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Neutron Flux Distribution of Slab Reactor Core using One-Dimensional Multi-Group Diffusion Equation in the Thermal Energy Region

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Abstract. Neutron flux distribution is an essential parameter in the neutronic analysis of the neutron transport in the nuclear reactor. The multi-group diffusion method is the approach commonly used to solve the neutron transport equation. Based on reactor types, the neutron energy range is classified into two regions, namely fast and thermal energy regions. This study presents the neutron flux distribution in the thermal region of the slab reactor core using a one-dimensional multi-group diffusion equation with the Gauss-Seidel iteration method. The reactor is designed in the form of a fast reactor, and it uses the macroscopic cross-sections of the calculation results in the U-PuN fuel cell level as initial input for 70 energy groups. The library data used is JFS-3-J33 70 energy groups, which is the library data of SLAROM computer codes from JAEA Japan. The computational program calculates the neutron flux distribution only in the thermal group of energy: The calculation of neutron flux distributions is executed using the Gauss-Seidel iteration method. The results indicated that the increase of group energy does not reflect the order of magnitude of a multi-group neutron flux distribution in the thermal region. In the case of neutron interaction with U-235 and Pu-239 nuclide, the pattern of neutron flux distribution for each energy group is not sequential, because both are fissile material. Contrary to the U-238 isotope, almost all group energies coincide with one another in the thermal region. The phenomenon occurs because of U-238 is a fertile material that cannot be directly fissioned.

INTRODUCTION

Neutron transport problems require a comprehensive solution to find out the distribution of neutrons as such variables of time, space, and energy in the reactor core. Identifying the distribution of neutrons, the distribution of space, time and energy of all nuclear reactions enables to be determined, bearing in mind the distribution of neutrons is crucial to the reactor power [1,2]. The neutron transport equation is the most crucial problem in the analysis of the nuclear reactor [3]. It is a complicated event whose behavior of the neutron can be described as a coupled integro-differential equation [4]. Integro-differential equations of neutron transport, written in the form of tightly packed neutrons with seven parameters of space, time, and energy, describe the situation of equilibrium among all nuclear processes that affect the number of neutron populations. Neutron transport equations are complicated to solve analytically unless a lot of simplifications are made based on the assumptions made. The most straightforward approach to solving neutron transport is the diffusion theory [5].

The spectrum of neutrons energy consists of two categories, namely, fast and thermal neutrons. The distribution of neutron in a slab medium with complex mechanisms are well simple modeled by a one-speed neutron transport equation [6]. The diffusion approach is very appropriate to be implemented to calculate the distribution of neutrons for a reactor that has large dimensions and does not pay attention to the geometric shape of the reactor core design.

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In a previous study [7], neutron flux distribution was presented in the one-dimensional diffusion equation based on the variation of spatial mesh in the slab of the reactor core using the Jacobi iteration method. The calculation of flux distribution uses a one-speed neutron diffusion equation in the fast energy region. The case resolved was quite simple, namely the geometry design of a slab reactor with a certain width and the spatial mesh in all regions changes in the *x*-axis direction.

In the current study, the neutron flux distribution is performed with a one-dimensional multi-group diffusion equation using the Gauss-Seidel iteration method in the slab geometry for the thermal energy region. Macroscopic scattering cross-section data in the calculation of nuclear fuel cells were obtained from the development of a computer program [8]. In the study of [7] and [8], the type of reactor selected was a fast reactor so that the distribution of neutron flux was mostly in the high energy region. Therefore, a review of the flux distribution in the thermal region is also important to consider, because the thermal region is region where the neutrons experience a slowdown. Energy dependence on the cross-section of nuclide constituen of the nuclear fuel cell is a significant concern in nuclear computing. To accurately represent several resonant nuclide reactions, more than one hundred thousand of energy points are needed. Because of the many energy and spatial points that must be calculated, most nuclear codes cannot handle this large amount of data [9]. For this reason, such an extensive range of energy is grouped into several energy groups called multi-group energy. The multi-group energy approach also has a significant impact on programming, especially in terms of computational time. The calculation of flux distribution is one way to find out the physical performance from the reactor core, commonly called reactor physics.

METHODOLOGY

Energy dependence cannot be separated from neutron flux. Nuclear reactors can be classified according to the typical energy of neutrons, which causes fission, namely fast and thermal reactors. Fast reactors are reactors where most of the neutron energy is in the range of 0.1 to 1 MeV. Neutrons remain high-energy because there are only a few materials that cause them to slow down. Otherwise, thermal reactors contain good neutron modifying agents, and most neutrons have an energy of less than 0.1 eV [10]. The neutron population then has a Maxwellian distribution that corresponds to the speed of the thermal neutron. Because of the neutron energy range is very large, the discretization of energy is made by dividing into many energy groups (multi-group energy) to see the pattern of neutron flux distribution. The general procedure used to select the energy structure is to divide the energy range into groups with similar properties in the region of fast and thermal energy [9]. The distribution of neutron energy is essential in the homogenization and collapsing of the nuclear fuel cell properties for use in multi-group energy neutron diffusion.

In this study, energy groups 1 to 19 represent the fast energy region, energy groups 20 to 37 represent the slowing down energy range and energy groups 38 to 70 represent the thermal energy range [11]. Some of the previous studies use the fast energy region data as well [4,7,8].

The multi-group diffusion equation is derived from the concept of neutron equilibrium, which describes the relationship between the rate of production, absorption, and leakage. The neutron transport equation as a function of time and energy group that describes the balance of the neutron population in the reactor as follows [4],

$$\begin{aligned} \frac{1}{v_g}\frac{\partial \phi_g(\vec{r},t)}{\partial t} &= \underbrace{\nabla \bullet D_g(r)\nabla \phi_g(\vec{r},t)}_{\text{leakage}} - \underbrace{\Sigma_{ag}(r)\phi_g(\vec{r},t)}_{\text{loss by absorption}} \\ &- \underbrace{\Sigma_{sg}(r)\phi_g(\vec{r},t)}_{\text{removal by scattering}} + \underbrace{\sum_{g'=1}^{G}\Sigma_{sg'g}(r)\phi_{g'}(\vec{r},t)}_{\text{Scattering into group } g} \\ &+ \underbrace{\chi_g}_{\substack{\text{fission}\\ \text{appearing}\\ \text{group } g}} \underbrace{\sum_{g'=1}^{G} v_{g'}\Sigma_{fg'}(r)\phi_{g'}(\vec{r},t)}_{\text{total fission production}} + \underbrace{S_{g'}^{\text{ext}}}_{\text{external source}} \end{aligned} \tag{1}$$

where g is an index of group energy, v_g is neutron speed, ϕ_g is neutron flux, and D_g is diffusion coefficient. In this study, it is necessary to simplify Eq. (1) by using the following assumptions:

1. Equation (1) satisfies steady-state conditions, in which the neutron flux is neither increasing nor decreasing in time [13] so that the left-hand side Eq. (1) becomes zero.
2. The one-dimensional multi-group diffusion equation depends on a single variable that is taken in the *x*-direction only.
3. The multi-group neutron diffusion coefficient depends only on group energy; dependence on position is ignored in the event of neutron leakage.
4. Neutron loss by absorption is neglected, then the second term right-hand side Eq. (1) vanished.
5. Only neutron removal by scattering process in the fuel material has been considered, and scattering into the group is ignored, then fourth term right-hand side Eq. (1) vanished.
6. Neutron fission production and external neutron source are considered the same as a source of neutrons.
7. The boundary conditions used are void boundary conditions, where the flux and neutron sources in the edge region of the reactor core are always zero.

Based on those assumptions, Eq. (1) can be written in the steady-state condition as

$$-\nabla \cdot D_g(x)\nabla \phi_g(x) + \sum_{sg} \phi_g(x) = S_g(x) \tag{2}$$

Equation (2) can be simplified into the one-dimensional multi-group diffusion equation as follows

$$-D_g(x)\nabla^2\phi_g(x) + \Sigma_{sg}\phi_g(x) = S_g(x) \tag{3}$$

Substitution of Laplacian operator for one-dimensional coordinate into Eq. (3) becomes

$$\frac{d^2\phi_g(x)}{dx^2} - \frac{\Sigma_{sg}}{D_g(x)}\phi_g(x) = -\frac{S_g(x)}{D_g(x)} \tag{4}$$

By using the central finite difference discretization model for general differential equations [14,15], Eq. (4) can be rewritten as

$$\frac{\phi_{(i+1)g} - 2\phi_{ig} + \phi_{(i-1)g}}{(\Delta x)^2} - \frac{\Sigma_{ag}}{D_{ig}}\phi_{ig} = -\frac{S_{ig}}{D_{ig}} \tag{5}$$

The Gauss-Seidel iteration method or successive relaxation [12] is applied to the Eq. (5), the neutron flux distribution becomes

$$\phi_{ig}{}^{n+1} = \frac{S_{ig}}{D_g} + \frac{\phi^n{}_{(i+1)g} + \phi^{n+1}{}_{(i-1)g}}{\Delta x^2} \tag{6}$$

where the convergence condition is

$$\left|\frac{\phi_{ig}^{n+1} - \phi_{ig}^n}{\phi_{ig}^{n+1}}\right| < \varepsilon \tag{7}$$

The nuclear fuel cell geometry is chosen in a cylindrical shape, which is divided into three regions: fuel, cladding, and coolant distributed in regions 1, 2, and 3, respectively, as shown in Figure 1.

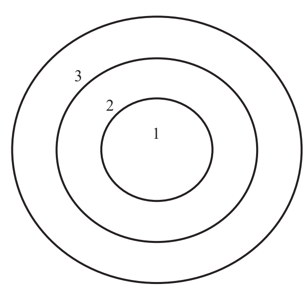

FIGURE 1. The regions of the nuclear fuel cell design.

Table 1 describes the parameters and specifications of the nuclear fuel cell design. The group constant library data used is JFS-3-J33 from JAEA (Japan Atomic Energy Agency). This library provides a constant group of SLAROM computer codes that contain 70 energy group structures for 383 nuclides and eight nuclides of integrated fission products [11].

TABLE 1. Parameters and specification of nuclear fuel cell design.

Parameters		Specification
Nuclear fuel cell material		U-PuN
Nuclear fuel cell geometry		Cylindrical
Structure material		Stainless steel
Coolant material		Pb-Bi
Fuel radius		0,35 cm
Cladding radius		0.46 cm
Coolant radius		0.57 cm
Temperature		1183 K
Fraction of Volume :	fuel	61.73%
	structure	19.40%
	coolant	18.87%

Figure 2 shows the arrangement of homogeneous U-PuN nuclear fuel cells forming a reactor slab with finite height and width *L*. The reactor core design is constructed using several spatial meshes Δx and void boundary conditions. The study focuses on the calculation of the neutron flux distribution using a one-dimensional multi-group diffusion equation in the *x*-direction only, so the height of the slab reactor is ignored. The macroscopic scattering cross-section in the reactor core only varies with energy group.

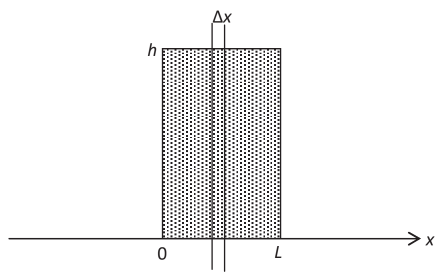

FIGURE 2. The design of the slab reactor core with height *h* and width L [7].

Based on the properties of Figure 2, the neutron flux and source at the edge point $x = 0$ and $x = L$ satisfies the boundary condition [16]

$$\phi_g(0) = \phi_g(L) = 0 \text{ and } S_g(0) = S_g(L) = 0 \tag{8}$$

In addition to specific requirements that must satisfy Eq. (1), calculation of neutron flux using multi-group diffusion also requires some assumptions:

- The macroscopic scattering cross-sections data of the nuclear fuel cell in the spatial direction is homogeneous [8], but it varies to the energy group.
- The flux calculation uses the flat flux approach so that there are no other sources in the reactor. Consequently, the reaction rate of fission and scattering acts as the source [13].

The calculation of the neutron flux distribution follows these computation procedures:

1. Determining the initial neutron flux and neutron source input for all spatial mesh and energy group.
2. Determining the neutron flux boundary and spatial variables of mesh based on Eq. (8) in the slab reactor core with length 20 cm in the thermal region.
3. Reading the multi-group macroscopic scattering cross-section of homogeneous U-PuN fuel cells that taken from the reference [8].
4. Calculating the neutron flux of U-235, U-238, and Pu-239 nuclide along the width of the slab reactor core as a variable of energy group using the Gauss-Seidel iteration method in Eq. (6).
5. Satisfying the condition of convergence iteration according to Eq.(7).

RESULTS AND DISCUSSIONS

Collision events between neutrons and the nucleus in the reactor core can cause a variety of reactions, such as scattering, fission reaction, neutron capture, and reactions process (*n*,*x*). Absorption of neutrons by nuclear fuel material results production of radioactive sources in the reactor. Figure 3 indicates that the multi-group neutron flux distribution of active material U-235 in the slab reactor core with a length of 20 cm reviewed in the thermal region. The distribution of neutron flux in the thermal region is represented by groups 46, 50, 55, 60, 65, and 70. The pattern of neutron flux distribution for each energy group is not sequential. For example, the current value of group 55 is higher than that of groups 46 and 50, although U-235 is a fissile material. The neutron flux patterns, along the width of the reactor core, have maximum values in the core center for the thermal multi-group energy of the system. This process occurs due to the absorption of a neutron by a fuel material which is very low, and the properties of neutron scattering is isotropic in all regions of the core [17]. Since the assumption used in this case is that there is no loss of neutrons by absorption or leakage, and the reactor is in a steady-state, the medium is considered homogenous during the slowing down process. It is also believed that the dependence of energy on spatial coordinates is an inseparable part. Assuming the absence of absorption requires a medium without absorbing neutrons with energy that higher than thermal energy. Because no neutrons lost in the non-absorbing medium in the steady-state conditions, neutrons that are undergoing a process of slowing down tend to be constant.

The multi-group scattering cross-section is not varying significantly in the thermal energy region [8]. The program was developed for a medium with certain limits where the distribution of neutron flux is not only a function of spatial coordinates but also an energy function. Nuclear reactors have finite dimensions where the distribution of neutron flux is a function of energy and spatial position, which is useful in studying the distribution of neutron flux by considering the neutron slowing in the thermal region [17].

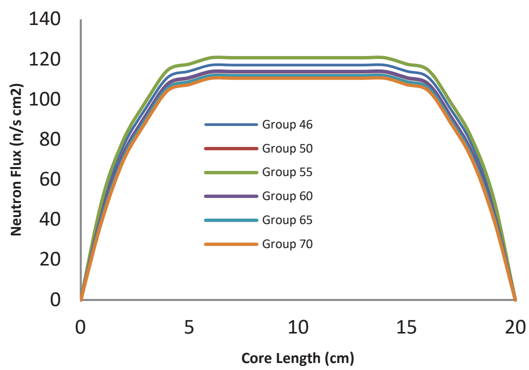

FIGURE 3. The multi-group neutron flux distribution of U-235 isotope in the thermal region

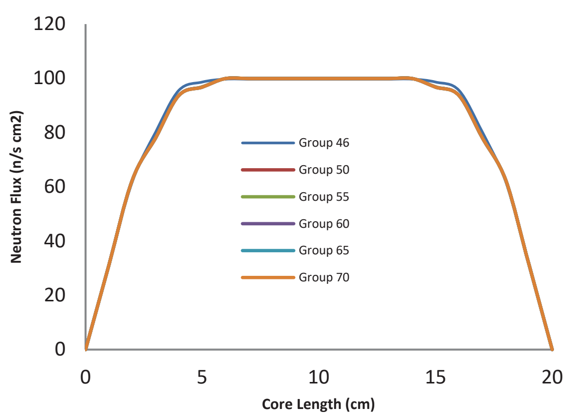

FIGURE 4. The multi-group neutron flux distribution of U-238 isotope in the thermal region

Figure 4 shows that the multi-group neutron flux distribution of the U-238 isotope almost all group energies coinciding with one another in the thermal region. The phenomenon occurs because U-238 is a fertile material that cannot be directly fissioned; it is not fissionable by thermal neutrons, but it can be converted into fissile material by enrichment process. Besides, this condition occurs because, in the thermal energy region, the macroscopic scattering cross-section of U-238 nuclide is smaller than one in the fast region.

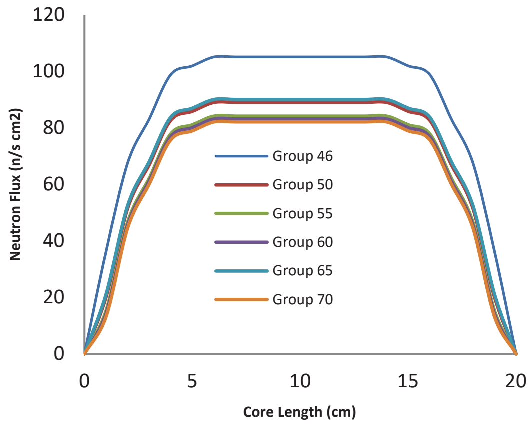

FIGURE 5. Multi-group neutron flux distribution of Pu-239 nuclide in the thermal region

Figure 5 indicates that Pu-239 has similar properties with those of U-235; both are fissile material. The pattern of neutron flux distribution for each energy group is also not sequential. The actual value of group 65 is higher than that of groups 50 and 55, although Pu-239 is a fissile material.

Neutrons that have high energy before slowing down will undergo fission process in both U-235 and U-238 nuclides in a thermal reactor. Because the proportion of U-238 is higher than that of U-235 and Pu-239 in nuclear fuel, the neutron energy that is above 1 MeV or in the fast energy region, most of the fission will be in the fertile material. Every single fission event will produce more than one neutron. Therefore there will be an increase in the number of neutrons in one fission process. This description is called the fast fission factor, which is the ratio of the total number of neutrons to the number of neutrons produced from thermal fission.

Figure 3, 4, and 5 show that the maximum value of multi-group neutron flux distribution of U-238 is very low compared to the distribution of fluxes at U-235 and Pu-239. The distribution of neutron fluxes in each energy group, i.e., fast, medium, and low energy groups, has a pattern of flux distribution that is almost the same in the fixed diameter of core 20 cm and a zero flux boundary condition on each wall. In the neutron flux distribution, it is assumed that in a place adjacent to the vacuum, there is no neutron entering. There are no reflectors on the slab reactor core design. Since there are no reflectors in the reactor core, the neutron flux at the edge boundary points of the core will be zero and satisfying the boundary conditions. Therefore, neutrons in the boundary point region will undergo leakage process, and the resulting of neutron flux does not change that concentrated throughout the core. The results will be better if extrapolation distances are used that can predict the presence of a neutron that exits or enters a vacuum interval. The neutron flux distribution profile occurs in the thermal region or low energy since the multi-group neutron scattering cross-sections have different characteristics between the fast reactor and thermal reactor. The neutron flux profile result follows the reference [13].

CONCLUSIONS

The increase of the group energy does not reflect the order of magnitude of a neutron flux distribution in the homogenous slab reactor system using a one-dimensional multi-group diffusion equation in the thermal region. However, in the spatial mesh, the distribution of neutron flux patterns remained similar to each other. Neutron flux calculation, in this case, use several assumptions, such as neutrons energy which is represented in multi-group energy, less absorption by materials in a medium, and neutron scattering which is isotropic in the system. Neutron flux in the boundary region always zeroes, because of the absence of a reflector. In the case of neutron interaction with U-235 nuclide, the pattern of neutron flux distribution for each energy group is not sequential. Similarly, with Pu-239, both are fissile material. Contrary to the U-238 isotope, almost all group energies coincide with one another in the thermal region. The phenomenon occurs because U-238 is a fertile material that cannot be directly fissioned. The neutron flux distribution profile in this study is in accordance with the reference.

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