemma 2.1 and Proposition 1.3 then we have the lower bound non isolated resolving number of $P_n \odot_2 K_m$ is $nr(P_n \odot_2 K_m) \ge |V(P_n \odot K_m)| nr(K_1 + K_m) = (nm+n)m = nm^2 + nm$. However, we can attain the sharpest lower bound. Furthermore, we prove that $nr(P_n \odot_2 K_m) \le nm^2 + nm$. Choosing $W \subset V(P_n \odot_2 K_m)$ with $W = \{x_i, x_{i,j_1}; 1 \le i \le n, 1 \le j_1 \le m\} \cup \{x_{i,j_1,j_2}, x_{i,j_2}; 1 \le i \le n, 1 \le j_1 \le m-1, 1 \le j_2 \le m-1\}$ is a non-isolated resolving set of $P_n \odot_2 K_m$ and the cardinality of non-isolated resolving set is $|W| = |\{x_i, x_{i,j_1}; 1 \le i \le n, 1 \le j_1 \le m\}| + |\{x_{i,j_1,j_2}, x_{i,j_2}; 1 \le i \le n, 1 \le j_1 \le m-1, 1 \le j_2 \le m-1\}| = nm^2 + nm$. Thus, the representation of vertices $v \in V(P_n \odot_2 K_m) - W$ respecting to W are as follows.
The vertex representation of x_{i,j_1,j_2} respect to W is

$$r(x_{i,j_1,j_2}|W) = (a, \underbrace{2, \underbrace{3,\ldots,3}_{m-1}, \ldots, 2, \underbrace{3,\ldots,3}_{m-1}}_{j_1-1}, \underbrace{1,\ldots,1}_{m}, \underbrace{2, \underbrace{3,\ldots,3}_{m-1}, \ldots, 2, \underbrace{3,\ldots,3}_{m-1}}_{m-j_1}, 2, \underbrace{3,\ldots,3}_{m-1}, c);\ 1 \le i \le n,\ 1 \le j_1 \le m,\ j_2 = m.$$

where

$$a = (t^1_{i-1}, \underbrace{t^2_{i-1},\ldots,t^2_{i-1}}_{m-1}, \underbrace{t^3_{i-1},\ldots,t^3_{i-1}}_{m}, \underbrace{t^4_{i-1},\ldots,t^4_{i-1}}_{m(m-1)}, \ldots, t^1_1, \underbrace{t^2_1,\ldots,t^2_1}_{m-1}, \underbrace{t^3_1,\ldots,t^3_1}_{m}, \underbrace{t^4_1,\ldots,t^4_1}_{m(m-1})$$

$$c = (t^1_1, \underbrace{t^2_1,\ldots,t^2_1}_{m-1}, \underbrace{t^3_1,\ldots,t^3_1}_{m}, \underbrace{t^4_1,\ldots,t^4_1}_{m(m-1}, \ldots, t^1_{n-i}, \underbrace{t^2_{n-i},\ldots,t^2_{n-i}}_{m-1}, \underbrace{t^3_{n-i},\ldots,t^3_{n-i}}_{m}, \underbrace{t^4_{n-i},\ldots,t^4_{n-i}}_{m(m-1)})$$

with $t^1_l = l+2$, $1 \le l \le n-i$; $t^2_l = l+3$, $1 \le l \le n-i$; $t^3_l = l+3$, $1 \le l \le n-i$; $t^4_l = l+4$, $1 \le l \le n-i$; $t^1_k = k+2$, $1 \le k \le i-1$; $t^2_k = k+3$, $1 \le k \le i-1$; $t^3_k = k+3$, $1 \le k \le i-1$; $t^4_k = k+4$, $1 \le k \le i-1$

The vertex representation of x_{i,j_2} respect to W is

$$r(x_{i,j_2}|W) = (a, \underbrace{1,\ldots,1}_{m}, \underbrace{2,\ldots,2}_{m}, \underbrace{3,\ldots,3}_{m(m-1)}, c);\ 1 \le i \le n,\ j_2 = m.$$

where

$$a = (t^1_{i-1}, \underbrace{t^2_{i-1},\ldots,t^2_{i-1}}_{m-1}, \underbrace{t^3_{i-1},\ldots,t^3_{i-1}}_{m}, \underbrace{t^4_{i-1},\ldots,t^4_{i-1}}_{m(m-1)}, \ldots, t^1_1, \underbrace{t^2_1,\ldots,t^2_1}_{m-1}, \underbrace{t^3_1,\ldots,t^3_1}_{m}, \underbrace{t^4_1,\ldots,t^4_1}_{m(m-1})$$

$$c = (t^1_1, \underbrace{t^2_1,\ldots,t^2_1}_{m-1}, \underbrace{t^3_1,\ldots,t^3_1}_{m}, \underbrace{t^4_1,\ldots,t^4_1}_{m(m-1}, \ldots, t^1_{n-i}, \underbrace{t^2_{n-i},\ldots,t^2_{n-i}}_{m-1}, \underbrace{t^3_{n-i},\ldots,t^3_{n-i}}_{m}, \underbrace{t^4_{n-i},\ldots,t^4_{n-i}}_{m(m-1)})$$

with $t^1_l = l+1$, $1 \le l \le n-i$; $t^2_l = l+2$, $1 \le l \le n-i$; $t^3_l = l+2$, $1 \le l \le n-i$; $t^4_l = l+3$, $1 \le l \le n-i$; $t^1_k = k+1$, $1 \le k \le i-1$; $t^2_k = k+2$, $1 \le k \le i-1$; $t^3_k = k+2$, $1 \le k \le i-1$; $t^4_k = k+3$, $1 \le k \le i-1$

It is clearly that every vertices $v \in V(P_n \odot_2 K_m) - W$ has the distinct representation respect to W. Furthermore, we need to shown that all vertices in non-isolated resolving set W without isolated vertex. All vertices in vertex set $W = \{x_i, x_{i,j_1}; 1 \le i \le n, 1 \le j_1 \le m\} \cup \{x_{i,j_1,j_2}, x_{i,j_2}; 1 \le i \le n, 1 \le j_1 \le m-1, 1 \le j_2 \le m-1\}$ without isolated vertex by the edge set $\{x_i x_{i,j_1}, x_{i,j_1} x_{i,j_1,j_2}, x_i x_{i,j_2}; 1 \le i \le n, 1 \le j_1 \le m, 1 \le j_2 \le m\}$ which all vertices in W induces subgraph $P_n \odot_2 K_m$ with pendant edges. Hence, $\langle W \rangle$ has no isolated vertices. So, the upper bound non-isolated resolving number of $P_n \odot_2 K_m$ is $nr(P_n \odot_2 K_m) \le nm^2 + nm$.

IOP Conf. Series: Journal of Physics: Conf. Series **1008** (2018) 012040 doi:10.1088/1742-6596/1008/1/012040

Hence, the lower bound non-isolated resolving number of $P_n \odot_2 K_m$ is $nr(P_n \odot_2 K_m) \geq nm^2 + nm$. It concludes that $nr(P_n \odot_2 K_m) = nm^2 + nm$

Theorem 2.3 *Let K_m and P_n be a connected graph of order $n \geq 2$ and $m \geq 3$, then non-isolated resolving number of $K_m \odot_2 P_n$ is $nr(K_m \odot_2 P_n) = (mn + m)(\lfloor \frac{n}{2} \rfloor + 1)$.*

Proof: Let $K_m \odot_2 P_n$ be a be a corona product of complete graph K_m and path P_n with vertex set $V(K_m \odot_2 P_n) = \{x_i; 1 \leq i \leq n\} \cup \{x_{i,j_1}; 1 \leq i \leq n, 1 \leq j_1 \leq m\} \cup \{x_{i,j_1,j_2}; 1 \leq i \leq n, 1 \leq j_1 \leq m, 1 \leq j_2 \leq m\} \cup \{x_{i,j_2}; 1 \leq i \leq n, 1 \leq j_2 \leq m\}$ and edge set $E(K_m \odot_2 P_n) = \{x_i x_{i+r}; 1 \leq i \leq n, 1 \leq r \leq n-i\} \cup \{x_i x_{i,j_1}, x_{i,j_1} x_{i,j_1,j_2}, x_i x_{i,j_2}; 1 \leq i \leq n, 1 \leq j_1 \leq m, 1 \leq j_2 \leq m\} \cup \{x_{i,j_1} x_{i,j_1+1}, x_{i,j_1,j_2} x_{i,j_1,j_2+1}, x_{i,j_2} x_{i,j_2+1}; 1 \leq i \leq n, 1 \leq j_1 \leq m-1, 1 \leq j_2 \leq m-1\}$ with the cardinality of vertex set $|V(K_m \odot_2 P_n)| = nm^2 + 2nm + n$ and the cardinality of edge set $|E(K_m \odot_2 P_n)| = 2nm^2 + 3mn - 2n + \frac{n^2-n}{2}$.

For $n \geq 2$ and $m \geq 3$, based on Lemma 2.1 and Proposition 1.3 then we have $nr(K_m \odot_2 P_n) \geq |V(K_m \odot P_n)| nr(K_1 + P_n) = (nm+m)(\lfloor \frac{n}{2} \rfloor + 1)$. However, we can attain the sharpest lower bound. Furthermore, we prove that $nr(K_m \odot_2 P_n) \leq (nm+m)(\lfloor \frac{n}{2} \rfloor + 1)$. Choosing $W \subset V(K_m \odot_2 P_n)$ with $W = \{x_i, x_{i,j_1}; 1 \leq i \leq n, 1 \leq j_1 \leq m\} \cup \{x_{i,j_1,j_2}, x_{i,j_2}; 1 \leq i \leq n, 1 \leq j_1 \leq m-1, 1 \leq j_2 \leq m-1, j_1 \text{ and } j_2 \text{ is odd}\}$ is a non-isolated resolving set of $K_m \odot_2 P_n$ and the cardinality of non-isolated resolving set is $|W| = |\{x_i, x_{i,j_1}; 1 \leq i \leq n, 1 \leq j_1 \leq m\}| + |\{x_{i,j_1,j_2}, x_{i,j_2}; 1 \leq i \leq n, 1 \leq j_1 \leq m-1, 1 \leq j_2 \leq m-1, j_1 \text{ and } j_2 \text{ is odd}\}| = (nm+m)(\lfloor \frac{n}{2} \rfloor + 1)$. Thus, the representation of vertices $v \in V(K_m \odot_2 P_n) - W$ respecting to W are as follows.

The vertex representation of x_{i,j_1,j_2} for n even respect to W is

$$r(x_{i,j_1,j_2}|W) = (a, t_{j_1-1}, \underbrace{t'_{j_1-1}, \dots, t'_{j_1-1}}_{\lfloor \frac{n}{2} \rfloor}, \dots, t_1, \underbrace{t'_1, \dots, t'_1}_{\lfloor \frac{n}{2} \rfloor}, 1, \underbrace{2, \dots, 2}_{\frac{j_2}{2}-1}, 1, 1, \underbrace{2, \dots, 2}_{\lfloor \frac{m}{2} \rfloor - \frac{j_2}{2} -}, t_1, \underbrace{t'_1, \dots, t'_1}_{\lfloor \frac{n}{2} \rfloor}, \dots,$$
$$t_{n-j_1}, \underbrace{t'_{n-j_1}, \dots, t'_{n-j_1}}_{\lfloor \frac{n}{2} \rfloor} 2, \underbrace{3, \dots, 3}_{\lfloor \frac{n}{2} \rfloor}, c); \; 1 \leq i \leq m, \; 1 \leq j_1 \leq n \; 1 \leq j_2 \leq n-1 \text{ and } j \text{ is even.}$$

with $t_l = l+1$, $1 \leq l \leq n-j_1$ and $t'_l = l+2$, $1 \leq l \leq n-j_1$, $1 \leq j_1 \leq n$; $t_k = k+1$, $1 \leq k \leq j_1 - 1$ and $t'_k = k+2$, $1 \leq k \leq j_1 - 1$, $1 \leq j_1 \leq n$.

The vertex representation of $x_{i,j_1,n}$ for n even respect to W is

$$r(x_{i,j_1,n}|W) = (a, t_{j_1-1}, \underbrace{t'_{j_1-1}, \dots, t'_{j_1-1}}_{\lfloor \frac{n}{2} \rfloor}, \dots, t_1, \underbrace{t'_1, \dots, t'_1}_{\lfloor \frac{n}{2} \rfloor}, 1, \underbrace{2, \dots, 2}_{\frac{n}{2}-1}, 1, t_1, \underbrace{t'_1, \dots, t'_1}_{\lfloor \frac{n}{2} \rfloor}, \dots, t_{n-j_1},$$
$$\underbrace{t'_{n-j_1}, \dots, t'_{n-j_1}}_{\lfloor \frac{n}{2} \rfloor} 2, \underbrace{3, \dots, 3}_{\lfloor \frac{n}{2} \rfloor}, c); \; 1 \leq i \leq m, \; 1 \leq j_1 \leq n, \; 1 \leq j_2 \leq n-1.$$

with $t_l = l+1$, $1 \leq l \leq n-j_1$ and $t'_l = l+2$, $1 \leq l \leq n-j_1$, $1 \leq j_1 \leq n$; $t_k = k+1$, $1 \leq k \leq j_1 - 1$ and $t'_k = k+2$, $1 \leq k \leq j_1 - 1$, $1 \leq j_1 \leq n$.

The vertex representation of x_{i,j_1,j_2} for n odd respect to W is

$$r(x_{i,j_1,j_2}|W) = (a, t_{j_1-1}, \underbrace{t'_{j_1-1}, \dots, t'_{j_1-1}}_{\lfloor \frac{n}{2} \rfloor}, \dots, t_1, \underbrace{t'_1, \dots, t'_1}_{\lfloor \frac{n}{2} \rfloor}, 1, \underbrace{2, \dots, 2}_{\frac{j_2}{2}-1}, 1, 1, \underbrace{2, \dots, 2}_{\lfloor \frac{n}{2} \rfloor - \frac{j_2}{2} -}, t_1, \underbrace{t'_1, \dots, t'_1}_{\lfloor \frac{n}{2} \rfloor}, \dots,$$
$$t_{n-j_1}, \underbrace{t'_{n-j_1}, \dots, t'_{n-j_1}}_{\lfloor \frac{n}{2} \rfloor} 2, \underbrace{3, \dots, 3}_{\lfloor \frac{n}{2} \rfloor}, c); \; 1 \leq i \leq m, \; 1 \leq j_1 \leq n \; 1 \leq j_2 \leq n-2 \text{ and } j \text{ is even.}$$

with $t_l = l+1$, $1 \leq l \leq n-j_1$ and $t'_l = l+2$, $1 \leq l \leq n-j_1$, $1 \leq j_1 \leq n$; $t_k = k+1$, $1 \leq k \leq j_1 - 1$ and $t'_k = k+2$, $1 \leq k \leq j_1 - 1$, $1 \leq j_1 \leq n$.

The vertex representation of $x_{i,j_1,n-1}$ and $x_{i,j_1,n}$ for n odd respect to W is

$$r(x_{i,j_1,m-1}|W) = (a, t_{j_1-1}, \underbrace{t'_{j_1-1}, \dots, t'_{j_1-1}}_{\lfloor \frac{n}{2} \rfloor}, \dots, t_1, \underbrace{t'_1, \dots, t'_1}_{\lfloor \frac{n}{2} \rfloor}, 1, \underbrace{2, \dots, 2}_{\lfloor \frac{n}{2} \rfloor - 1}, 1, t_1, \underbrace{t'_1, \dots, t'_1}_{\lfloor \frac{n}{2} \rfloor}, \dots, t_{n-j_1},$$
$$\underbrace{t'_{n-j_1}, \dots, t'_{n-j_1}}_{\lfloor \frac{n}{2} \rfloor} 2, \underbrace{3, \dots, 3}_{\lfloor \frac{n}{2} \rfloor}, c); \; 1 \leq i \leq m, \; 1 \leq j_1 \leq n, \; 1 \leq j_2 \leq n-1.$$

IOP Conf. Series: Journal of Physics: Conf. Series **1008** (2018) 012040 doi:10.1088/1742-6596/1008/1/012040

$r(x_{i,j_1,n}|W) = (a, t_{j_1-1}, \underbrace{t'_{j_1-1}, \dots, t'_{j_1-1}}_{\lfloor\frac{n}{2}\rfloor}, \dots, t_1, \underbrace{t'_1, \dots, t'_1}_{\lfloor\frac{n}{2}\rfloor}, 1, \underbrace{2, \dots, 2}_{\lfloor\frac{n}{2}\rfloor}, t_1, \underbrace{t'_1, \dots, t'_1}_{\lfloor\frac{n}{2}\rfloor}, \dots, t_{n-j_1},$
$\underbrace{t'_{n-j_1}, \dots, t'_{n-j_1}}_{\lfloor\frac{n}{2}\rfloor} 2, \underbrace{3, \dots, 3}_{\lfloor\frac{n}{2}\rfloor}, c)$; $1 \le i \le m$, $1 \le j_1 \le n$, $1 \le j_2 \le n-1$.
with $t_l = l+1$, $1 \le l \le n-j_1$ and $t'_l = l+2$, $1 \le l \le n-j_1$, $1 \le j_1 \le n$; $t_k = k+1$, $1 \le k \le j_1-1$ and $t'_k = k+2$, $1 \le k \le j_1-1$, $1 \le j_1 \le n$.
where

$$a = \underbrace{(3, \underbrace{4, \dots, 4}_{\lfloor\frac{n}{2}\rfloor}, \underbrace{4, \dots, 4}_{n}, \underbrace{5, \dots, 5}_{n\lfloor\frac{n}{2}\rfloor}, \dots, 3, \underbrace{4, \dots, 4}_{\lfloor\frac{n}{2}\rfloor}, \underbrace{4, \dots, 4}_{n}, \underbrace{5, \dots, 5}_{n\lfloor\frac{n}{2}\rfloor})}_{i-1}$$

$$c = \underbrace{(3, \underbrace{4, \dots, 4}_{\lfloor\frac{n}{2}\rfloor}, \underbrace{4, \dots, 4}_{n}, \underbrace{5, \dots, 5}_{n\lfloor\frac{n}{2}\rfloor}, \dots, 3, \underbrace{4, \dots, 4}_{\lfloor\frac{n}{2}\rfloor}, \underbrace{4, \dots, 4}_{n}, \underbrace{5, \dots, 5}_{n\lfloor\frac{n}{2}\rfloor})}_{m-i}$$

The vertex representation of x_{i,j_2} for n even respect to W is
$r(x_{i,j_2}|W) = (a, 1, \underbrace{2, \dots, 2}_{\frac{j_2}{2}-1}, 1, 1, \underbrace{2, \dots, 2}_{\lfloor\frac{n}{2}\rfloor-\frac{j_2}{2}-1}, \underbrace{2, \dots, 2}_{n}, \underbrace{3, \dots, 3}_{n(\lfloor\frac{n}{2}\rfloor)}, c)$; $1 \le i \le m$, $1 \le j_2 \le n-1$ and j_2 is even.
The vertex representation of $x_{i,n}$ for n even respect to W is
$r(x_{i,n}|W) = (a, 1, \underbrace{2, \dots, 2}_{\frac{n}{2}}, 1, c)$; $1 \le i \le m$, $1 \le j_1 \le n$, $1 \le j_2 \le n-1$.

The vertex representation of x_{i,j_2} for m odd respect to W is
$r(x_{i,j_2}|W) = (a, 1, \underbrace{2, \dots, 2}_{\frac{j_2}{2}-1}, 1, 1, \underbrace{2, \dots, 2}_{\lfloor\frac{n}{2}\rfloor-\frac{j_2}{2}-}, \underbrace{2, \dots, 2}_{n}, \underbrace{3, \dots, 3}_{n(\lfloor\frac{n}{2}\rfloor)}, c)$; $1 \le i \le m$, $1 \le j_2 \le n-2$ and j_2 is even.
The vertex representation of $x_{i,n-1}$ and $x_{i,n}$ for n odd respect to W is
$r(x_{i,n-1}|W) = (a, 1, \underbrace{2, \dots, 2}_{\lfloor\frac{n}{2}\rfloor-1}, 1, \underbrace{2, \dots, 2}_{n}, \underbrace{3, \dots, 3}_{n(\lfloor\frac{n}{2}\rfloor)}, c)$; $1 \le i \le m$.
$r(x_{i,n}|W) = (a, 1, \underbrace{2, \dots, 2}_{\lfloor\frac{n}{2}\rfloor}, \underbrace{2, \dots, 2}_{n}, \underbrace{3, \dots, 3}_{n(\lfloor\frac{n}{2}\rfloor)}, c)$; $1 \le i \le m$.
where

$$a = \underbrace{(2, \underbrace{3, \dots, 3}_{\lfloor\frac{n}{2}\rfloor}, \underbrace{3, \dots, 3}_{n}, \underbrace{4, \dots, 4}_{n\lfloor\frac{n}{2}\rfloor}, \dots, 2, \underbrace{3, \dots, 3}_{\lfloor\frac{n}{2}\rfloor}, \underbrace{3, \dots, 3}_{n}, \underbrace{4, \dots, 4}_{n\lfloor\frac{n}{2}\rfloor})}_{i-1}$$

$$c = \underbrace{(2, \underbrace{3, \dots, 3}_{\lfloor\frac{n}{2}\rfloor}, \underbrace{3, \dots, 3}_{n}, \underbrace{4, \dots, 4}_{n\lfloor\frac{n}{2}\rfloor}, \dots, 2, \underbrace{3, \dots, 3}_{\lfloor\frac{n}{2}\rfloor}, \underbrace{3, \dots, 3}_{n}, \underbrace{4, \dots, 4}_{n\lfloor\frac{n}{2}\rfloor})}_{m-i}$$

It is clearly that every vertices $v \in V(K_m \odot_2 P_n) - W$ has the distinct representation respect to W. Furthermore, we need to shown that all vertices in non-isolated resolving set W without isolated vertex. All vertices in vertex set $W = \{x_i, x_{i,j_1}; 1 \le i \le n, 1 \le j_1 \le m\} \cup \{x_{i,j_1,j_2}, x_{i,j_2}; 1 \le i \le n, 1 \le j_1 \le m-1, 1 \le j_2 \le m-1, j_1 \text{ and } j_2 \text{ is odd}\}$ without isolated vertex by the edge set $\{x_i x_{i,j_1}, x_{i,j_1} x_{i,j_1,j_2}, x_i x_{i,j_2}; 1 \le i \le n, 1 \le j_1 \le m, 1 \le j_2 \le m\}$ which all vertices in W induces subgraph $K_m \odot_2 P_n$ with pendant edges. Hence, $\langle W \rangle$ has no isolated vertices. So, the upper bound non-isolated resolving number of $K_m \odot_2 P_n$ is $nr(K_m \odot_2 P_n) \le (nm+m)(\lfloor\frac{n}{2}\rfloor+1)$.

Hence, the lower bound non-isolated resolving number of $K_m \odot_2 P_n$ is $nr(K_m \odot_2$

IOP Conf. Series: Journal of Physics: Conf. Series **1008** (2018) 012040 doi:10.1088/1742-6596/1008/1/012040

$P_n) \geq (nm+m)(\lfloor \frac{n}{2} \rfloor + 1)$. It concludes that $nr(K_m \odot_2 P_n) = (nm+m)(\lfloor \frac{n}{2} \rfloor + 1)$ □

Theorem 2.4 *Let K_n and K_m be a connected graph of order $n, m \geq 3$, then non-isolated resolving number of $K_n \odot_2 K_m$ is $nr(K_n \odot_2 K_m) = nm^2 + nm$.*

Proof: Let $K_n \odot_2 K_m$ be a be a corona product of complete graph K_n and K_m with vertex set $V(K_n \odot_2 K_m) = \{x_i; 1 \leq i \leq n\} \cup \{x_{i,j_1}; 1 \leq i \leq n, 1 \leq j_1 \leq m\} \cup \{x_{i,j_1,j_2}; 1 \leq i \leq n, 1 \leq j_1 \leq m, 1 \leq j_2 \leq m\} \cup \{x_{i,j_2}; 1 \leq i \leq n, 1 \leq j_2 \leq m\}$ and edge set $E(K_n \odot_2 K_m) = \{x_i x_{i+r}; 1 \leq i \leq n, 1 \leq r \leq n-i\} \cup \{x_i x_{i,j_1}, x_{i,j_1} x_{i,j_1,j_2}, x_i x_{i,j_2}; 1 \leq i \leq n, 1 \leq j_1 \leq m, 1 \leq j_2 \leq m\} \cup \{x_{i,j_1} x_{i,j_1+r_1}, x_{i,j_1,j_2} x_{i,j_1,j_2+r_2}, x_{i,j_2} x_{i,j_2+r_2}; 1 \leq i \leq n, 1 \leq j_1 \leq m, 1 \leq j_2 \leq m, 1 \leq r_1 \leq m-j_1$ and $1 \leq r_2 \leq m-j_2\}$ with the cardinality of vertex set $|V(K_n \odot_2 K_m)| = nm^2 + 2nm + n$ and the cardinality of edge set $|E(K_n \odot_2 K_m)| = 2nm^2 + mn + mn(\frac{m^2-m}{2}) + \frac{n^2-n}{2}$.

For $n \geq 2$ and $m \geq 3$, based on Lemma 2.1 and Proposition 1.3 then we have $nr(K_n \odot_2 K_m) \geq |V(K_n \odot K_m)| nr(K_1 + K_m) = (nm+n)m = nm^2 + nm$. However, we can attain the sharpest lower bound. Furthermore, we prove that $nr(K_n \odot_2 K_m) \leq nm^2 + nm$. Choosing $W \subset V(K_n \odot_2 K_m)$ with $W = \{x_i, x_{i,j_1}; 1 \leq i \leq n, 1 \leq j_1 \leq m\} \cup \{x_{i,j_1,j_2}, x_{i,j_2}; 1 \leq i \leq n, 1 \leq j_1 \leq m-1, 1 \leq j_2 \leq m-1\}$ is a non-isolated resolving set of $K_n \odot_2 K_m$ and the cardinality of non-isolated resolving set is $|W| = |\{x_i, x_{i,j_1}; 1 \leq i \leq n, 1 \leq j_1 \leq m\}| + |\{x_{i,j_1,j_2}, x_{i,j_2}; 1 \leq i \leq n, 1 \leq j_1 \leq m-1, 1 \leq j_2 \leq m-1\}| = nm^2 + nm$. Thus, the representation of vertices $v \in V(K_n \odot_2 K_m) - W$ respecting to W are as follows.

The vertex representation of x_{i,j_1,j_2} respect to W is

$$r(x_{i,j_1,m}|W) = (a, \underbrace{2, \underbrace{3,\ldots,3}_{m-1}, \ldots, 2, \underbrace{3,\ldots,3}_{m-1}}_{j_1-1}, \underbrace{1,\ldots,1}_{m}, \underbrace{2, \underbrace{3,\ldots,3}_{m-1}, \ldots, 2, \underbrace{3,\ldots,3}_{m-1}}_{m-j_1}, 2, \underbrace{3,\ldots,3}_{m-1}, c);\ 1 \leq i \leq n,\ 1 \leq j_1 \leq m,\ j_2 = m.$$

where

$$a = (\underbrace{3, \underbrace{4,\ldots,4}_{m-1}, \underbrace{4,\ldots,4}_{m}, \underbrace{5,\ldots,5}_{m(m-1)}, \ldots, 3, \underbrace{4,\ldots,4}_{m-1}, \underbrace{4,\ldots,4}_{m}, \underbrace{5,\ldots,5}_{m(m-1)}}_{i-1})$$

$$c = (\underbrace{3, \underbrace{4,\ldots,4}_{m-1}, \underbrace{4,\ldots,4}_{m}, \underbrace{5,\ldots,5}_{m(m-1)}, \ldots, 3, \underbrace{4,\ldots,4}_{m-1}, \underbrace{4,\ldots,4}_{m}, \underbrace{5,\ldots,5}_{m(m-1)}}_{n-i})$$

The vertex representation of x_{i,j_2} respect to W is

$$r(x_{i,j_2}|W) = (a, \underbrace{1,\ldots,1}_{m}, \underbrace{2,\ldots,2}_{m}, \underbrace{3,\ldots,3}_{m(m-1)}, c);\ 1 \leq i \leq n,\ j_2 = m.$$

where

$$a = (\underbrace{2, \underbrace{3,\ldots,3}_{m-1}, \underbrace{3,\ldots,3}_{m}, \underbrace{4,\ldots,4}_{m(m-1)}, \ldots, 2, \underbrace{3,\ldots,3}_{m-1}, \underbrace{3,\ldots,3}_{m}, \underbrace{4,\ldots,4}_{m(m-1)}}_{i-1})$$

$$c = (\underbrace{2, \underbrace{3,\ldots,3}_{m-1}, \underbrace{3,\ldots,3}_{m}, \underbrace{4,\ldots,4}_{m(m-1)}, \ldots, 2, \underbrace{3,\ldots,3}_{m-1}, \underbrace{3,\ldots,3}_{m}, \underbrace{4,\ldots,4}_{m(m-1)}}_{n-i})$$

It is clearly that every vertices $v \in V(K_n \odot_2 K_m) - W$ has the distinct representation respect to W. Furthermore, we need to shown that all vertices in non-isolated resolving set W without isolated vertex. All vertices in vertex set $W = \{x_i, x_{i,j_1}; 1 \leq i \leq n, 1 \leq j_1 \leq m\} \cup \{x_{i,j_1,j_2}, x_{i,j_2}; 1 \leq i \leq n, 1 \leq j_1 \leq m-1, 1 \leq j_2 \leq m-1\}$ without isolated vertex by the edge set $\{x_i x_{i,j_1}, x_{i,j_1} x_{i,j_1,j_2}, x_i x_{i,j_2}; 1 \leq i \leq n, 1 \leq j_1 \leq m, 1 \leq j_2 \leq m\}$

which all vertices in W induces subgraph $K_n \odot_2 K_m$ with pendant edges. Hence, $\langle W \rangle$ has no isolated vertex. So, the upper bound non-isolated resolving number of $K_n \odot_2 K_m$ is $nr(K_n \odot_2 K_m) \leq nm^2 + nm$. Hence, the lower bound non-isolated resolving number of $K_n \odot_2 K_m$ is $nr(K_n \odot_2 K_m) \geq nm^2 + nm$. It concludes that $nr(K_n \odot_2 K_m) = nm^2 + nm$ □

3. Conclusion

The results show that the non-isolated resolving number attain the best lower bound. There are some open problem as follows

Open Problem 1 *Find the non-isolated resolving number of $G \odot_k H$ with $k \geq 3$ for G, H are connected graph except path P_n and complete graph K_n.*

Acknowledgement

We gratefully acknowledge the support from CGANT - University of Jember of year 2018.

References

[1] Gross J L, Yellen J and Zhang P 2014 *Handbook of graph Theory* Second Edition CRC Press Taylor and Francis Group
[2] Chartrand G and Lesniak L 2000 *Graphs and digraphs 3rd* ed (London: Chapman and Hall)
[3] Slater P J 1975 Leaves of trees. in:*Proc. 6th Southeast Conf. Comb., Graph Theory, Comput. Boca Rotan* No. 14 549-559
[4] Harary F and Melter R A 1976 On The metric dimension of a graph, *Ars Combin* No.2 191-195
[5] Chitra P J B and Arumugam S 2015 Resolving Sets without Isolated Vertices, *Procedia Computer Science* **74** 38-42
[6] Chartrand G, Eroh L, Johnson M A and Oellermann O R 2000 Resolvability in graphs and the metric dimension of a graph *Discrete Appl. Math* No.105 99-113
[7] Saenpholphat V and Zhang P 2002 Connected partition dimension of a graphs *Discussiones Mathematicae Graph Theory* **22** 305-323
[8] Baca M, Baskoro E T, Salman A N M, Saputro S W and Suprijanto D 2011 The Metric Dimension of Regular Bipartite Graphs *Bull. Math. Soc. Sci. Math. Roumanie, Tome* **54**(102) 15-28
[9] Iswadi H, Baskoro E T and Simanjuntak R 2011 On the Metric Dimension of Corona Product Graphs, *Far East Journal of Mathematical Sciences* **52** 155 170
[10] Rodriguez-Velazquez J A and Fernau H 2013 On the (Adjacency) Metric Dimension of Corona and Strong Product Graph and Their Local Variants *Combinatorial And Computational Results* Arxiv: 1309.2275.v1 (Math Co)
[11] Simanjuntak R, Uttunggadewa S and Saputro S W 2013 Metric Dimension of Amalgamation of Graphs arXiv:1312.0191v2 [math.CO]
[12] Saputro S W, Suprijanto D, Baskoro E T and Salman A N M 2012 The Metric Dimension of a Graph Composition Products With Star *J. Indones. Math. Soc.* **18** 85-92
[13] Yero I G, Kuziak D and Rodriguez-Velazquez J A 2010 On the Metric Dimension of Corona Product Graphs *Combinatorial and Computational Results* arXiv:1009.2586v2[math CO]
[14] Yunika S M 2017 *Analisa Dimensi Metrik dengan Himpunan Pembeda Tak-Terisolasi Graf Hasil Operasi Korona* Thesis
[15] Dafik, Agustin I H, Surahmat, Syafrizal Sy and Alfarisi R 2017 On non-isolated resolving number of some graph operations *Far East Journal of Mathematical Sciences* **102** (2) 2473 - 2492
[16] Alfarisi R, Dafik, Agustin I H, Kristiana A I 2017 On non-isolated resolving number of graphs with pendant edges *Journal Interconnection Network* Submitted
[17] Frucht R and Harary F 1970 On the corona of two graphs *Aequationes Math* **4** 322-325
[18] Furmanczyk H, Kubale M and Mkrtchyan V V 2012 Equitable Colorings of Corona Multiproducts of Graphs, arXiv:1210.6568v1 [cs.DM]