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Kristiana Wijaya, Edy Tri Baskoro, Asep Iqbal Taufik, Denny Riama Silaban

Let G, H, and F be simple graphs. The notation $F \longrightarrow (G, H)$ means that any red-blue coloring of all edges of F contains a red copy of G or a blue copy of H. The graph F satisfying this property is called a Ramsey (G, H)-graph. A Ramsey (G, H)-graph is called minimal if for each edge $e \in E(F)$, there exists...

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Maya Nabila, Edy Tri Baskoro, Hilda Assiyatun

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On Ramsey (mK_2, P_4)-Minimal Graphs

Asep Iqbal Taufik, Denny Riama Silaban, Kristiana Wijaya

Let F, G, and H be simple graphs. The notation F → (G, H) means that any red-blue coloring of all edges of F will contain either a red copy of G or a blue copy of H. Graph F is a Ramsey (G, H)-minimal if F → (G, H) but for each e ∈ E(F), (F − e) ↛ (G, H). The set $\mathscr{R}$(G, H) consists of all Ramsey (G,...

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Spectrum of Unicyclic Graph

Budi Rahadjeng, Dwi Nur Yunianti, Raden Sulaiman, Agung Lukito

Let G be a simple graph with n vertices and let A(G) be the (0, 1)-adjacency matrix of G. The characteristic polynomial of the graph G with respect to the adjacency matrix A(G), denoted by χ(G, λ) is a determinant of (λI − A(G)), where I is the identity matrix. Suppose that λ1 ≥ λ2 ≥ ⋯ ≥ λn are the adjacency...

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Distinguishing Number of the Generalized Theta Graph

Andi Pujo Rahadi, Edy Tri Baskoro, Suhadi Wido Saputro

A generalized theta graph is a graph constructed from two distinct vertices by joining them with l (>=3) internally disjoint paths of lengths greater than one. The distinguishing number D(G) of a graph G is the least integer d such that G has a vertex labelling with d labels that is preserved only...

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Inne Singgih

An edge magic total (EMT) labeling of a graph G = (V, E) is a bijection from the set of vertices and edges to a set of numbers defined by $\lambda: V \cup E \rightarrow \{1,2, ..., |V| + |E|\}$ with the property that for every $xy \in E$, the weight of xy equals to a constant k, that is, $\lambda(x) + \lambda(y) + \lambda(xy) = k$ for some integer...

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Further Result of *H*-Supermagic Labeling for Comb Product of Graphs

Ganesha Lapenangga P., Aryanto, Meksianis Z. Ndii

Let G = (V, E) and H = (V', E') be a connected graph. H-magic labeling of graph G is a bijective function $f: V(G) \cup E(G) \rightarrow \{1,2, ..., |V(G)| + |E(G)|\}$ such that for every subgraph H'of G isomorphic to H, $\sum v \in V(H')\ f(v) + \sum e \in E(H')\ f(e) = k$. Moreover, it is H-supermagic labeling if $f(V) = \{1,2 ,..., |V|\}$....

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Ikhsanul Halikin, Hafif Komarullah

A graph labelling with a condition at distance two was first introduced by Griggs and Robert. This labelling is also known as L(2,1)-labelling. Let G = (V, E) be a non-multiple graph, undirected, and connected. An L(2,1)-labelling on a graph is defined as a mapping from the vertex set V(G) to the set...

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On the Minimum Span of Cone, Tadpole, and Barbell Graphs

Hafif Komarullah, Ikhsanul Halikin, Kiswara Agung Santoso

Let G be a simple and connected graph with p vertices and q edges. An L(2,1)-labelling on the graph G is a function f: V(G) → {0,1, ..., k} such that every two vertices with a distance one receive labels that differ by at least two, and every two vertices at distance two receive labels that differ by at...

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L(2,1) Labeling of Lollipop and Pendulum Graphs

Kusbudiono, Irham Af'idatul Umam, Ikhsanul Halikin, Mohamat Fatekurohman

One of the topics in graph labeling is L(2,1) labeling which is an extension [redacted] graph labeling. Definition of L(2,1) labeling is a function that maps the set [redacted] vertices in the graph to non-negative integers such that every two vertices u, v that have a distance one must have a label with a difference...

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Magic and Antimagic Decomposition of Amalgamation of Cycles

Sigit Pancahayani, Annisa Rahmita Soemarsono, Dieky Adzkiya, Musyarofah

Consider G = (V, E) as a finite, simple, connected graph with vertex set V and edge set E. G is said to be a decomposable graph if there exists a collection of subgraphs of G, say $\mathscr{H} = \{Hi | 1 \leq i \leq n\}$ such that for every $i \neq j$, Hi is isomorphic to Hj, ∪ i=1n Hi = G and should satisfy that E(Hi) ∩ E(Hj)...

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A Minimum Coprime Number *for Amalgamation of Wheel*

Hafif Komarullah, Slamin, Kristiana Wijaya

Let G be a simple graph of order n. A coprime labeling of a graph G is a vertex labeling of G with distinct positive integers from the set {1, 2, ..., k} for some $k \geq n$ such that any adjacent labels are relatively prime. The minimum value of k for which G has a coprime labelling, denoted as $\mathfrak{pr}(G)$, is...

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Rainbow Connection Number of Shackle Graphs

M. Ali Hasan, Risma Yulina Wulandari, A.N.M. Salman

Let G be a simple, finite and connected graph. For a natural number k, we define an edge coloring c: E(G) → {1,2,..., k} where two adjacent edges can be colored the same. A u – v path (a path connecting two vertices u and v in V(G)) is called a rainbow path if no two edges of path receive the same color....

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Local Antimagic Vertex Coloring of Corona Product Graphs $P_n \circ P_k$

Setiawan, Kiki Ariyanti Sugeng

Let G = (V, E) be a graph with vertex set V and edge set E. A bijection map f : E → {1,2, ..., |E|} is called a local antimagic labeling if, for any two adjacent vertices u and v, they have different vertex sums, i.e. w(u) ≠ w(v), where the vertex sum w(u) = Σe ∈ E(u) f(e), and E(u) is the set of edges...

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Local Antimagic Vertex Coloring of Gear Graph

Masdaria Natalina Br Silitonga, Kiki Ariyanti Sugeng

Let G = (V, E) be a graph that consist of a vertex set V and an edge set E. The local antimagic labeling f of a graph G with edge-set E is a bijection map from E to {1, 2, ..., |E|} such that w(u) ≠ w(v), where w(u) = ∑e ∈ E(u) f(e) and E(u) is the set of edges incident to u. In this labeling, every vertex...

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Implementations of Dijkstra Algorithm for Searching the Shortest Route of Ojek Online and a Fuzzy Inference System for Setting the Fare Based on Distance and Difficulty of Terrain (Case Study: in Semarang City, Indonesia)

Vani Natali Christie Sebayang, Isnaini Rosyida

Ojek Online is a motorcycle taxi that is usually used by people that need a short time for traveling. It is one of the easiest forms of transportation, but there are some obstacles in hilly areas such as Semarang City. The fare produced by online motorcycle taxis is sometimes not in accordance with the...

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Mohamad Hasan

During the deposition process, many factors play a role in the dynamics of the system including materials' characteristics and media onto which the materials dropped. The stick-slip model has been applied to simulate the depositions of polydisperse granular materials. As the size of the materials varied,...

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Natanael Karjanto

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Agustina Pradjaningsih, Fatmawati, Herry Suprajitno

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Generalization of Chaos Game on Polygon

Kosala D. Purnomo

The original chaos game has been applied to the triangular attractor points. With the rules for selecting attractor points randomly, the points generated in large iterations will form like a Sierpinski triangle. Several studies have developed it on the attractor points of quadrilaterals, pentagons, and...

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High Order Three-Steps Newton Raphson-like Schemes for Solving Nonlinear Equation Systems

Rizki Multazamil Fatahillah, M Ziaul Arif, Rusli Hidayat, Kusbudiono, Ikhsanul Halikin

This study proposes several new 3-steps schemes based on the Newton-Raphson method for solving non-linear equation systems. The proposed schemes are analysed and formulated based on the Newton-Raphson method and the Newton-cotes open form numerical integration method. In general, the schemes can be considered...

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Symmetry Functions with Respect to Any Point in R^n and Their Properties

Firdaus Ubaidillah

A function f : R → R is said to be an odd function if f(−x) = −f(x) for every x in R. The graph of an odd function is symmetric with respect to the origin, that is the point (0,0). The aims of this paper are to generalize odd functions on Rn and introduce symmetry functions with respect to any point...

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Root Water Uptake Process for Different Types of Soil in Unsteady Infiltration from Periodic Trapezoidal Channels

Millatuz Zahroh, Imam Solekhudin

This study involves a non-linear partial differential equation known as Richard's Equation. An unsteady infiltration from trapezoidal periodic irrigation channel with root-water uptake is considered as the problem. To solve the problem, A set of transformations, Kirchhoff transformation, dimensionless...

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Contact Tracking with Social Network Analysis Graph

Alvida Mustika Rukmi, Wildan Zakky, M. Lutfhi Shahab

In 2020, the world is facing a Covid-19 virus pandemic. The fields of epidemiology and networks are needed in dealing with its spread. Individual (contact) tracing is an important control measure in the spread of infectious diseases. The network of contacts describes the potential pathways for the spread...

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Learning Materials Development of Parametric Curves and Surfaces for Modeling the Objects Using Maple on Differential Geometry Courses

Kusno, Abduh Riski

Modeling industrial objects needs the formulas of curves and surfaces to construct a precise shape of real goods and simulate some process of form

creations. For this reason, the study of the equations of curves and surfac[redacted]
for objects modeling is essential for resulting in a required shape and fea[redacted]

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Hanging Rotera Modeling by Joining Deformation Result of Space Geometry Objects

Een Ubaningrum, Bagus Juliyanto, Ahmad Kamsyakawuni, Firdaus Ubaidillah

The hanging rotera is a small lamp covered by a glass lid with a light source from a burning candle or LED (Light Emitting Diode) candle and hung on a support pole that is hooked to the rotera connector. The purpose of this paper are to obtain a models of the various and symmetrical components of the...

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Learning Content Development in Modeling Creative Industry Objects Using Real Function Formulas Supported with Maple

Kusno, Bagus Juliyanto, Kiswara Agung Santoso

Creative industries are a national strategic commodity to support international marketing. For this reason, modeling creative industry objects are essential for resulting in various shapes and features of the goods. This paper presents to develop learning content in modeling creative goods supported...

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SHINY OFFICE-R: A Web-based Data Mining Tool for Exploring and Visualizing Company Profiles

I Made Tirta, Mohamad Fatekurahman, Khairul Anam, Bayu Taruna Widjaja Putra

The profile of institutions or companies are often measured internally, nationally and internationally using several indicators that may be changed over time. We develop SHINY OFFICE-R a Web-GUI (Graphical User Interface) using R software to explore and visualize data on institution performance/ profile....

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Bayesian Accelerated Failure Time Model and Its Application to Preeclampsia

Dennis Alexander, Sarini Abdullah

Preeclampsia (PE) often described as new-onset hypertension and proteinuria during the third trimester of pregnancy. PE, is one of the most feared complications of pregnancy because it can progress rapidly to serious complications, including death of both mother and fetus. It is important to get a better...

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Dinagusti Sianturi, Alvida Rukmi

The COVID-19 pandemic has impact in every sector of life. Studies of the impact of the COVID-19 pandemic on stock trading are also being develop[redacted] Indonesia regarding to the number of industries affected by the pandemic. This research aims to provide information about the results of the correlation...

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Classification of Bank Deposit Using Naïve Bayes Classifier (NBC) and *K*–Nearest Neighbor (*K*-NN)

Muhammad Hafidh Effendy, Dian Anggraeni, Yuliani Setia Dewi, Alfian Futuhul Hadi

Banks are financial institutions whose activities are to collect funds from the public in the form of deposits (saving deposit, demand deposit, and time deposit) and distribute them to the public in the form of credit or other forms. Deposits are an alternative for customers because the interest offered...

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Information Retrieval Using Matrix Methods

Case Study: Three Popular Online News Sites in Indonesia

Ferry Wiranto, I Made Tirta

This research is part of data mining, a sub-section of information retrieval and text mining. This research focuses on finding an approach to getting relevant documents online news documents with a specific threshold value and improving computing performance to get relevant documents with large documents....

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Statistical Downscaling Technique Using Response Based Unit Segmentation-Partial Least Square (REBUS-PLS) for Monthly Rainfall Forecasting

Izdihar Salsabila, Alfian Futuhul Hadi, I Made Tirta, Yuliani Setia Dewi, Firdaus Ubaidillah, Dian Anggraeni

One of the newest forecasting techniques today is the Statistical Downscaling (SDs) technique. The SDs technique is a procedure for inferring high-resolution information from low-resolution variables. Forecasting rainfall using the SDs technique is to build a function that can predict the value of a...

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Hurdle Regression Modelling on the Number of Deaths from Chronic Filariasis Cases in Indonesia

Nur Kamilah Sa'diyah, Ani Budi Astuti, Maria Bernadetha T. Mitakda

Poisson regression is one of the model to explain the functional relationship between response variable in the form of count and predictor variable. An important assumption in Poisson Regression analysis is equidispersion. In certain cases, where response variable consists of too many zeros, causing...

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Multiple Discriminant Analysis Altman Z-Score, Multiple Discriminant Analysis Stepwise and K-Means Cluster for Classification of Financial Distress Status in Manufacturing Companies Listed on the Indonesia Stock Exchange in 2019

Hazrina Ishmah, Solimun, Maria Bernadetha Theresia Mitakda

This study uses the MDA (Multiple Discriminant Analysis) Altman Z-Score to predict the status of financial distress in manufacturing companies listed on the Indonesia Stock Exchange in 2019. MDA Stepwise model is used to prove that the variables used in the MDA Altman Z-Score method are the best variables...

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Naive Bayes Classifier (NBC) for Forecasting Rainfall in Banyuwangi District Using Projection Pursuit Regression (PPR) Method

Ana Ulul Azmi, Alfian Futuhul Hadi, Yuliani Setia Dewi, I Made Tirta, Firdaus Ubaidillah, Dian Anggraeni

Rainfall is one of the climates that has a big influence on life, such as aviation, plantations, and agriculture. Remote areas like Banyuwangi Regency are most likely to lack information on weather and climate data. Rainfall information in the future is also very decisive for the community in carrying...

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Analysis of Factors Affecting Poverty Depth Index in Papua Province Using Panel Data Regression

Rufina Indriani, Erma Oktania Permatasari

One of the main problems in Papua Province is poverty, because the Pove[redacted] Depth Index (P1) in Papua Province is greater than other province, which [redacted] 7.17 in 2019. This value is bigger than the Poverty Depth Index in Indonesia which was only 1.55. This study will analyse the factors that affect the...

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Projection Pursuit Regression on Statistical Downscaling Using Artificial Neural Network and Support Vector Regression for Rainfall Forecasting in Jember

Chandrika Desyana Putri, Ema Fahma Farikha, Alfian Futuhul Hadi, Yuliani Setia Dewi, I Made Tirta, Firdaus Ubaidillah, Dian Anggraeni

Information about rainfall is very necessary for the country of Indonesia which bears the title of an agricultural country. This is because the agricultural sector is very vulnerable to climate change, where rainfall is one indicator of climate change-related to crops. Therefore, an accurate rainfall...

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Weather Forecasting at BMKG Office Lumajang City Using Markov Chain Method

Ummi Masrurotul Jannah, Mohamat Fatekurohman

Weather forecasting is one of the important factors in everyday life, because it can affect the activities carried out by the community. Weather forecasting refers to a series of activities carried out to produce a set of information about weather conditions. One method that can be used to model these...

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DOPE: MDC-2 Scheme Using PRESENT

Anjeli Lutfiani, Bety Hayat Susanti

Modification Detection Code (MDC) as an unkeyed hash function is designed to provide data integrity. Manipulation Detection Codes (MDC-2) is one of double-length (2n-bit) hash-values that requires 2 block cipher operations per block of hash input where the output size of the hash function is twice the...

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Primary Key Encryption Using Hill Cipher Chain (Case Study: STIE Mandala PMB Site)

Muhamat Abdul Rohim, Kiswara Agung Santoso, Alfian Futuhul Hadi

The condition of the world experiencing the COVID-19 pandemic has resulted in some daily activities limited by health protocols. The Indonesian government's policy in the academic field has forced STIE Mandala Jember, as one of the private universities, to implement online-based new student admissions....

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A Modification of ECDSA to Avoid the Rho Method Attack

Amira Zahra, Kiki Ariyanti Sugeng

Elliptic Curve Digital Signature Algorithm (ECDSA) is a digital signature [redacted] algorithm that utilizes an elliptic curve. ECDSA consists of three steps, w[redacted] are key generation, signature generation, and verification algorithm. ECDSA is used on Bitcoin transactions to generate the user's public key, private...

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Image Authentication Based on Magic Square

Kiswara Agung Santoso, Maulidyah Lailatun Najah, Moh. Hasan

Image is a digital media that is easy to change, so it is susceptible to being used for crime. Image changes may be affected by the unstable internet during transmission or deliberate manipulation of images for specific purposes. Hence, we need a tool to determine the authenticity of the image. One strategy...

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Pattern Recognition of Batik Madura Using Backpropagation Algorithm

Abduh Riski, Ega Bandawa Winata, Ahmad Kamsyakawuni

Since October 2, 2009, UNESCO has acknowledged batik as one of Indonesia's intellectual properties. Throughout the archipelago, distinct and diverse batik motifs have emerged and been produced with the passage of time; Madura batik is one of them. The Backpropagation Algorithm is used to recognize Madura...

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Hybrid Cat-Particle Swarm Optimization Algorithm on Bounded Knapsack Problem with Multiple Constraints

Kiswara Agung Santoso, Muhammad Bagus Kurniawan, Ahmad Kamsyakawuni, Abduh Riski

Optimization problems have become interesting problems to discuss, including the knapsack problem. There are many types and variations of knapsack problems. In this paper, the authors introduce a new hybrid metaheuristic algorithm to solve the modified bounded knapsack problem with multiple constraints...

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High Order Three-Steps Newton Raphson-like Schemes for Solving Nonlinear Equation Systems

Rizki Multazamil Fatahillah, M Ziaul Arif*, Rusli Hidayat, Kusbudiono, Ikhsanul Halikin

Department of Mathematics, FMIPA, University of Jember
**Corresponding author. Email: ziaul.fmipa@unej.ac.id*

ABSTRACT
This study proposes several new 3-steps schemes based on the Newton-Raphson method for solving non-linear equation systems. The proposed schemes are analysed and formulated based on the Newton-Raphson method and the Newton-cotes open form numerical integration method. In general, the schemes can be considered as a predictor and corrector principles. In the first and the third steps, the Newton-Raphson method is applied. Furthermore, Newton-cotes Open Form numerical integration modification is operated in the second step of the proposed schemes. The convergence analysis of the proposed schemes is given. It shows that the proposed scheme provides the 8th order of convergence. The performance of the proposed schemes is compared and assessed with several numerical examples.

Keywords: *Nonlinear equation systems, Newton-Raphson method, Newton-cotes open form.*

1. INTRODUCTION

Finding the exact or nearly exact solution of the non-linear equation system is the most common problem in mathematics. Since the exact solution, called the analytical solution is problematic in some mathematical sense, the numerical solution provides the solution that produces a nearly exact solution of the problems. The algorithm is considered to be effective and efficient if it has a high order of convergence. It means the algorithm can produce the solution faster. Several modifications of algorithms have been introduced and investigated [1-7]. However, discovering the other high order algorithm is highly possible.

Some modifications and new schemes for solving have been investigated for decades. Farida in [1] proposed some 3-steps Newton method schemes by utilizing the numerical integration to modify the steps of the schemes. The schemes have an order of convergence 6. Furthermore, Frontini and Sormani [2], denoted FSM, found three-steps 5th and 6th-order of convergence scheme based on predictor-corrector principle. Moreover, Darvishi *et al.* [3-4] investigated and proposed two new methods: Newton-like 3rd-order of convergence method and super cubic iterative approach to solve non-linear equation system. Khirallah and Hafiz developed 3rd-order Newton-family and Jarrat methods to solve non-linear equation systems in 2012 [7], called KHM methods.

There are still many possibilities to develop and modify non-linear equation system solving methods to get a higher-order convergence. The modification method can be conducted by using either one-step or multi-step methods. In this paper, the first and third steps are the Newton-Raphson method. Furthermore, the modification of the 3-steps method has been done by employing numerical integration Newton-cotes Open Form six and seven points method as the second step of the proposed method.

2. METHODS

2.1 System of Nonlinear Equations (SNLE)

The general form of non-linear equation system is

$$\mathbf{F}(x_1, x_2, \dots, x_m) = (f_1(x_1, x_2, \dots, x_m), f_2(x_1, x_2, \dots, x_m), \dots, f_m(x_1, x_2, \dots, x_m)).$$

Where f_i is the non-linear function which is mapping $\mathbb{R}^m$ into $\mathbb{R}^m$. The system of the function F is mapping $\mathbb{R}^m$ into $\mathbb{R}^m$. Furthermore, the solution of the system is x^*, *if* $F(x^*) = 0$.

The solution of the non-linear equation systems can be obtained by using iterative methods. The most common iterative method is the Newton-Raphson method. However, the modifications of the Newton-Raphson method have been investigated in some references.

2.2 Numerical Integration

In this paper, the numerical integration method is used for modifying the non-linear equation system schemes. High order numerical integration Newton-Cotes open forms is employed as the second step of the proposed method. The numerical integrations Newton-Cotes open forms 4, 5, 6 and 7-points are modified in this paper.

3. RESULTS

3.1. Modification of the Schemes

Let X be the solution of the differentiated function and considered as the numerical solution of the equation system $F(x) = 0$, for $F: D \subseteq \mathbb{R}^n \to \mathbb{R}^n$ is the mapping that continues in the set D convex, $F(x)$ has unique roots in D, $(x) = (f_1(x), f_2(x), \dots, f_n(x))T$

$x = (x_1, x_2, \dots, x_n)^T$ and $f_i : \mathbb{R}^n \to \mathbb{R}$ is a nonlinear function, then:

$$F(x) = F(x_i) + \int_{x_i}^{x} F'(t)\, dt. \quad (1)$$

If the integral of the Equation (1) is approximated with numerical integration method, Newton-Cotes open form 6-points, it can be written as

$$\int_{x_0}^{x_7} F'(x)dx = \frac{7h}{1440}\left[611F'\left(\frac{6x_0 + x_7}{7}\right) - 453F'\left(\frac{5x_0 + 2x_7}{7}\right) + 562F'\left(\frac{4x_0 + 3x_7}{7}\right) + 562F'\left(\frac{3x_0 + 4x_7}{7}\right) - 453F'\left(\frac{2x_0 + 5x_7}{7}\right) + 611F'\left(\frac{x_0 + 6x_7}{7}\right)\right]. \quad (2)$$

Supposed that $x_0 = x_i$ and $x_7 = x$, $h = \frac{x - x_i}{7}$, and supposed that $F(x)$ is a system of nonlinear equations. It can be assumed that a vector (x^*) is a solution of $F(x)$, such that $F(x^*) = 0$. By substituting Equation (2) to Equation (1), the implicit scheme of solving the non-linear equation systems can be formed,

$$x = x_i - 1440\left[611F'\left(\frac{6x_i + x}{7}\right) - 453F'\left(\frac{5x_i + 2x}{7}\right) + 562F'\left(\frac{4x_i + 3x}{7}\right) + 562F'\left(\frac{3x_i + 4x}{7}\right) - 453F'\left(\frac{2x_i + 5x}{7}\right) + 611F'\left(\frac{x_i + 6x}{7}\right)\right]^{-1} F(x_i) \quad (3)$$

The order of convergence of Equation (3) can be increased by modifying the interpolation points using the formula $\frac{(N-M)x_i + Mx_i}{N}$ where N and M are integers, see [1] for the details of the interpolation points. Once the linear equation of the interpolation points has been determined, the non-unique solution of the linear equation can be found. Consequently, the new interpolation points of Equation (3) are such equations below,

$w_1 = \frac{3x_i + 4y_i}{7}$, $w_2 = \frac{-2x_i + 9y_i}{7}$, $w_3 = \frac{-x_i + 8y_i}{7}$, $w_4 = \frac{x_i + 6y_i}{7}$.,
$w_5 = \frac{2x_i + 5y_i}{7}$, dan $w_6 = \frac{-3x_i + 10y_i}{7}$

Finally, Equation (3) can be written using the new interpolation points as follows,

$$x = x_i - 1440\left[611F'\left(\frac{3x_i + 4x}{7}\right) - 453F'\left(\frac{-2x_i + 9x}{7}\right) + 562F'\left(\frac{-x_i + 8x}{7}\right) + 562F'\left(\frac{x_i + 6x}{7}\right) - 453F'\left(\frac{2x_i + 5x}{7}\right) + 611F'\left(\frac{-3x_i + 10x}{7}\right)\right]^{-1} F(x_i) \quad (4)$$

Equation (4) is an implicit equation. However, since the converged solution is expected in this case, we can assume the Equation (4) is an explicit equation if the value of x on the right-hand side is estimated with the prediction step from the Newton-Raphson scheme. Furthermore, by adding one step in the following step 2 (Equation (4), the new 3-steps Newton-Raphson-like scheme can be considered as the predictor-corrector technique as follows

$$\begin{aligned} y_n &= x_n - F'(x_n)^{-1}F(x_n), \\ Z_n &= y_n - 1440\left[611F'\left(\frac{3x_n + 4y_n}{7}\right) - 453F'\left(\frac{-2x_n + 9y_n}{7}\right) + 562F'\left(\frac{-x_n + 8y_n}{7}\right) + 562F'\left(\frac{x_n + 6y_n}{7}\right) - 453F'\left(\frac{2x_n + 5y_n}{7}\right) + 611F'\left(\frac{-3x_n + 10y_n}{7}\right)\right]^{-1} F(y_n), \quad (5) \\ x_{n+1} &= Z_n - F'(Z_n)^{-1}F(Z_n). \end{aligned}$$

Moreover, let the integral of the Equation (1) be approximated with numerical integration Newton-Cotes open form 7-points method. To increase the order of the convergence, we apply a similar manner to find the new interpolation points. Furthermore, the new 3-steps Newton-Raphson-like scheme can be determined as follows,

$$\begin{aligned} y_n &= x_n - F'(x_n)^{-1}F(x_n) \\ Z_n &= y_n - 945\left[460F'\left(\frac{16x_n - 8y_n}{8}\right) - 954F'\left(\frac{-3x_n + 11y_n}{8}\right) + 2196F'\left(\frac{25x_n - 17y_n}{8}\right) - 2496F'\left(\frac{23x_n - 15y_n}{8}\right) + 2196F'\left(\frac{-2x_n + 10y_n}{8}\right) - 954F'\left(\frac{3x_n + 5y_n}{8}\right) + 460F'\left(\frac{-x_n + 9y_n}{8}\right)\right]^{-1} F(y_n) \quad (6) \end{aligned}$$

$$x_{n+1} = Z_n - F'(Z_n)^{-1}F(Z_n)$$

By using numerical integration method Newton-Cotes open form 4-points to estimate the integration part of Equation (1), the following third 3-steps Newton-Raphson-like has been developed.

$$y_n = x_n - F'(x_n)^{-1}F(x_n)$$
$$Z_n = y_n - 24\left[11F'\left(\frac{7x_n - y_n}{6}\right) + F'\left(\frac{2x_n + 4y_n}{6}\right) + F'\left(\frac{-2x_n + 8y_n}{6}\right) + 11F'\left(\frac{-7x_n + 13y_n}{6}\right)\right]^{-1} F(y_n) \quad (7)$$
$$x_{n+1} = Z_n - F'(Z_n)^{-1}F(Z_n)$$

Lastly, the integral of the Equation (1) can be approximated with numerical integration method Newton-Cotes open form 5-points. The modification of the numerical integration equation above yields the fourth 3-steps Newton-Raphson-like scheme below,

$$y_n = x_n - F'(x_n)^{-1}F(x_n)$$
$$z_n = y_n - 20\left[11F'\left(\frac{28x_n - 20y_n}{8}\right) - 14F'\left(\frac{11x_n - 3y_n}{8}\right) + 26F'\left(\frac{-5x_n + 13y_n}{8}\right) - 14F'\left(\frac{19x_n - 11y_n}{8}\right) + 11F'\left(\frac{22x_n - 14y_n}{8}\right)\right]^{-1} F(y_n) \quad (8)$$
$$x_{n+1} = Z_n - F'(Z_n)^{-1}F(Z_n)$$

Four new 3-steps schemes have been established. Furthermore, the convergence analysis of all proposed schemes is explained in Section 3.2.

3.2 Convergence Analysis

Algorithm #1

Theorem 1: Let x^* be a simple solution of the differentiable function $F(x)$ and given the initial value x_i, the three-steps scheme defined in Equation (5) has order of convergence 8.

Proof.

Step 1:

$$y_n = x_n - \frac{F(x_n)}{F'(x_n)}. \quad (9)$$

By using the Taylor series, it can be obtained,

$$F(x_n) = F(x^*) + F'(x^*)(x_n - x^*) + F''(x^*)\frac{(x_n-x^*)^2}{2!} + F'''(x^*)\frac{(x_n-x^*)^3}{3!} + \cdots \quad (10)$$

Since $F(x^*) = 0$, by assuming $V_k = \frac{1}{k!}\cdot\frac{F^{(k)}(x^*)}{F'(x^*)}$, $k = 2, 3, \ldots$ and $E_n = x_n - x^*$, then Equation (10) become,

$$F(x_n) = F'(x^*)\left[E_n + V_2{E_n}^2 + V_3{E_n}^3 + V_4{E_n}^4 + V_5{E_n}^5 + V_6{E_n}^6 + V_7{E_n}^7 + V_8{E_n}^8 + O\left(\|{E_n}^9\|\right)\right]. \quad (11)$$

Furthermore, the derivative of Equation (11) is

$$F'(x_n) = F'(x^*)\left[I + 2V_2E_n + 3V_3{E_n}^2 + 4V_4{E_n}^3 + V_5{E_n}^4 + 6V_6{E_n}^5 + 7V_7{E_n}^6 + 8V_8{E_n}^7 + O\left(\|{E_n}^8\|\right)\right]. \quad (12)$$

From Equation (11) and Equation (12), we obtain,

$$\frac{F(x_n)}{F'(x_n)} = E_n - V_2{E_n}^2 + 2(V_2^2 - V_3){E_n}^3 + (7V_2V_3 - 3V_4 - 4V_2^3){E_n}^4 + O\left(\|{E_n}^5\|\right). \quad (13)$$

Substituting Equation (13) to Equation (9), it can be simplified as follows, since $E_n = x_n - x^*$,

$$y_n = x^* + V_2{E_n}^2 + 2(V_3 - V_2^2){E_n}^3 + (7V_2V_3 - 3V_4 - 4V_2^3){E_n}^4 + O\left(\|{E_n}^5\|\right). \quad (14)$$

Step 2:

$$Z_n = y_n - 1440\left[611F'\left(\frac{3x_n+4y_n}{7}\right) - 453F'\left(\frac{-2x_n+9y_n}{7}\right) + 562F'\left(\frac{-x_n+8y_n}{7}\right) + 562F'\left(\frac{x_n+6y_n}{7}\right) - 453F'\left(\frac{2x_n+5y_n}{7}\right) + 611F'\left(\frac{-3x_n+10y_n}{7}\right)\right]^{-1} F(y_n). \quad (15)$$

By using the Taylor series, is can be obtained

$$F(y_n) = F'(x^*)\left[(y_n - x^*) + V_2(y_n - x^*)^2 + V_3(y_n - x^*)^3 + O\left(\|{E_n}^4\|\right)\right]. \quad (16)$$

Next, by substituting Equation (14) to Equation (16), we have,

$$F(y_n) = F'(x^*)\left[V_2{E_n}^2 + 2(V_3 - V_2^2){E_n}^3 + (7V_2V_3 - 3V_4 - 4V_2^3){E_n}^4 + V_2^3{E_n}^4 + O\left(\|{E_n}^5\|\right)\right]. \quad (17)$$

By using the Taylor series, we obtain:

$$F(w_r) = F'(x^*)\left[(w_r - x^*) + V_2(w_r - x^*)^2 + V_3(w_r - x^*)^3 + O\left(\|{E_n}^4\|\right)\right]. \quad (18)$$

and the derivative of Equation (18) is:

$$F'(w_r) = F'(x^*)[I + 2V_2(w_r - x^*) + 3V_3(w_r - x^*)^2] + O\left(\|{E_n}^3\|\right). \quad (19)$$

where r = 1, 2, 3, 4, 5, and 6, and $w_1 = \frac{3x_i+4y_i}{7}$, $w_2 = \frac{-2x_i+9y_i}{7}$, $w_3 = \frac{-x_i+8y_i}{7}$, $w_4 = \frac{x_i+6y_i}{7}$., $w_5 = \frac{2x_i+5y_i}{7}$, and $w_6 = \frac{-3x_i+10y_i}{7}$.

Defined T = $V_2{E_n}^2 + 2(V_3 - V_2^2){E_n}^3 + (7V_2V_3 - 3V_4 - 4V_2^3){E_n}^4 + O\left(\|{E_n}^5\|\right)$ in Equation (14), we obtain

$$y_n = x^* + \mathrm{T}. \quad (20)$$

Substitute Equation (20) to w_1, w_2, w_3, w_4, w_5, and w_6, it can be obtained,

$w_1 = \frac{3x_i+4(x^*+T)}{7}$, $w_2 = \frac{-2x_i+9(x^*+T)}{7}$, $w_3 = \frac{-x_i+8(x^*+T)}{7}$, $w_4 = \frac{x_i+6(x^*+T)}{7}$, $w_5 = \frac{2x_i+5(x^*+T)}{7}$, and $w_6 = \frac{-3x_i+10(x^*+T)}{7}$.

Next, the new, w_1, w_2, w_3, w_4, w_5, and w_6 are substituted to Equation (19). Supposed that P_{nm} is a coefficient. We will obtain,

$$F'(w_1) = F'(x^*)\left[I + \frac{6}{7}V_2E_n + P_{11}{E_n}^2 + P_{12}{E_n}^3 + P_{13}{E_n}^4 + O(\|{E_n}^5\|)\right]$$

$$F'(w_2) = F'(x^*)\left[I - \frac{4}{7}V_2E_n + P_{21}{E_n}^2 + P_{22}{E_n}^3 + P_{23}{E_n}^4 + O(\|{E_n}^5\|)\right]$$

$$F'(w_3) = F'(x^*)\left[I - \frac{2}{7}V_2E_n + P_{31}{E_n}^2 + P_{32}{E_n}^3 + P_{33}{E_n}^4 + O(\|{E_n}^5\|)\right]$$

$$F'(w_4) = F'(x^*)\left[I + \frac{2}{7}V_2E_n + P_{41}{E_n}^2 + P_{42}{E_n}^3 + P_{43}{E_n}^4 + O(\|{E_n}^5\|)\right]$$

$$F'(w_5) = F'(x^*)\left[I + \frac{4}{7}V_2E_n + P_{51}{E_n}^2 + P_{52}{E_n}^3 + P_{53}{E_n}^4 + O(\|{E_n}^5\|)\right].$$

$$F'(w_6) = F'(x^*)\left[I - \frac{6}{7}V_2E_n + P_{61}{E_n}^2 + P_{62}{E_n}^3 + P_{63}{E_n}^4 + O(\|{E_n}^5\|)\right].$$

Then,

$$611F'(w_1) - 453F'(w_2) + 562F'(w_3) + 562F'(w_4) - 453F'(w_5) + 611F'(w_6) = 1440F'(x^*)[I + P_1{E_n}^2 + P_2{E_n}^3 + P_3{E_n}^4 + O(\|{E_n}^5\|)]. \quad (21)$$

We assume,

$611P_{11}$- $453P_{21}$+ $562P_{31}$ + $562P_{41}$ - $453P_{51}$ + $611P_{61}$ = P_1 is a coefficient of ${E_n}^2$

$611P_{12}$- $453P_{22}$+ $562P_{32}$ + $562P_{42}$ - $453P_{52}$ + $611P_{62}$ = P_2 is a coefficient of ${E_n}^3$

$611P_{13}$- $453P_{23}$+ $562P_{33}$ + $562P_{43}$ - $453P_{53}$ + $611P_{63}$ = P_3 is a coefficient of ${E_n}^4$

Substitute Equation (14) and Equation (21) to Equation (15), it is obtained

$$Z_n = x^* + P_{101}{E_n}^4 + P_{102}{E_n}^5 + P_{103}{E_n}^6 + P_{104}{E_n}^7 + P_{105}{E_n}^8 + O(\|{E_n}^9\|). \quad (22)$$

Suppose the coefficient of ${E_n}^4$ is P_{101}, ${E_n}^5$ is P_{102}, ${E_n}^6$ is P_{103}, ${E_n}^7$ is P_{104}, and ${E_n}^8$ is P_{105}

Step 3:

$$x_{n+1} = Z_n - \frac{F(Z_n)}{F'(Z_n)} \quad (23)$$

By using the Taylor series, it is obtained

$$F(Z_n) = F'(x^*)\left[(Z_n - x^*) + V_2(Z_n - x^*)^2 + V_3(Z_n - x^*)^3 + O(\|{E_n}^4\|)\right]. \quad (24)$$

Equation (22) is substituted to Equation (24)

$$F(Z_n) = F'(x^*)\left[P_{101}{E_n}^4 + P_{102}{E_n}^5 + P_{103}{E_n}^6 + P_{104}{E_n}^7 + P_{105}{E_n}^8 + V_2P_{101}^2{E_n}^8 + O(\|{E_n}^9\|)\right]. \quad (25)$$

The derivatives Equation (24) is,

$$F'(Z_n) = F'(x^*)\left[I + 2V_2(Z_n - x^*) + 3V_3(Z_n - x^*)^2 + O(\|{E_n}^3\|)\right]. \quad (26)$$

Equation (22) is substituted to Equation (26). We have

$$F'(Z_n) = F'(x^*)\left[I + 2V_2(P_{101}{E_n}^4 + P_{102}{E_n}^5 + P_{103}{E_n}^6 + P_{104}{E_n}^7 + O(\|{E_n}^8\|)\right]. \quad (27)$$

From Equation (25) and Equation (27). It is obtained

$$\frac{F(Z_n)}{F'(Z_n)} = P_{101}{E_n}^4 + P_{102}{E_n}^5 + P_{103}{E_n}^6 + P_{104}{E_n}^7 + P_{105}{E_n}^8 - V_2P_{101}^2{E_n}^8 + O(\|{E_n}^9\|). \quad (28)$$

Substitute Equation (22) and Equation (28) to Equation (23)

$$x_{n+1} = x^* + P_{101}{E_n}^4 + P_{102}{E_n}^5 + P_{103}{E_n}^6 + P_{104}{E_n}^7 + P_{105}{E_n}^8 + O(\|{E_n}^9\|) - (P_{101}{E_n}^4 + P_{102}{E_n}^5 + P_{103}{E_n}^6 + P_{104}{E_n}^7 + P_{105}{E_n}^8 - V_2P_{101}^2{E_n}^8 + O(\|{E_n}^9\|))$$

$$x_{n+1} = x^* + V_2P_{101}^2{E_n}^8 + O(\|{E_n}^9\|) \quad (29)$$

If $x_{n+1} - x^* = E_{n+1}$, finally we will obtain,

$$E_{n+1} = O(\|{E_n}^8\|),$$

which shows that Algorithm #1 (Equation (5)) has order of convergence 8, the required results.

Algorithm #2

Theorem 2: Let x^* be a simple solution of the differentiable function $F(x)$ and given the initial value x_i, the three-steps scheme defined in Equation (6) has order of convergence 8.

Proof:

Step 1: It is the same as step 1 of Algorithm #1 (Equation (14)).

Step 2:

It has similar behavior as Algorithm #1. We can find the Taylor series of $F(y_n)$ and $F'(w_r)$ in step 2 of Equation (6), wherewhere r = 1, 2, 3, 4, 5, 6 and 7, and $w_1 = \frac{16x_n-8y_n}{8}$, $w_2 = \frac{-3x_n+11y_n}{8}$, $w_3 = \frac{25x_n-17y_n}{8}$, $w_4 = \frac{23x_n-15y_n}{8}$, $w_5 = \frac{-2x_n+10y_n}{8}$, $w_6 = \frac{3x_n+5y_n}{8}$, and $w_7 = \frac{-x_n+9y_n}{8}$.

Then substitute Equation (20) to $w_1, w_2, w_3, w_4, w_5, w_6$, and w_7, it is obtained,

$w_1 = \frac{16x_n - 8(x^* + T)}{8}$, $w_2 = \frac{-3x_n + 11(x^* + T)}{8}$, $w_3 = \frac{25x_n - 17(x^* + T)}{8}$, $w_4 = \frac{23x_n - 15(x^* + T)}{8}$, $w_5 = \frac{-2x_n + 10(x^* + T)}{8}$, $w_6 = \frac{3x_n + 5(x^* + T)}{8}$ and $w_7 = \frac{-x_n + 9(x^* + T)}{8}$.

Further, the new w_1, w_2, w_3, w_4, w_5, w_6, and w_7 are used to expand the Taylor series of $F'(w_r)$. Supposed that Q_{nm} is a coefficient. We will obtain,

$$F'(w_1) = F'(x^*)\left[I + 4V_2E_n + Q_{11}{E_n}^2 + Q_{12}{E_n}^3 + Q_{13}{E_n}^4 + O(\|{E_n}^5\|)\right]$$

$$F'(w_2) = F'(x^*)\left[I - \frac{3}{4}V_2E_n + Q_{21}{E_n}^2 + Q_{22}{E_n}^3 + Q_{23}{E_n}^4 + O(\|{E_n}^5\|)\right]$$

$$F'(w_3) = F'(x^*)\left[I + \frac{25}{4}V_2E_n + Q_{31}{E_n}^2 + Q_{32}{E_n}^3 + Q_{33}{E_n}^4 + O(\|{E_n}^5\|)\right]$$

$$F'(w_4) = F'(x^*)\left[I + \frac{23}{4}V_2E_n + Q_{41}{E_n}^2 + Q_{42}{E_n}^3 + Q_{43}{E_n}^4 + O(\|{E_n}^5\|)\right]$$

$$F'(w_5) = F'(x^*)\left[I - \frac{1}{2}V_2E_n + Q_{51}{E_n}^2 + Q_{52}{E_n}^3 + Q_{53}{E_n}^4 + O(\|{E_n}^5\|)\right]$$

$$F'(w_6) = F'(x^*)\left[I + \frac{3}{4}V_2E_n + Q_{61}{E_n}^2 + Q_{62}{E_n}^3 + Q_{63}{E_n}^4 + O(\|{E_n}^5\|)\right]$$

$$F'(w_7) = F'(x^*)\left[I - \frac{1}{4}V_2E_n + Q_{71}{E_n}^2 + Q_{72}{E_n}^3 + Q_{73}{E_n}^4 + O(\|{E_n}^5\|)\right]$$

Then,

$$460F'(w_1) - 954F'(w_2) + 2196F'(w_3) - 2496F'(w_4) + 2196F'(w_5) - 954F'(w_6) + 460F'(w_7) = 945F'(x^*)\left[I + Q_1{E_n}^2 + Q_2{E_n}^3 + Q_3{E_n}^4 + O(\|{E_n}^5\|)\right]. \quad (30)$$

We assume,

$460Q_{11}$ - $954Q_{21}$ + $2196Q_{31}$ - $2496Q_{41}$ + $2196Q_{51}$ - $954Q_{61}$ + $460Q_{71}$= Q_1 is a coefficient of ${E_n}^2$

$460Q_{12}$ - $954Q_{22}$ + $2196Q_{32}$ - $2496Q_{42}$ + $2196Q_{52}$ - $954Q_{62}$ + $460Q_{72}$= Q_2 is a coefficient of ${E_n}^3$

$460Q_{13}$ - $954Q_{23}$ + $2196Q_{33}$ - $2496Q_{43}$ + $2196Q_{53}$ - $954Q_{63}$ + $460Q_{73}$= Q_3 is a coefficient of ${E_n}^4$

Substitute Equation (14) and Equation (30) to step 2 of Equation (6), it is obtained

$$Z_n = x^* + Q_{101}{E_n}^4 + Q_{102}{E_n}^5 + Q_{103}{E_n}^6 + Q_{104}{E_n}^7 + Q_{105}{E_n}^8 + O(\|{E_n}^9\|). \quad (31)$$

Suppose the coefficient of ${E_n}^4$ is Q_{101}, ${E_n}^5$ is Q_{102}, ${E_n}^6$ is Q_{103}, ${E_n}^7$ is Q_{104}, and ${E_n}^8$ is Q_{105}.

Step 3: We can easily find the Taylor series of $F(Z_n)$ and $F'(Z_n)$ of step 3 in Equation (6) since Z_n has been defined in Equation (31). Furthermore, we can also obtain

$$\frac{F(Z_n)}{F'(Z_n)} = Q_{101}{E_n}^4 + Q_{102}{E_n}^5 + Q_{103}{E_n}^6 + Q_{104}{E_n}^7 + Q_{105}{E_n}^8 - V_2Q_{101}^2{E_n}^8 + O(\|{E_n}^9\|). \quad (32)$$

Finally, Equation (31) and Equation (32) are substituted to step 3 of Equation (6) we obtain,

$$x_{n+1} = x^* + Q_{101}{E_n}^4 + Q_{102}{E_n}^5 + Q_{103}{E_n}^6 + Q_{104}{E_n}^7 + Q_{105}{E_n}^8 + O(\|{E_n}^9\|) - (Q_{101}{E_n}^4 + Q_{102}{E_n}^5 + Q_{103}{E_n}^6 + Q_{104}{E_n}^7 + Q_{105}{E_n}^8 - V_2Q_{101}^2{E_n}^8 + O(\|{E_n}^9\|))$$

$$x_{n+1} = x^* + V_2Q_{101}^2{E_n}^8 + O(\|{E_n}^9\|)$$

If $x_{n+1} - x^* = E_{n+1}$, Finally we have

$$E_{n+1} = O(\|{E_n}^8\|).$$

which shows that Algorithm #2 (Equation (6)) has order of convergence 8, the required results.

Algorithm #3

Theorem 3: Let x^* be a simple solution of the differentiable function $F(x)$ and given the initial value x_i, the three-steps scheme defined in Equation (7) has order of convergence 8.

Proof:

Step 1: It is the same as step 1 of Algorithm #1 (Equation (14)).

Step 2: We can find the Taylor series of $F(y_n)$ and $F'(w_r)$ in step 2 of Equation (7), where *r* = 1, 2, 3, and 4 and $w_1 = \frac{7x_n - y_n}{6}$, $w_2 = \frac{2x_n + 4y_n}{6}$, $w_3 = \frac{-2x_n + 8y_n}{6}$, and $w_4 = \frac{-7x_n + 13y_n}{6}$.

Then substitute Equation (20) to w_1, w_2, w_3, and w_4 , it is obtained

$w_1 = \frac{7x_n - (x^* + T)}{6}$, $w_2 = \frac{2x_n + 4(x^* + T)}{6}$, $w_3 = \frac{-2x_n + 8(x^* + T)}{6}$, and $w_4 = \frac{-7x_n + 13(x^* + T)}{6}$.

The new w_1, w_2, w_3, and w_4 are used to expand the Taylor series of $F'(w_r)$. Supposed that L_{nm} is a coefficient. We will obtain,

$$F'(w_1) = F'(x^*)\left[I + \frac{7}{3}V_2E_n + L_{11}{E_n}^2 + L_{12}{E_n}^3 + L_{13}{E_n}^4 + O(\|{E_n}^5\|)\right]$$

$$F'(w_2) = F'(x^*)\left[I + \frac{2}{3}V_2E_n + L_{21}{E_n}^2 + L_{22}{E_n}^3 + L_{23}{E_n}^4 + O(\|{E_n}^5\|)\right]$$

$$F'(w_3) = F'(x^*)\left[I - \frac{2}{3}V_2E_n + L_{31}{E_n}^2 + L_{32}{E_n}^3 + L_{33}{E_n}^4 + O(\|{E_n}^5\|)\right]$$

$F'(w_4) = F'(x^*)\left[I - \frac{7}{3}V_2E_n + L_{41}{E_n}^2 + L_{42}{E_n}^3 + L_{43}{E_n}^4 + O\left(\|{E_n}^5\|\right)\right].$

Then,

$$11F'(w_1) + F'(w_2) + F'(w_3) + 11F'(w_4) = 24F'(x^*)\left[I + L_1{E_n}^2 + L_2{E_n}^3 + L_3{E_n}^4 + O\left(\|{E_n}^5\|\right)\right]. \quad (33)$$

It is assumed,

$11L_{11}+ L_{21}+ L_{31}+11L_{41} = L_1$ is a coefficient of ${E_n}^2$

$11L_{12}+ L_{22}+ L_{32}+11L_{42} = L_2$ is a coefficient of ${E_n}^3$

$11L_{13}+ L_{23}+ L_{33}+11L_{43} = L_3$ is a coefficient oft ${E_n}^4$

Substitute Equation (14) and Equation (33) to step 2 of Equation (7), it is obtained

$$Z_n = x^* + L_{101}{E_n}^4 + L_{102}{E_n}^5 + L_{103}{E_n}^6 + L_{104}{E_n}^7 + L_{105}{E_n}^8 + O\left(\|{E_n}^9\|\right). \quad (34)$$

Suppose the coefficient of ${E_n}^4$ is L_{101}, ${E_n}^5$ is L_{102}, ${E_n}^6$ is L_{103}, ${E_n}^7$ is L_{104}, and ${E_n}^8$ is L_{105}

Step 3: We can easily find the Taylor series of $F(Z_n)$ and $F'(Z_n)$ of step 3 in Equation (7) since Z_n has been defined in Equation (34). Furthermore, we can also obtainWe can easily find the Taylor series of $F(Z_n)$ and $F'(Z_n)$ of step 3 in Equation (6) since Z_n has been defined by Equation (31). Furthermore, we can also obtain

$$\frac{F(Z_n)}{F'(Z_n)} = L_{101}{E_n}^4 + L_{102}{E_n}^5 + L_{103}{E_n}^6 + L_{104}{E_n}^7 + L_{105}{E_n}^8 - V_2L_{101}^2{E_n}^8 + O\left(\|{E_n}^9\|\right). \quad (35)$$

Finally, Equation (34) and Equation (35) are substituted to step 3 of Equation (7) we obtain,

$$x_{n+1} = x^* + L_{101}{E_n}^4 + L_{102}{E_n}^5 + L_{103}{E_n}^6 + L_{104}{E_n}^7 + L_{105}{E_n}^8 + O\left(\|{E_n}^9\|\right) - (L_{101}{E_n}^4 + L_{102}{E_n}^5 + L_{103}{E_n}^6 + L_{104}{E_n}^7 + L_{105}{E_n}^8 - V_2L_{101}^2{E_n}^8 + O\left(\|{E_n}^9\|\right))$$

$$x_{n+1} = x^* + V_2L_{101}^2{E_n}^8 + O\left(\|{E_n}^9\|\right) \quad (36)$$

If $x_{n+1} - x^* = E_{n+1}$ then we have $E_{n+1} = O\left(\|{E_n}^8\|\right)$.

which shows that Algorithm #3 (Equation (7)) has order of convergence 8, the required results.

Algorithm #4

Theorem 4: Let x^* be a simple solution of the differentiable function $F(x)$ and given the initial value x_i, the three-steps scheme defined in Equation (8) has order of convergence 8.

Proof:

Step 1: It is the same as step 1 of Algorithm #1 (Equation (14)).

Step 2: We can find the Taylor series of $F(y_n)$ and $F'(w_r)$ in step 2 of Equation (8), where r = 1, 2, 3, 4, and 5, and $w_1 = \frac{28x_n-20y_n}{8}$, $w_2 = \frac{11x_n-3y_n}{8}$, $w_3 = \frac{-5x_n+13y_n}{8}$, $w_4 = \frac{19x_n-11y_n}{8}$, dan $w_5 = \frac{22x_n-14y_n}{8}$.

Then substitute Equation (20) to w_1, w_2, w_3, w_4 and w_5 , it is obtained

$w_1 = \frac{28x_n-20(x^*+T)}{8}$, $w_2 = \frac{11x_n-3(x^*+T)}{8}$, $w_3 = \frac{-5x_n+13(x^*+T)}{8}$, $w_4 = \frac{19x_n-11(x^*+T)}{8}$, and $w_5 = \frac{22x_n-14(x^*+T)}{8}$.

Now, the new w_1, w_2, w_3, w_4 and w_5 are used to expand the Taylor series of $F'(w_r)$. Supposed that M_{nm} is a coefficient. We will obtain,

$F'(w_1) = F'(x^*)\left[I + 7V_2E_n + M_{11}{E_n}^2 + M_{12}{E_n}^3 + M_{13}{E_n}^4 + O\left(\|{E_n}^5\|\right)\right]$

$F'(w_2) = F'(x^*)\left[I + \frac{11}{4}V_2E_n + M_{21}{E_n}^2 + M_{22}{E_n}^3 + M_{23}{E_n}^4 + O\left(\|{E_n}^5\|\right)\right]$

$F'(w_3) = F'(x^*)\left[I - \frac{5}{4}V_2E_n + M_{31}{E_n}^2 + M_{32}{E_n}^3 + M_{33}{E_n}^4 + O\left(\|{E_n}^5\|\right)\right]$

$F'(w_4) = F'(x^*)\left[I + \frac{19}{4}V_2E_n + M_{41}{E_n}^2 + M_{42}{E_n}^3 + M_{43}{E_n}^4 + O\left(\|{E_n}^5\|\right)\right]$

$F'(w_5) = F'(x^*)\left[I + \frac{11}{2}V_2E_n + M_{51}{E_n}^2 + M_{52}{E_n}^3 + M_{53}{E_n}^4 + O\left(\|{E_n}^5\|\right)\right]$

Then,

$$11F'(w_1) - 14F'(w_2) + 26F'(w_3) - 14F'(w_4) + 11F'(w_5) = 20F'(x^*)\left[I + M_1{E_n}^2 + M_2{E_n}^3 + M_3{E_n}^4 + O\left(\|{E_n}^5\|\right)\right]. \quad (37)$$

It is assumed,

$11M_{11}$ - $14M_{21}$ + $26M_{31}$ - $14M_{41}$ +$11M_{51}$ = M_1 is a coefficient of ${E_n}^2$

$11M_{12}$ - $14M_{22}$ + $26M_{32}$ - $14M_{42}$ +$11M_{52}$ = M_2 is a coefficient of ${E_n}^3$

$11M_{13}$ - $14M_{23}$ + $26M_{33}$ - $14M_{43}$ +$11M_{53}$ = M_3 is a coefficient of ${E_n}^4$

Substitute Equation (14) and Equation (37) to step 2 of Equation (8), it is obtained

$$Z_n = x^* + M_{101}{E_n}^4 + M_{102}{E_n}^5 + M_{103}{E_n}^6 + M_{104}{E_n}^7 + M_{105}{E_n}^8 + O\left(\|{E_n}^9\|\right). \quad (38)$$

Suppose the coefficient of ${E_n}^4$ is M_{101}, ${E_n}^5$ is M_{102}, ${E_n}^6$ is M_{103}, ${E_n}^7$ is M_{104}, and ${E_n}^8$ is M_{105}.

Step 3: Since Z_n has been defined in Equation (38), we can find the convergence of the algorithm using similar way as previous algorithm as follows

$Z_n \quad x_{n+1} = x^* + M_{101}{E_n}^4 + M_{102}{E_n}^5 + M_{103}{E_n}^6 + M_{104}{E_n}^7 + M_{105}{E_n}^8 + O(\|{E_n}^9\|) - (M_{101}{E_n}^4 + M_{102}{E_n}^5 + M_{103}{E_n}^6 + M_{104}{E_n}^7 + M_{105}{E_n}^8 - V_2 M_{101}^2 {E_n}^8 + O(\|{E_n}^9\|))$

$$x_{n+1} = x^* + V_2 M_{101}^2 {E_n}^8 + O(\|{E_n}^9\|)$$

If $x_{n+1} - x^* = E_{n+1}$, then, $E_{n+1} = O(\|{E_n}^8\|)$.

which shows that Algorithm #3 (Equation (7)) has order of convergence 8, the required results.

3.3 Numerical Results

In this study, three simple non-linear equation systems examples are given. The examples are solved with the other multistep methods, such as FSM, KHM, Farida's best algorithm [1], and the proposed algorithm. The convergence criteria or error tolerance (ε) is $\varepsilon \leq 10^{-15}$ and the maximum iteration is 50. The comparison results are shown in Tables 1 - 3 below.

Example 1. Initial value (1, 1, 1)

$$2x^2 + y - z^2 - 10 = 0$$

$$3x^2 + 6y - z^2 - 2 = 0$$

$$x^2 - 5y + 6z^2 - 4 = 0$$

Example 2. initial value (-1, 1, -1)

$$10x + \sin(x + y) - 1 = 0$$

$$8y - (\cos(z - y))^2 - 1 = 0$$

$$12z + \sin z - 1 = 0$$

Example 3. initial value (0, 0, 0)

$$15x + y^2 - 4z - 13 = 0$$

$$x^2 + 10y - e^{-z} - 11 = 0$$

$$y^3 - 25z + 22 = 0$$

Table 1. Comparison results of Example 1

Method	Iter	Solution	The function value
KHM1	5	x=2.183031149571622, y=2.046875000000000, z=1.256234452640111	f_1=-0.00000000035x10^(-5), f_2=-0.00000000071x10^(-5), f_3=-0.00000000071x10^(-5)
FSM	6	x=2.183031149571622, y=2.046875000000000, z=1.256234452640111	f_1=-0.00000000035x10^(-5), f_2=-0.00000000071x10^(-5), f_3=-0.00000000071x10^(-5)
The best Farida's method	5	x=2.183031149571622, y=2.046875000000000, z=1.256234452640111	f_1=-0.00000000035x10^(-5), f_2=-0.00000000071x10^(-5), f_3=-0.00000000071x10^(-5)
Algorithm #1	3	x=2.183031149571622, y=2.046875000000000, z=1.256234452640111	f_1=-0.00000000035x10^(-5), f_2=-0.00000000035x10^(-5), f_3=-0.00000000035x10^(-5)
Algorithm #2	3	x=2.183031149571622, y=2.046875000000000, z=1.256234452640111	f_1=-0.00000000035x10^(-5), f_2=-0.00000000035x10^(-5), f_3=-0.00000000035x10^(-5)
Algorithm #3	3	x=2.183031149571622, y=2.046875000000000, z=1.256234452640111	f_1=0.000000000355x10^(-5), f_2=0.000000000355x10^(-5), f_3=-0.00000000035x10^(-5)
Algorithm #4	3	x=2.183031149571622, y=2.046875000000000, z=1.256234452640111	f_1=-0.00000000035x10^(-5), f_2=-0.00000000035x10^(-5), f_3=-0.00000000035x10^(-5)

Table 2. Comparison results of Example 2

Method	Iter	Solution	The function value
KHM1	4	x=0.068978349172667, y=0.246442418609183, z=0.076928911987537	f_1=0.000000000488x10^(-5), f_2=0.000000000044x10^(-5), f_3=0.000000000044x10^(-5)
FSM	6	x=0.068978349172667, y=0.246442418609183, z=0.076928911987537	f_1=0.000000000488x10^(-5), f_2=0.000000000044x10^(-5), f_3=0.000000000044x10^(-5)
The best Farida's method	4	x=0.068978349172667, y=0.246442418609183, z=0.076928911987537	f_1=0.000000000488x10^(-5), f_2=0.000000000044x10^(-5), f_3=0.000000000044x10^(-5)
Algorithm #1	3	x=0.068978349172667, y=0.246442418609183, z=0.076928911987537	f_1=0.000000000488x10^(-5), f_2=0.000000000044x10^(-5), f_3=0.000000000044x10^(-5)
Algorithm #2	3	x=0.068978349172667, y=0.246442418609183, z=0.076928911987537	f_1=0.000000000488x10^(-5), f_2=0.000000000044x10^(-5), f_3=0.000000000044x10^(-5)
Algorithm #3	3	x=0.068978349172667, y=0.246442418609183, z=0.076928911987537	f_1=0.000000000488x10^(-5), f_2=0.000000000044x10^(-5), f_3=0.000000000044x10^(-5)
Algorithm #4	3	x=0.068978349172667, y=0.246442418609183, z=0.076928911987537	f_1=0.000000000488x10^(-5), f_2=0.000000000044x10^(-5), f_3=0.000000000044x10^(-5)

Table 3. Comparison results of Example 3

Method	Iter	Solution	The function value
KHM1	4	x=1.042149560576938, y=1.031091271839402, z=0.923848154879368	f_1=-0.0000000005x10^(-5), f_2=-0.0000000001x10^(-5), f_3=-0.0000000007x10^(-5)
FSM	5	x=1.042149560576938, y=1.031091271839402, z=0.923848154879368	f_1=-0.0000000005x10^(-5), f_2=-0.0000000001x10^(-5), f_3=-0.0000000007x10^(-5)
The best Farida's method	4	x=1.042149560576938, y=1.031091271839402, z=0.923848154879368	f_1=-0.0000000005x10^(-5), f_2=-0.0000000001x10^(-5), f_3=-0.0000000007x10^(-5)
Algorithm #1	3	x=1.042149560576938, y=1.031091271839402, z=0.923848154879368	f_1=-0.0000000005x10^(-5), f_2=-0.0000000001x10^(-5), f_3=-0.0000000007x10^(-5)
Algorithm #2	3	x=1.042149560576938, y=1.031091271839402, z=0.923848154879368	f_1=-0.0000000005x10^(-5), f_2=-0.0000000001x10^(-5), f_3=-0.0000000007x10^(-5)
Algorithm #3	3	x=1.042149560576938, y=1.031091271839402, z=0.923848154879368	f_1=-0.0000000005x10^(-5), f_2=-0.0000000001x10^(-5), f_3=-0.0000000007x10^(-5)
Algorithm #4	3	x=1.042149560576938, y=1.031091271839402, z=0.923848154879368	f_1=-0.0000000005x10^(-5), f_2=-0.0000000001x10^(-5), f_3=-0.0000000007x10^(-5)

Tables 1 - 3 show that the proposed algorithm converge faster than the references in use.

4. CONCLUSION

In this study, we propose four new three-steps Newton-Raphson-like algorithms. It has been proven that the proposed algorithms have the order of convergence 8. Numerical examples show that the proposed algorithms converge faster than the other multistep algorithms, such as the FSM method, KHM method, and Farida's best method.

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