# Theoretical Analysis of Integral Neutron Transport Equation using Collision Probability Method with QuadraticFlux Approach 

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#### Abstract

Theoretical analysis of integral neutron transport equation using collision probability (CP) method with quadratic flux approach has been carried out. In general, the solution of the neutron transport using the CP method is perfomed with the flat flux approach. In this research, the CP method is implemented in the cylindrical nuclear fuel cell with the spatial of mesh being conducted into non flat flux approach. It means that the neutron flux at any point in the nuclear fuel cell are considered different each other followed the distribution pattern of quadratic flux. The result is presented here in the form of quadratic flux that is better understanding of the real condition in the cell calculation and as a starting point to be applied in computational calculation.


## INTRODUCTION

In nuclear reactor analysis, the integral transport equation is commonly used as a starting point for describing the exact conditions inside the core [1]. One of the most complicated steps in order to estimate the neutron parameters is to solve neutron transport equation in nuclear reactor core [2]. For this reason, the development of advanced reactor core designs requires the computer program that allows more accurate solution of neutron transport equation. Collision probability ( CP ) method starts from the integral neutron transport equation which is usually used to calculate the flux fine structure in a heterogeneous medium or sub region of the reactor. This method assumed that the cell is a part of a repeating pattern of cells as a net of leakage into or out of the cell. For simple argument, it is called flat flux approach, that isneutron flux is assumed flat in all region [3]. This approach explains that neutron flux is assumed constant in each region. The CP metode is flexibel to determine the form of complex cell geometry [4] and it also can be calculated the CP, escape dan transmision matrix [5].

New concepts are introduced in the previous works [6]. Neutronflux in the integral transport equation can be added a spatial distribution function that described the real condition inside the cell. Neutron flux in each region is different each other because of an existing of the linear spatial function. This assumption is called as non flat flux (NFF) approach. The calculation resultof neutron flux distribution in each region of cylindrical using 6 meshes of NFF approach is equivalent with 24 meshes using NFF aprroach [7].

In this short note, the mathematical background to support a completed solving of neutron transport equation based on NFFapproach is performed by introducing quadratic spatial distribution function in the simple cylindrical annular cell. The result will be applicated to computational implementation in the future research.

## MATHEMATICAL MODEL

The neutron angular flux at position $r$, with energy $E$ satisfies the integral neutron transport equation for probability of neutron born or travelling in region $i$ of cell will have its next collision either in the same region $i$ or in some other region $j$ is given by [3]

$$
\begin{equation*}
\Sigma_{j}(E) \int_{V_{j}} \varphi(\vec{r}, E) d r=\sum_{j} \int_{V_{j}} d r_{j} \int_{V_{i}} d r_{i}^{\prime}\left[\int_{0}^{\infty} d E \Sigma_{s}\left(\vec{r}^{\prime}, E^{\prime} \rightarrow E\right) \varphi\left(\vec{r}^{\prime}, E^{\prime}\right)+S\left(\vec{r}^{\prime}, E\right)\right] P\left(\vec{r}^{\prime} \rightarrow \vec{r}, E\right) \tag{1}
\end{equation*}
$$

Neutron flux as an energy and spatial dependent variable distribution function is expressed by

$$
\begin{equation*}
\varphi(\bar{r}, E)=\psi(E) \phi(\bar{r}) \tag{2}
\end{equation*}
$$

By insertingthe equation (2) into (1) becomes to

$$
\begin{equation*}
\Sigma_{j}(E) \int_{V_{j}} \psi(E) \phi(\vec{r}) d r=\sum_{j} \int_{V_{j}} d r_{j} \int_{V_{i}} d r_{j} \prime^{\prime}\left[\int_{0}^{\infty} d E \Sigma_{s}\left(\vec{r}^{\prime}, E^{\prime} \rightarrow E\right) \psi\left(E^{\prime}\right) \phi\left(\vec{r}^{\prime}\right)+S\left(\vec{r}^{\prime}, E\right)\right] P\left(\vec{r}^{\prime} \rightarrow \vec{r}, E\right) \tag{3}
\end{equation*}
$$

A quadratic spatial distribution function is defined by

$$
\begin{equation*}
\phi(r)=a+b r+c r^{2} \tag{4}
\end{equation*}
$$

Variablesof quadratic function $a$, band $c$ in the equation (4) are determined by using quadratic interpolation of two meshes in each different flux of region that shows in Figure 1. For one dimensional case, the boundary condition satisfies the following equation

$$
\begin{equation*}
\phi(r)=\phi_{i-1} \text { for } r=r_{i-1} \phi(r)=\phi_{i} \text { for } r=r_{i}, \quad \text { and } \quad \phi(r)=\phi_{i+1} \text { for } r=r_{i+1} \tag{5}
\end{equation*}
$$



Figure 1. Element of neutron flux with the quadratic interpolation between two meshes.

## RESULTS AND DISCUSSIONS

Variables of quadratic function $a, b$ and $c$ are determined by using Kramers matrix rule. Substituting equation (5) into (4) are obtained the variables

$$
\begin{align*}
& a=\frac{\phi_{i-1}\left(r_{i} r_{i+1}^{2}-r_{i}^{2} r_{i+1}\right)+\phi_{i}\left(r_{i-1}^{2} r_{i+1}-r_{i-1} r_{i+1}^{2}\right)+\phi_{i+1}\left(r_{i-1} r_{i}^{2}-r_{i-1}^{2} r_{i}\right)}{r_{i} r_{i+1}^{2}+r_{i-1} r_{i}^{2}+r_{i-1}^{2} r_{i+1}-r_{i-1}^{2} r_{i}-r_{i}^{2} r_{i+1}-r_{i-1} r_{i+1}^{2}} \\
& b=\frac{\phi_{i-1}\left(r_{i}^{2}-r_{i+1}^{2}\right)+\phi_{i}\left(r_{i+1}^{2}-r_{i-1}^{2}\right)+\phi_{i+1}\left(r_{i-1}^{2}-r_{i}^{2}\right)}{r_{i} r_{i+1}^{2}+r_{i-1} r_{i}^{2}+r_{i-1}^{2} r_{i+1}-r_{i-1}^{2} r_{i}-r_{i}^{2} r_{i+1}-r_{i-1} r_{i+1}^{2}} \tag{6}
\end{align*}
$$

$$
c=\frac{\phi_{i-1}\left(r_{i+1}-r_{i}\right)+\phi_{i}\left(r_{i-1}-r_{i+1}\right)+\phi_{i+1}\left(r_{i}-r_{i-1}\right)}{r_{i} r_{i+1}^{2}+r_{i-1} r_{i}^{2}+r_{i-1}^{2} r_{i+1}-r_{i-1}^{2} r_{i}-r_{i}^{2} r_{i+1}-r_{i-1} r_{i+1}^{2}}
$$

Equation (6) is subtituted to equation (4) gives the quadratic flux in each mesh. According to quadratic flux in equation (4), the neutron transport equation that depend on energy becomes to

$$
\begin{align*}
& \Sigma_{j}(E) \varphi_{i-1}(E)\left(\frac{1}{2} r_{j+1}^{2} r_{i} r_{i+1}^{2}-\frac{1}{2} r_{j+1}^{2} r_{i}^{2} r_{i+1}-\frac{1}{2} r_{j-1}^{2} r_{i} r_{i+1}^{2}+\frac{1}{2} r_{j-1}^{2} r_{i}^{2} r_{i+1}+\frac{1}{3} r_{j+1}^{3} r_{i}^{2}-\frac{1}{3} r_{j+1}^{3} r_{i+1}^{2}-\frac{1}{3} r_{j-1}^{3} r_{i}^{2}+\frac{1}{3} r_{j-1}^{3} r_{i+1}^{2}\right. \\
& \left.+\frac{1}{4} r_{j+1}^{4} r_{i+1}-\frac{1}{4} r_{j+1}^{4} r_{i}-\frac{1}{4} r_{j-1}^{4} r_{i+1}+\frac{1}{4} r_{j-1}^{4} r_{i}\right)+\Sigma_{j}(E) \varphi_{i}(E)\left(\frac{1}{2} r_{j+1}^{2} r_{i-1}^{2} r_{i+1}-\frac{1}{2} r_{j+1}^{2} r_{i-1} r_{i+1}^{2}-\frac{1}{2} r_{j-1}^{2} r_{i-1}^{2} r_{i+1}+\frac{1}{2} r_{j-1}^{2} r_{i-1} r_{i+1}^{2}\right. \\
& \left.+\frac{1}{3} r_{j+1}^{3} r_{i+1}^{2}-\frac{1}{3} r_{j+1}^{3} r_{i-1}^{2}-\frac{1}{3} r_{j-1}^{3} r_{i+1}^{2}+\frac{1}{3} r_{j-1}^{3} r_{i-1}^{2}+\frac{1}{4} r_{j+1}^{4} r_{i-1}-\frac{1}{4} r_{j+1}^{4} r_{i+1}-\frac{1}{4} r_{j-1}^{4} r_{i-1}+\frac{1}{4} r_{j-1}^{4} r_{i+1}\right) \\
& +\sum_{j}(E) \varphi_{i}(E)\left(\frac{1}{2} r_{j+1}^{2} r_{i-1} r_{i}^{2}-\frac{1}{2} r_{j+1}^{2} r_{i-1}^{2} r_{i}-\frac{1}{2} r_{j-1}^{2} r_{i-1} r_{i}^{2}+\frac{1}{2} r_{j-1}^{2} r_{i-1}^{2} r_{i}+\frac{1}{3} r_{j+1}^{3} r_{i-1}^{2}-\frac{1}{3} r_{j+1}^{3} r_{i}^{2}\right. \\
& \left.-\frac{1}{3} r_{j-1}^{3} r_{i-1}^{2}+\frac{1}{3} r_{j-1}^{3} r_{i}^{2}+\frac{1}{4} r_{j+1}^{4} r_{i}-\frac{1}{4} r_{j+1}^{4} r_{i-1}-\frac{1}{4} r_{j-1}^{4} r_{i}+\frac{1}{4} r_{j-1}^{4} r_{i-1}\right) \\
& =\sum_{i} P_{i j} \sum_{g} \sum_{s i, g^{\prime} \rightarrow g} \varphi_{i-1}(E)\left(\frac{1}{4} r_{i} r_{i+1}^{4}-\frac{1}{6} r_{i}^{2} r_{i+1}^{3}-\frac{1}{2} r_{i-1}^{2} r_{i} r_{i+1}^{2}+\frac{1}{2} r_{i-1}^{2} r_{i}^{2} r_{i+1}-\frac{1}{12} r_{i+1}^{5}-\frac{1}{3} r_{i-1}^{3} r_{i}^{2}+\frac{1}{3} r_{i-1}^{3} r_{i+1}^{2}-\frac{1}{4} r_{i-1}^{4} r_{i+1}+\frac{1}{4} r_{i-1}^{4} r_{i}\right) \\
& +\sum_{i} P_{i j} \sum_{g} \sum_{s i, g^{\prime} \rightarrow g} \varphi_{i}(E)\left(\frac{1}{6} r_{i-1}^{2} r_{i+1}^{3}-\frac{1}{4} r_{i-1} r_{i+1}^{4}-\frac{1}{4} r_{i-1}^{4} r_{i+1}+\frac{1}{6} r_{i-1}^{3} r_{i+1}^{2}+\frac{1}{12} r_{i+1}^{5}+\frac{1}{12} r_{i-1}^{5}\right) \\
& +\sum_{i} P_{i j} \sum_{g} \sum_{s i, g^{\prime} \rightarrow g} \varphi_{i+1}(E)\left(\frac{1}{2} r_{i-1} r_{i}^{2} r_{i+1}^{2}-\frac{1}{2} r_{i-1}^{2} r_{i} r_{i+1}^{2}-\frac{1}{6} r_{i-1}^{3} r_{i}^{2}+\frac{1}{4} r_{i-1}^{4} r_{i}+\frac{1}{3} r_{i-1}^{2} r_{i+1}^{3}-\frac{1}{3} r_{i}^{2} r_{i+1}^{3}-\frac{1}{12} r_{i-1}^{5}+\frac{1}{4} r_{i} r_{i+1}^{4}-\frac{1}{4} r_{i-1} r_{i+1}^{4}\right) \\
& +\frac{1}{2} \sum_{i} P_{i j} S_{i}(E)\left(r_{i} r_{i+1}^{4}+r_{i-1} r_{i}^{2} r_{i+1}^{2}+r_{i-1}^{2} r_{i+1}^{3}-2 r_{i-1}^{2} r_{i} r_{i+1}^{2}-r_{i}^{2} r_{i+1}^{3}-r_{i-1} r_{i+1}^{4}-r_{i-1}^{3} r_{i}^{2}-r_{i-1}^{4} r_{i+1}+r_{i-1}^{4} r_{i}+r_{i-1}^{2} r_{i}^{2} r_{i+1}+r_{i-1}^{3} r_{i+1}^{2}\right) \tag{7}
\end{align*}
$$

where index $i$ or $j$ indicates the spatial region $i$ or $j$ and index $g$ denotes the energy group.If the neutron energy range is divided into multigrup energy, the average flux in each spatial region and energy intervaldenotes the flux $\phi_{i g}$. Because the energy width in the group $g$ and $g^{\prime}$ is same, therefore the neutron flux in each energy grup $g$ and $g$ ' also the same, thenequation (7) reduces to

$$
\begin{align*}
& \Sigma_{j g} \varphi_{i-1 g}\left(\frac{1}{2} r_{j+1}^{2} r_{i} r_{i+1}^{2}-\frac{1}{2} r_{j+1}^{2} r_{i}^{2} r_{i+1}-\frac{1}{2} r_{j-1}^{2} r_{i} r_{i+1}^{2}+\frac{1}{2} r_{j-1}^{2} r_{i}^{2} r_{i+1}+\frac{1}{3} r_{j+1}^{3} r_{i}^{2}-\frac{1}{3} r_{j+1}^{3} r_{i+1}^{2}-\frac{1}{3} r_{j-1}^{3} r_{i}^{2}+\frac{1}{3} r_{j-1}^{3} r_{i+1}^{2}\right. \\
& \left.+\frac{1}{4} r_{j+1}^{4} r_{i+1}-\frac{1}{4} r_{j+1}^{4} r_{i}-\frac{1}{4} r_{j-1}^{4} r_{i+1}+\frac{1}{4} r_{j-1}^{4} r_{i}\right)+\sum_{j g} \varphi_{i g}\left(\frac{1}{2} r_{j+1}^{2} r_{i-1}^{2} r_{i+1}-\frac{1}{2} r_{j+1}^{2} r_{i-1} r_{i+1}^{2}-\frac{1}{2} r_{j-1}^{2} r_{i-1}^{2} r_{i+1}+\frac{1}{2} r_{j-1}^{2} r_{i-1} r_{i+1}^{2}\right. \\
& \left.+\frac{1}{3} r_{j+1}^{3} r_{i+1}^{2}-\frac{1}{3} r_{j+1}^{3} r_{i-1}^{2}-\frac{1}{3} r_{j-1}^{3} r_{i+1}^{2}+\frac{1}{3} r_{j-1}^{3} r_{i-1}^{2}+\frac{1}{4} r_{j+1}^{4} r_{i-1}-\frac{1}{4} r_{j+1}^{4} r_{i+1}-\frac{1}{4} r_{j-1}^{4} r_{i-1}+\frac{1}{4} r_{j-1}^{4} r_{i+1}\right) \\
& +\sum_{j g} \varphi_{i g}\left(\frac{1}{2} r_{j+1}^{2} r_{i-1} r_{i}^{2}-\frac{1}{2} r_{j+1}^{2} r_{i-1}^{2} r_{i}-\frac{1}{2} r_{j-1}^{2} r_{i-1} r_{i}^{2}+\frac{1}{2} r_{j-1}^{2} r_{i-1}^{2} r_{i}+\frac{1}{3} r_{j+1}^{3} r_{i-1}^{2}-\frac{1}{3} r_{j+1}^{3} r_{i}^{2}\right. \\
& \left.-\frac{1}{3} r_{j-1}^{3} r_{i-1}^{2}+\frac{1}{3} r_{j-1}^{3} r_{i}^{2}+\frac{1}{4} r_{j+1}^{4} r_{i}-\frac{1}{4} r_{j+1}^{4} r_{i-1}-\frac{1}{4} r_{j-1}^{4} r_{i}+\frac{1}{4} r_{j-1}^{4} r_{i-1}\right) \\
& =\sum_{i} P_{i j} \sum_{g} \Sigma_{s i g} \varphi_{i-1 g}\left(\frac{1}{4} r_{i} r_{i+1}^{4}-\frac{1}{6} r_{i}^{2} r_{i+1}^{3}-\frac{1}{2} r_{i-1}^{2} r_{i} r_{i+1}^{2}+\frac{1}{2} r_{i-1}^{2} r_{i}^{2} r_{i+1}-\frac{1}{12} r_{i+1}^{5}-\frac{1}{3} r_{i-1}^{3} r_{i}^{2}+\frac{1}{3} r_{i-1}^{3} r_{i+1}^{2}-\frac{1}{4} r_{i-1}^{4} r_{i+1}+\frac{1}{4} r_{i-1}^{4} r_{i}\right) \tag{8}
\end{align*}
$$

$$
\begin{aligned}
& +\sum_{i} P_{i j} \sum_{g} \Sigma_{s i g} \varphi_{i g}\left(\frac{1}{6} r_{i-1}^{2} r_{i+1}^{3}-\frac{1}{4} r_{i-1} r_{i+1}^{4}-\frac{1}{4} r_{i-1}^{4} r_{i+1}+\frac{1}{6} r_{i-1}^{3} r_{i+1}^{2}+\frac{1}{12} r_{i+1}^{5}+\frac{1}{12} r_{i-1}^{5}\right) \\
& +\sum_{i} P_{i j} \sum_{g} \Sigma_{s i g} \varphi_{i+1 g}\left(\frac{1}{2} r_{i-1} r_{i}^{2} r_{i+1}^{2}-\frac{1}{2} r_{i-1}^{2} r_{i} r_{i+1}^{2}-\frac{1}{6} r_{i-1}^{3} r_{i}^{2}+\frac{1}{4} r_{i-1}^{4} r_{i}+\frac{1}{3} r_{i-1}^{2} r_{i+1}^{3}-\frac{1}{3} r_{i}^{2} r_{i+1}^{3}-\frac{1}{12} r_{i-1}^{5}+\frac{1}{4} r_{i}^{4} r_{i+1}^{4}-\frac{1}{4} r_{i-1} r_{i+1}^{4}\right) \\
& +\frac{1}{2} \sum_{i} P_{i j} S_{i g}\left(r_{i} r_{i+1}^{4}+r_{i-1} r_{i}^{2} r_{i+1}^{2}+r_{i-1}^{2} r_{i+1}^{3}-2 r_{i-1}^{2} r_{i+1}^{2} r_{i+1}^{2}-r_{i}^{2} r_{i+1}^{3}-r_{i-1} r_{i+1}^{4}-r_{i-1}^{3} r_{i}^{2}-r_{i-1}^{4} r_{i+1}+r_{i-1}^{4} r_{i}+r_{i-1}^{2} r_{i}^{2} r_{i+1}+r_{i-1}^{3} r_{i+1}^{2}\right)
\end{aligned}
$$

By using mathematical simplification, equation (8) can be expressed as a simpler equation as follows

$$
\begin{equation*}
\alpha_{i-1 g} \varphi_{i-1 g}+\beta_{i g} \varphi_{i g}+\gamma_{i+1 g} \varphi_{i+1 g}=Q_{i g} \tag{9}
\end{equation*}
$$

where

$$
\begin{aligned}
& \alpha_{i-1, g}=\left\{\sum _ { j g } \left(\frac{1}{2} r_{j+1}^{2} r_{i} r_{i+1}^{2}-\frac{1}{2} r_{j+1}^{2} r_{i}^{2} r_{i+1}-\frac{1}{2} r_{j-1}^{2} r_{i} r_{i+1}^{2}+\frac{1}{2} r_{j-1}^{2} r_{i}^{2} r_{i+1}+\frac{1}{3} r_{j+1}^{3} r_{i}^{2}-\frac{1}{3} r_{j+1}^{3} r_{i+1}^{2}-\frac{1}{3} r_{j-1}^{3} r_{i}^{2}+\frac{1}{3} r_{j-1}^{3} r_{i+1}^{2}+\frac{1}{4} r_{j+1}^{4} r_{i+1}\right.\right. \\
& \left.-\frac{1}{4} r_{j+1}^{4} r_{i}-\frac{1}{4} r_{j-1}^{4} r_{i+1}+\frac{1}{4} r_{j-1}^{4} r_{i}\right)+\sum_{i, g} P_{i j g} \sum_{s i g}\left(\frac{1}{4} r_{i} r_{i+1}^{4}-\frac{1}{6} r_{i}^{2} r_{i+1}^{3}-\frac{1}{2} r_{i-1}^{2} r_{i} r_{i+1}^{2}+\frac{1}{2} r_{i-1}^{2} r_{i}^{2} r_{i+1}-\frac{1}{12} r_{i+1}^{5}-\frac{1}{3} r_{i-1}^{3} r_{i}^{2}\right. \\
& \left.\left.+\frac{1}{3} r_{i-1}^{3} r_{i+1}^{2}-\frac{1}{4} r_{i-1}^{4} r_{i+1}+\frac{1}{4} r_{i-1}^{4} r_{i}\right)\right\} \\
& \beta_{i, g}=\left\{\sum _ { j g } \left(\frac{1}{2} r_{j+1}^{2} r_{i-1}^{2} r_{i+1}-\frac{1}{2} r_{j+1}^{2} r_{i-1} r_{i+1}^{2}-\frac{1}{2} r_{j-1}^{2} r_{i-1}^{2} r_{i+1}+\frac{1}{2} r_{j-1}^{2} r_{i-1} r_{i+1}^{2}+\frac{1}{3} r_{j+1}^{3} r_{i+1}^{2}-\frac{1}{3} r_{j+1}^{3} r_{i-1}^{2}-\frac{1}{3} r_{j-1}^{3} r_{i+1}^{2}+\frac{1}{3} r_{j-1}^{3} r_{i-1}^{2}\right.\right. \\
& \left.\left.+\frac{1}{4} r_{j+1}^{4} r_{i-1}-\frac{1}{4} r_{j+1}^{4} r_{i+1}-\frac{1}{4} r_{j-1}^{4} r_{i-1}+\frac{1}{4} r_{j-1}^{4} r_{i+1}\right)+\sum_{i, g} P_{i j g} \sum_{s i g}\left(\frac{1}{6} r_{i-1}^{2} r_{i+1}^{3}-\frac{1}{4} r_{i-1} r_{i+1}^{4}-\frac{1}{4} r_{i-1}^{4} r_{i+1}+\frac{1}{6} r_{i-1}^{3} r_{i+1}^{2}+\frac{1}{12} r_{i+1}^{5}+\frac{1}{12} r_{i-1}^{5}\right)\right\} \\
& \gamma_{i+1, g}=\left\{\sum _ { j g } \left(\frac{1}{2} r_{j+1}^{2} r_{i-1} r_{i}^{2}-\frac{1}{2} r_{j+1}^{2} r_{i-1}^{2} r_{i}-\frac{1}{2} r_{j-1}^{2} r_{i-1} r_{i}^{2}+\frac{1}{2} r_{j-1}^{2} r_{i-1}^{2} r_{i}+\frac{1}{3} r_{j+1}^{3} r_{i-1}^{2}-\frac{1}{3} r_{j+1}^{3} r_{i}^{2}-\frac{1}{3} r_{j-1}^{3} r_{i-1}^{2}+\frac{1}{3} r_{j-1}^{3} r_{i}^{2}+\frac{1}{4} r_{j+1}^{4} r_{i}\right.\right. \\
& \left.-\frac{1}{4} r_{j+1}^{4} r_{i-1}-\frac{1}{4} r_{j-1}^{4} r_{i}+\frac{1}{4} r_{j-1}^{4} r_{i-1}\right) \\
& \left.+\sum_{i, g} P_{i j g} \sum_{s i g}\left(\frac{1}{2} r_{i-1} r_{i}^{2} r_{i+1}^{2}-\frac{1}{2} r_{i-1}^{2} r_{i} r_{i+1}^{2}-\frac{1}{6} r_{i-1}^{3} r_{i}^{2}+\frac{1}{4} r_{i-1}^{4} r_{i}+\frac{1}{3} r_{i-1}^{2} r_{i+1}^{3}-\frac{1}{3} r_{i}^{2} r_{i+1}^{3}-\frac{1}{12} r_{i-1}^{5}+\frac{1}{4} r_{i} r_{i+1}^{4}-\frac{1}{4} r_{i-1} r_{i+1}^{4}\right)\right\} \\
& Q_{i g}=\frac{1}{2} \sum_{i, g} P_{i j g} S_{i g}\left(r_{i} r_{i+1}^{4}+r_{i-1} r_{i}^{2} r_{i+1}^{2}+r_{i-1}^{2} r_{i+1}^{3}-2 r_{i-1}^{2} r_{i} r_{i+1}^{2}-r_{i}^{2} r_{i+1}^{3}-r_{i-1} r_{i+1}^{4}-r_{i-1}^{3} r_{i}^{2}-r_{i-1}^{4} r_{i+1}+r_{i-1}^{4} r_{i}+r_{i-1}^{2} r_{i}^{2} r_{i+1}+r_{i-1}^{3} r_{i+1}^{2}\right) .
\end{aligned}
$$

By using the Gauss elimination method, neutron flux in each region $i$ with energy group $g$ in equation (9) may be written as

$$
\begin{equation*}
\phi_{i g}=\frac{1}{\beta_{i, g}}\left(Q_{i g}-\alpha_{i-1, g} \phi_{i-1 g}-\gamma_{i+1, g} \phi_{i+1 g}\right) \tag{10}
\end{equation*}
$$

Equation (10) gives a balance equation to calculate the multiplication factor as a eigen value [8]

$$
\begin{equation*}
\Sigma_{j} V_{j} \phi_{i g}=\frac{1}{k_{e f f}} \sum_{i} V_{i} P_{i j} S_{i} \tag{11}
\end{equation*}
$$

Equation (11) is a starting point to compute collision probabilities, especially to treat the cylindrical cell calculation. It is a basic formulation to obtain the solving of integral transport equation using NFF approximation. The calculation of the probability $P_{i j}$ is straight forward involving exponential integral, Bickley-Naylor function
and other function. It is more complicated to calculate $P_{i j}$ for sophisticated system such as an array of pin cell or more complex geometry. For computational works, the integral parameters such as a collision probability matrix, the multiplication factor and the neutron flux are taken into account in nuclear data that used in the nuclear reactor core calculation. Consequently, the calculation of neutron flux distribution will be more accurate according to the selected function in the non flat flux approach.

## CONCLUSIONS

Theoretical analysis to solve the integral neutron transport equation based on non flat flux approach by inserting quadratic flux spatial distribution function requires a lot of variables in accordance with the selected function. The neutron flux at any point in the cylindrical nuclear fuel cell are considered different each other followed the distribution pattern of quadratic flux. The result is performed in the form of quadratic flux that is better understanding of the real condition in the cell calculation. The research is as a starting point to be implemented in computational calculation in the future study.

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