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On Ramsey (mK_2, P_4)-Minimal Graphs

Asep Iqbal Taufik, Denny Riama Silaban, Kristiana Wijaya

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On Ramsey Minimal Graphs for a 3-Matching Versus a Path on Five Vertices

Kristiana Wijaya, Edy Tri Baskoro, Asep Iqbal Taufik, Denny Riama Silaban

Let G, H, and F be simple graphs. The notation F ⟶ (G, H) means that any red-blue coloring of all edges of F contains a red copy of G or a blue copy of H. The graph F satisfying this property is called a Ramsey (G, H)-graph. A Ramsey (G, H)-graph is called minimal if for each edge e ∈ E(F), there exists...

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Let G, A, and B be simple graphs. The notation G → (A, B) means that for any red-blue coloring of the edges of G, there is a red copy of A or a blue copy of B in G. A graph G is called a Ramsey graph for (A, B) if G → (A, B). Additionally, if the graph G satisfies that G − e ↛ (A, B), for any e ∈ E(G),...

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On Ramsey (mK_2, P_4)-Minimal Graphs

Asep Iqbal Taufik, Denny Riama Silaban, Kristiana Wijaya

Let F, G, and H be simple graphs. The notation F → (G, H) means that any red-blue coloring of all edges of F will contain either a red copy of G or a blue copy of H. Graph F is a Ramsey (G, H)-minimal if F → (G, H) but for each $e \in$ E(F), $(F - e) \nrightarrow$ (G, H). The set $\mathscr{R}$(G, H) consists of all Ramsey (G,...

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Let G be a simple graph with n vertices and let A(G) be the (0, 1)-adjacency matrix of G. The characteristic polynomial of the graph G with respect to the adjacency matrix A(G), denoted by $\chi(G, \lambda)$ is a determinant of $(\lambda I - A(G))$, where I is the identity matrix. Suppose that $\lambda 1 \geq \lambda 2 \geq \cdots \geq \lambda n$ are the adjacency...

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Andi Pujo Rahadi, Edy Tri Baskoro, Suhadi Wido Saputro

A generalized theta graph is a graph constructed from two distinct vertices by joining them with l (>=3) internally disjoint paths of lengths greater than one. The distinguishing number D(G) of a graph G is the least integer d such that G has a vertex labelling with d labels that is preserved only...

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Inne Singgih

An edge magic total (EMT) labeling of a graph G = (V, E) is a bijection from the set of vertices and edges to a set of numbers defined by $\lambda: V \cup E \rightarrow \{1,2, ..., |V| + |E|\}$ with the property that for every $xy \in E$, the weight of xy equals to a constant k, that is, $\lambda(x) + \lambda(y) + \lambda(xy) = k$ for some integer...

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Ganesha Lapenangga P., Aryanto, Meksianis Z. Ndii

Let G = (V, E) and H = (V', E') be a connected graph. H-magic labeling of graph G is a bijective function $f: V(G) \cup E(G) \rightarrow \{1,2, ..., |V(G)| + |E(G)|\}$ such that for every subgraph H'of G isomorphic to H, $\sum v \in V(H')\ f(v) + \sum e \in E(H')\ f(e) = k$. Moreover, it is H-supermagic labeling if $f(V) = \{1,2 ,..., |V|\}$....

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Labelling of Generalized Friendship, Windmill, and Torch Graphs with a Condition at Distance Two

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A graph labelling with a condition at distance two was first introduced by Griggs and Robert. This labelling is also known as L(2,1)-labelling. Let G = (V, E) be a non-multiple graph, undirected, and connected. An L(2,1)-labelling on a graph is defined as a mapping from the vertex set V(G) to the set...

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On the Minimum Span of Cone, Tadpole, and Barbell Graphs

Hafif Komarullah, Ikhsanul Halikin, Kiswara Agung Santoso

Let G be a simple and connected graph with p vertices and q edges. An L(2,1)-labelling on the graph G is a function f: V(G) → {0,1, ..., k} such that every two vertices with a distance one receive labels that differ by at least two, and every two vertices at distance two receive labels that differ by at...

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L(2,1) Labeling of Lollipop and Pendulum Graphs

Kusbudiono, Irham Af'idatul Umam, Ikhsanul Halikin, Mohamat Fatekurohman

One of the topics in graph labeling is L(2,1) labeling which is an extension of graph labeling. Definition of L(2,1) labeling is a function that maps the set of vertices in the graph to non-negative integers such that every two vertices u, v that have a distance one must have a label with a difference...

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Magic and Antimagic Decomposition of Amalgamation of Cycles

Sigit Pancahayani, Annisa Rahmita Soemarsono, Dieky Adzkiya, Musyarofah

Consider G = (V, E) as a finite, simple, connected graph with vertex set V and edge set E. G is said to be a decomposable graph if there exists a collection of subgraphs of G, say $\mathscr{H}$ = {Hi|1 ≤ i ≤n} such that for every i ≠ j, Hi is isomorphic to Hj, ∪ i=1n Hi = G and should satisfy that E(Hi) ∩ E(Hj)...

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A Minimum Coprime Number *for Amalgamation of Wheel*

Hafif Komarullah, Slamin, Kristiana Wijaya

Let G be a simple graph of order n. A coprime labeling of a graph G is a vertex labeling of G with distinct positive integers from the set {1, 2, ..., k} for some k ≥ n such that any adjacent labels are relatively prime. The minimum value of k for which G has a coprime labelling, denoted as pr(G), is...

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Rainbow Connection Number of Shackle Graphs

M. Ali Hasan, Risma Yulina Wulandari, A.N.M. Salman

Let G be a simple, finite and connected graph. For a natural number k, we define an edge coloring c: E(G) → {1,2,..., k} where two adjacent edges can be colored the same. A u − v path (a path connecting two vertices u and v in V(G)) is called a rainbow path if no two edges of path receive the same color....

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Local Antimagic Vertex Coloring of Corona Product Graphs $P_n \circ P_k$

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Let G = (V, E) be a graph with vertex set V and edge set E. A bijection map f : E → {1,2, ..., |E|} is called a local antimagic labeling if, for any two adjacent vertices u and v, they have different vertex sums, i.e. w(u) ≠ w(v), where the vertex sum w(u) = Σe ∈ E(u) f(e), and E(u) is the set of edges...

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Local Antimagic Vertex Coloring of Gear Graph

Masdaria Natalina Br Silitonga, Kiki Ariyanti Sugeng

Let G = (V, E) be a graph that consist of a vertex set V and an edge set E. The local antimagic labeling f of a graph G with edge-set E is a bijection map from E to {1, 2, ..., |E|} such that w(u) ≠ w(v), where w(u) = ∑e ∈ E(u) f(e) and E(u) is the set of edges incident to u. In this labeling, every vertex...

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Implementations of Dijkstra Algorithm for Searching the Shortest Route of Ojek Online and a Fuzzy Inference System for Setting the Fare Based on Distance and Difficulty of Terrain (Case Study: in Semarang City, Indonesia)

Vani Natali Christie Sebayang, Isnaini Rosyida

Ojek Online is a motorcycle taxi that is usually used by people that need a short time for traveling. It is one of the easiest forms of transportation, but there are some obstacles in hilly areas such as Semarang City. The fare produced by online motorcycle taxis is sometimes not in accordance with the...

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On Ramsey (mK_2, P_4)-Minimal Graphs

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ABSTRACT

Let F, G, and H be simple graphs. The notation $F \to (G, H)$ means that any red-blue coloring of all edges of F will contain either a red copy of G or a blue copy of H. Graph F is a Ramsey (G, H)-minimal if $F \to (G, H)$ but for each $e \in E(F)$, $(F - e) \nrightarrow (G, H)$. The set $\mathcal{R}(G, H)$ consists of all Ramsey (G, H)-minimal graphs. Let mK_2 be matching with m edges and P_n be a path on n vertices. In this paper, we construct all disconnected Ramsey minimal graphs, and found some new connected graphs in $\mathcal{R}(3K_2, P_4)$. Furthermore, we also construct new Ramsey minimal graphs in $\mathcal{R}((m+1)K_2, P_4)$ from Ramsey minimal graphs in $\mathcal{R}(mK_2, P_4)$ for $m \geq 4$, by subdivision operation.

Keywords: *Matching, Path, Ramsey minimal graphs, Subdivision.*

1. INTRODUCTION

Let F, G, and H be simple graphs. The notation $F \to (G, H)$ means that in any red-blue coloring of F, there exists a red copy of G or a blue copy of H as a subgraph. A (G, H)-coloring of F is a red-blue coloring of F such that neither a red G nor a blue H occurs. A graph F is said to be a Ramsey (G, H)-minimal if $F \to (G, H)$ but for any $e \in E(F)$, there exists a (G, H)-coloring on graph $F - e$. The set of all Ramsey (G, H)-minimal graphs is denoted by $\mathcal{R}(G, H)$.

The determination and the characterization of all graphs F belonging to $\mathcal{R}(G, H)$ are the main problems in Ramsey (G, H)-minimal graphs. Some papers discuss the problem of determining all graphs in $\mathcal{R}(G, H)$. Burr et al. [1] proved that if H is any graph then $\mathcal{R}(mK_2, H)$ is a finite set. One of challenging problems in Ramsey Theory is to characterize all graphs in the set $\mathcal{R}(mK_2, H)$ for a given graph H.

Let K_n, C_n, and P_n be a complete graph, a cycle, and a path on n vertices, respectively. The characterization of Ramsey minimal graphs belonging to $\mathcal{R}(2K_2, K_4)$ can be seen in [2, 3]. The set $\mathcal{R}(2K_2, P_3)$ is determined by Mengersen and Oeckermann [4]. Mushi and Baskoro [5] determined all graphs in $\mathcal{R}(3K_2, P_3)$. Furthermore, the set $\mathcal{R}(4K_2, P_3)$ given by Wijaya et al. [6].

Wijaya et al. [7] showed that the cycle C_s belongs to $\mathcal{R}(mK_2, P_n)$ if and only if $s \in [mn - n + 1 \leq s \leq mn - 1]$. Recently Wijaya *et al*. [8] constructed a family of Ramsey (mK_2, P_4) minimal graphs from Ramsey $\big((m-1)K_2, P_4\big)$ minimal graph by doing 4 times subdivision on any edge belongs to a cycle in a Ramsey (mK_2, P_4)-minimal graph. Furthermore, Wijaya et al. [9] constructed a class of disconnected Ramsey (mK_2, H)-minimal graphs from a union of two or more connected graphs. Motivated by result in [9], in this paper, we focus on determining all disconnected graphs in $\mathcal{R}(3K_2, P_4)$, and found some connected graphs belonging to Ramsey $(3K_2, P_4)$-minimal. In addition, we also construct some graph in $\mathcal{R}((m+1)K_2, P_4)$ by doing subdivisions to graphs in $\mathcal{R}(mK_2, P_4)$ for m ≥ 4.

2. PRELIMINARIES

Let $G = (V, E)$ be graph. If $U \subseteq V$, then $G - U$ is a graph obtained from G by deleting vertices in U and all incident edges. If $H \subseteq G$, then $G - E(H)$ is a graph obtained from G by deleting edges in H. When $U = \{v\}$ and $E(H) = \{e\}$, for simplicity, we write $G - v$ and $G - e$, respectively.

Lemma 1 and 2 provide the necessary and sufficient conditions for any graph in $\mathcal{R}(3K_2, H)$ for any graph H.

Lemma 1. [9, 10] For any fixed graph H, the graph $F \longrightarrow (3K_2, H)$ holds if and only if the following four conditions are satisfied: (i) $F - \{u, v\} \supseteq H$ for each $u, v \in V(F)$, (ii) $F - u - E(K_3) \supseteq H$ for each $u \in V(F)$

and a triangle K_3 in F, (iii) $F - E(2K_3) \supseteq H$ for every two triangles in F, (iv) $F - E(S_5) \supseteq H$ for every induced subgraph with 5 vertices S in F.

Lemma 2. [9, 10] Let H be a simple graph. Suppose F is a Ramsey $(3K_2, H)$-graph. F is said to be minimal if for each $e \in E(F)$ satisfy $(F - e) \nrightarrow (3K_2, H)$, that is (i) $(F - e) - \{u, v\} \not\supseteq H$ for each $u, v \in V(F)$, ii) $F - u - E(K_3) \not\supseteq H$ for each $u \in V(F)$ and a triangle K_3 in F, (iii) $F - E(2K_3) \not\supseteq H$ for every two triangles in F, (iv) $F - E(S_5) \not\supseteq H$ for every induced subgraph with 5 vertices S in F.

Any graph satisfying all conditions in Lemma 1 and 2 is a Ramsey $(3K_2, H)$-minimal graph. The condition stated in Lemma 2 is called the **minimality property** of a graph in $\mathcal{R}(3K_2, H)$. In [10], Wijaya et al. defined $SF(e, t)$ as a t times subdivision of edge e in the connected graph F, and gave Theorem 3. Moreover, Baskoro and Yulianti [7] gave Theorem 4.

Theorem 3. Let F be a connected graph and $m \geq 2$ be an integer. Suppose α is one non-pendant edge of F. If $F \in \mathcal{R}(mK_2, P_4)$, then $SF(\alpha, 4) \in \mathcal{R}\big((m+1)K_2, P_4\big)$.

Theorem 4. [7] $\mathcal{R}(2K_2, P_4) = \{2P_4, C_7, C_6, C_5, C_4^+\}$, where C_4^+ is a C_4 with additional two pendant vertices as in Figure 1

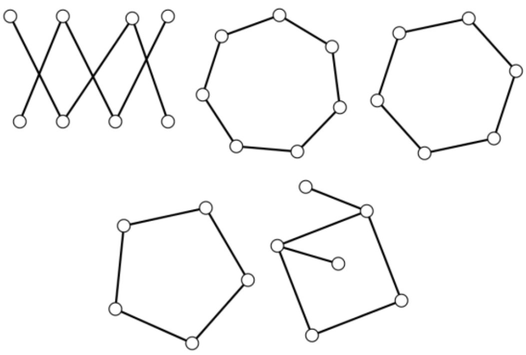

Figure 1 All graphs in $\mathcal{R}(2K_2, P_4)$.

3. MAIN RESULTS

3.1 Disconnected Graph in $\mathcal{R}(3K_2, P_4)$

In this section, we give all disconnected graphs belonging to $\mathcal{R}(3K_2, P_4)$.

Theorem 5. $G \cup P_4 \in \mathcal{R}(3K_2, P_4)$ if and only if $G \in \mathcal{R}(2K_2, P_4)$.

Proof. ($\Leftarrow$) We will show that for any $G \in \mathcal{R}(2K_2, P_4)$, then $G \cup P_4 \in \mathcal{R}(3K_2, P_4)$. Since $G \in \mathcal{R}(2K_2, P_4)$, then $G \rightarrow (2K_2, P_4)$ and $G - e \nrightarrow (2K_2, P_4)$ for any $e \in E(G)$. Since $G \rightarrow (2K_2, P_4)$, by coloring all edges incident to any vertex in G produces a blue copy of P_4 subset of G. Thus, any red coloring of two independent edges in G produces blue copy of P_4 subset of $G \cup P_4$. Moreover, any red coloring of one edge in G and one edge in P_4 produces a blue copy of P_4 subset of $G \cup P_4$. Hence, $G \cup P_4 \rightarrow (3K_2, P_4)$. Let $e_1 \in E(G)$ and $e_2 \in E(P_4)$. Since $G - e_1 \nrightarrow (2K_2, P_4)$, there exists a red-blue coloring of $G - e_1$ where a red K_2 occurs and blue P_4 cannot be found. Therefore, there exists a red-blue coloring on $G \cup P_4 - e_1$ where neither a red $3K_2$ nor a blue P_4 occurs. Moreover, any red coloring of two independent edges in $G \subset G \cup P_4 - e_2$ produces red-blue coloring of $G \cup P_4 - e_2$ where neither a red $3K_2$ nor a blue P_4 occurs. Hence, $G \cup P_4 - e \nrightarrow (3K_2, P_4)$. Since $G \cup P_4 \rightarrow (3K_2, P_4)$ and $G \cup P_4 - e \nrightarrow (3K_2, P_4)$ for any $e \in E(G)$, then $G \cup P_4 \in \mathcal{R}(3K_2, P_4)$.

($\Rightarrow$) If $G \cup P_4 \in \mathcal{R}(3K_2, P_4)$, then $G \in \mathcal{R}(2K_2, P_4)$. For a contradiction, suppose that $G \notin \mathcal{R}(2K_2, P_4)$. Then, we have two cases.

Case 1. Suppose $G \nrightarrow (2K_2, P_4)$. Then there exist a $(2K_2, P_4)$-coloring of G. Extend the coloring to color $G \cup P_4$ and color the edges of P_4 by red. Thus, there exist a $(3K_2, P_4)$-coloring of $G \cup P_4$, which contradicts the fact that $G \cup P_4 \in \mathcal{R}(3K_2, P_4)$.

Case 2. Suppose $G \rightarrow (2K_2, P_4)$, but G is not minimal. It means there exists a graph $H \in \mathcal{R}(2K_2, P_4)$ where $G \supset H$. Thus $G \cup P_4 \supset H \cup P_4$. Since $H \in \mathcal{R}(2K_2, P_4)$, then $H \cup P_4 \in \mathcal{R}(3K_2, P_4)$ by the first case, which contradicts to the minimality of $G \cup P_4$.

Therefore, from two cases above, we conclude that $G \cup P_4 \in \mathcal{R}(3K_2, P_4)$ if and only if $G \in \mathcal{R}(2K_2, P_4)$. ■

Theorem 6. Let H be a disconnected graph in $\mathcal{R}(3K_2, P_4)$. Therefore, one component of H must be isomorphic to P_4.

Proof. Suppose to the contrary that $H = H_1 \cup H_2$ and none of H_1 or H_2 is isomorphic to P_4. Since there is no component in H isomorphic to P_4, there is no component P_4 in either H_1 and H_2. Every vertex in H is in a connected subgraph containing a P_4. Then, both H_1 and H_2 contain P_4. Therefore, there will be edges $e_1 \in E(H_1)$ and $e_2 \in E(H_2)$ such that $P_4 \subseteq H_1 - e_1$ and $P_4 \subseteq H_2 - e_2$. Since $H \in \mathcal{R}(3K_2, P_4)$, there exist a $(3K_2, P_4)$-coloring of $H - e_1$ and $H - e_2$, say J_1 and J_2, respectively. Under J_1, $H_1 - e_1$ must contain at least one red edge and H_2 must have a $(2K_2, P_4)$-coloring. Since if it is not the case, $H - e_1$ would contain a red $3K_2$ or blue P_4, a contradiction to the minimality of H. Moreover, under J_2, $H_2 - e_2$ must contain at least one red edge and H_1 must have a $(2K_2, P_4)$-coloring. We conclude that we will obtain a $(3K_2, P_4)$-coloring of H if we color H by using J_1 on H_2 and J_2 on H_1, which contradicts to the minimality of H.

Therefore, if H is a disconnected graph in $\mathcal{R}(3K_2, P_4)$. Then, one component of H must be isomorphic to P_4. ■

Theorem 7. The graphs $C_5 \cup P_4$, $C_6 \cup P_4$, $C_7 \cup P_4$, $C_4^+ \cup P_4$ and $3P_4$ are the only disconnected graphs in $\mathcal{R}(3K_2, P_4)$.

Proof. Using Theorem 6, if F is a disconnected graph in $\mathcal{R}(3K_2, P_4)$, then F must have a component isomorphic to P_4. Furthermore, Theorem 5 states that the other component of F must be a member of the set $\mathcal{R}(2K_2, P_4)$. Moreover, Theorem 4 determined all graphs in $\mathcal{R}(2K_2, P_4)$. Therefore, the graphs $C_5 \cup P_4$, $C_6 \cup P_4$, $C_7 \cup P_4$, $C_4^+ \cup P_4$ and $3P_4$ are the only disconnected graphs in $\mathcal{R}(3K_2, P_4)$. ■

3.2 Some Connected Graphs in $\mathcal{R}(3K_2, P_4)$

In this section, we determine some connected graphs other than the cycle belonging to $\mathcal{R}(3K_2, P_4)$. First, we show that a graph F_1, depicted in Fig. 2, is a Ramsey $(3K_2, P_4)$-minimal graph.

Proposition 8. Let F_1 be a graph as depicted in Fig. 2. The graph F_1 is a Ramsey $(3K_2, P_4)$-minimal graph.

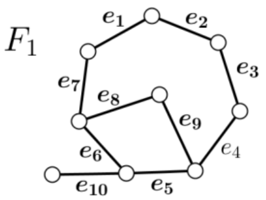

Figure 2 The graph $F_1 \in (3K_2, P_4)$.

Proof. First, we show that for any red-blue coloring of F_1 contains a red $3K_2$ or a blue P_4. We can see that $F_1 - \{u, v\}$ always contains a path P_4 for any $u, v \in V(F_1)$. It can be verified that $F_1 - E(S_5) \supseteq H$ for every induced subgraph with 5 vertices S in F_1. Since F_1 has no triangle, then by Lemma 1 we have that $F_1 \rightarrow (3K_2, P_4)$. Next, we prove the minimality property. For any edge e we will show that $(F_1 - e) \nrightarrow (3K_2, P_4)$. If e is one of the dashed edges in Fig. 3, then each red-blue coloring in Fig. 3 is the $(3K_2, P_4)$-coloring on $F_1 - e$. Therefore $F_1 \in \mathcal{R}(3K_2, P_4)$. ■

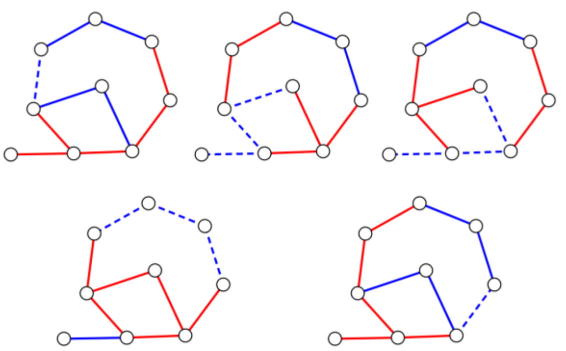

Figure 3 The $(3K_2, P_4)$-colorings on $F_1 - e$ if e is one of the dashed edges.

Suppose $V(C_n) = \{v_1, v_2, \ldots, v_{n-1}, v_n\}$ is the vertex-set of C_n. We define a graph C_n^a as a graph obtained from C_n by adding a pendant vertex, say v_{n+1}, adjacent to v_a for $a \in [1, n]$. A graph $C_n^{a,b}$ is obtained from C_n by adding two pendant vertices, say v_{n+1} and v_{n+2}, adjacent to v_a and v_b, respectively, for $a, b \in [1, n]$. Moreover, following Wijaya et al. in [10], we define special graphs with certain circumference. Let *a, b, c, d, e, f, g* and *h* be eight integers. Graph $C_n[(a, b), (c, d)]$ is obtained from C_n by adding two new edges $v_a v_b$ and $v_c v_d$. Graph $C_n[(a, b), (c, d), (e, f)]$ is obtained from C_n by adding three new edges $v_a v_b$, $v_c v_d$, and $v_e v_f$. Graph $C_n[(a, b), (c, d), (e, f), (g, h)]$ is obtained from C_n by adding four new edges $v_a v_b$, $v_c v_d$, $v_e v_f$, and $v_g v_h$. Now, consider graphs $C_6[(1,4), (2,5), (2,6), (3,5)]$, $C_7^5[(1,3), (2,6), (5,7)]$ $C_7[(1,5), (3,7)]$, $C_7^7[(2,6), (3,7)]$, $C_8[(2,7), (4,7), (6,8)]$ $C_6^{3,4}[(1,4), (3,6)]$ as depicted in Fig. 4. We will show that those graphs are Ramsey $(3K_2, P_4)$-minimal.

Theorem 9. All graphs in Fig. 4 are Ramsey $(3K_2, P_4)$-minimal graphs.

Proof. Let F be any graph in Fig. 4. It is easy to see that F satisfies all the conditions in Lemma 1. Then, $F \longrightarrow (3K_2, P_4)$ holds. Now, we will show the minimality property of F. Let e be any edge in F. If e is one of the dashed edges, then a $(3K_2, P_4)$-coloring on $F - e$ is provided in Figures 5, 6, 7. 8, 9 and 10 respectively for all cases. ■

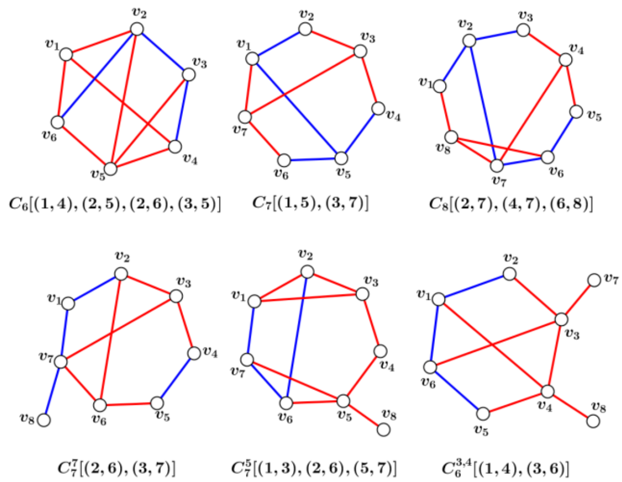

Figure 4 Six non-isomorphic graphs belonging to $\mathcal{R}(3K_2, P_4)$ which is obtained from C_n with some cords or pendant vertices or combination both.

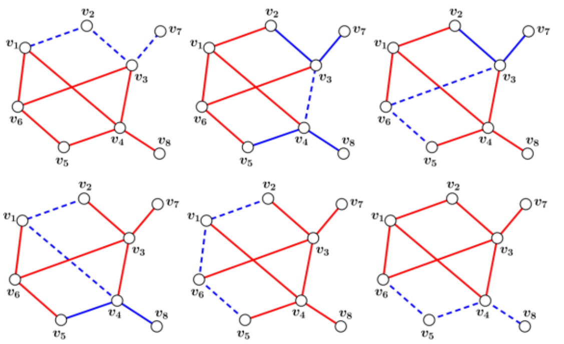

Figure 5 The $(3K_2, P_4)$-colorings on $C_6^{3,4}[(1,4),(3,6)] - e$ if e is one of the dashed edges.

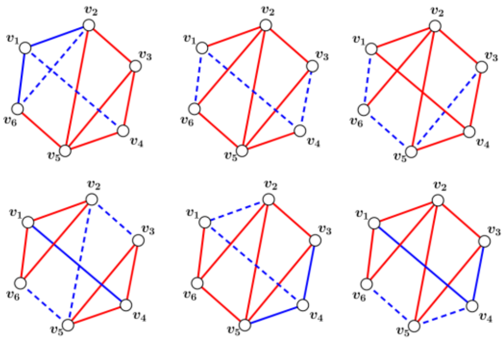

Figure 6 The $(3K_2, P_4)$ -colorings on $C_6[(1,4),(2,5),(2,6),(3,5)] - e$ if e is one of the dashed edges.

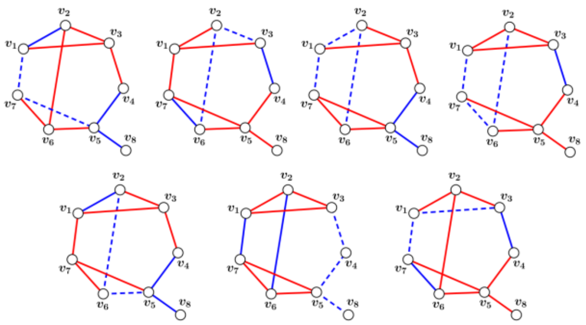

Figure 7 The $(3K_2, P_4)$ -colorings on $C_7^5[(1,3),(2,6),(5,7)] - e$ if e is one of the dashed edges.

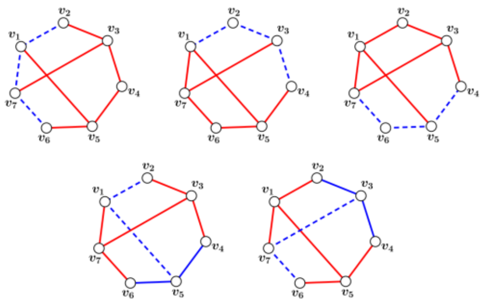

Figure 8 The $(3K_2, P_4)$ -colorings on $C_7[(1,5),(3,7)] - e$ if e is one of the dashed edges.

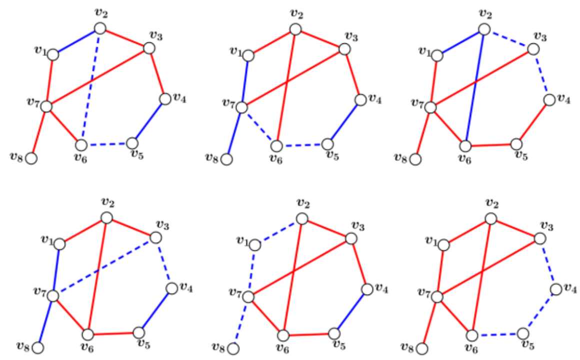

Figure 9 The $(3K_2, P_4)$ -colorings on $C_7^7[(2,6),(3,7)] - e$ if e is one of the dashed edges.

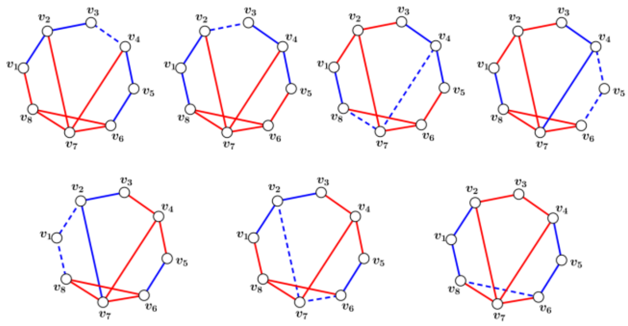

Figure 10 The $(3K_2, P_4)$ -colorings on $C_8[(2,7),(4,7),(6,8)] - e$ if e is one of the dashed edges.

3.3 Some New Family of Ramsey (mK_2, P_4)-Minimal Graphs

Recall that $SF(e,t)$ is a subdivision t times of edge e. In the previous section, it has been shown that $F_1 \in \mathcal{R}(3K_2, P_4)$. According to Theorem 3, if we subdivide (4 times) any non-pendant edge of F_1, then we obtain three non-isomorphism graphs belonging to $\mathcal{R}(4K_2, P_4)$, namely $SF_1(e_1, 4)$, $SF_1(e_5, 4)$, and $SF_1(e_8, 4)$ as depicted in Fig.11 (4 vertices, green vertex). The proof of the minimality of a graph $SF_1(e_5, 4)$ can be seen in Fig.12, while the minimality of the other graphs can be represented in the same way.

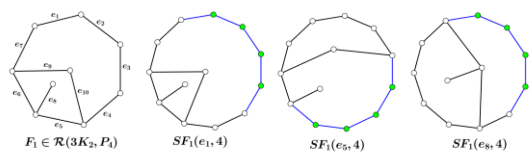

Figure 11 Three non-isomorphism graphs belonging to $(3K_2, P_4)$ are obtained by subdividing four times (4 green vertices) a non-pendant edge of F_1.

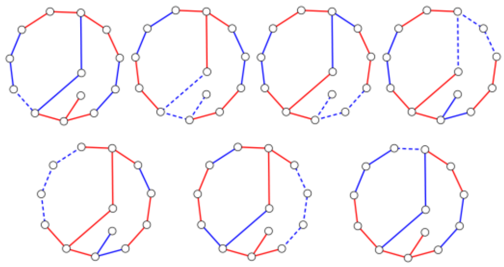

Figure 12 The $(4K_2, P_4)$-colorings on $SF_1(e_5, 4) - e$ if e is one of the dashed edges.

Now, we consider graph $C_7[(1,5),(3,7)]$. Since every edge in $C_7[(1,5),(3,7)]$ is non-pendant, then according to Theorem 3, the subdivision (4 times) on any edge of $C_7[(1,5),(3,7)]$ will produce three non-isomorphism graphs in $\mathcal{R}(4K_2, P_4)$. By repeating this process for the resulting graphs, we obtain Corollary 10.

Corollary 10. Let $m \geq 4$ be an integer. Then, the graphs $C_{4m-5}[(1,5),(3,7)]$, $C_{4m-5}[(1,4m-7),(4m-9,4m-5)]$, and $C_{4m-5}[(1,4m-7),(3,4m-5)]$ are in $\mathcal{R}(mK_2, P_4)$.

Proof. Let $\{v_1, v_2, \dots, v_7\}$ be the vertex-set of $C_7[(1,5),(3,7)]$. The subdivision (4 vertices) on the edge $e = v_1v_2$ will result $C_{11}[(1,9),(7,11)]$. Since $C_7[(1,5),(3,7) \in \mathcal{R}(3K_2, P_4)$, then by Theorem 3, we have that $C_{11}[(1,9),(7,11)] \in \mathcal{R}(4K_2, P_4)$. Furthermore, by subdividing (4 vertices) the edge $e = v_1v_2$ of $C_{11}[(1,9),(7,11)]$, we obtain $C_{15}[(1,13),(11,15)]$. By Theorem 3, we have that $C_{15}[(1,13),(11,15)] \in \mathcal{R}(5K_2, P_4)$. By continuing this process and applying it to the resulting graph, then we obtain the graph $C_{4m-5}[(1,4m-7),(4m-9,4m-5)]$. By Theorem 3, $C_{4m-5}[(1,4m-7),(4m-9,4m-5)] \in \mathcal{R}(mK_2, P_4)$. Next, by subdivision (4 vertices) on the edge $e = v_3v_4$ of the graph $C_7[(1,5),(3,7)]$, repeatedly, and apply Theorem 3, we obtain $C_{4m-5}[(1,4m-7),(3,4m-5)] \in \mathcal{R}(mK_2, P_4)$. By doing the same way to the edge $e = v_7v_1$, we obtain $C_{4m-5}[(1,5),(3,7)] \in \mathcal{R}(mK_2, P_4)$. ■

In the same way, we can construct some other graphs in $\mathcal{R}(mK_2, P_4)$ from some graph in $\mathcal{R}(3K_2, P_4)$, namely, $C_6^{3,4}[(1,4),(3,6)]$, $C_6[(1,4),(2,5),(2,6),(3,5)]$, $C_7^5[(1,3),(2,6),(5,7)]$, $C_7^7[(2,6),(3,7)]$, and $C_8[(2,7),(4,7),(6,8)]$. Therefore, we have Corollary 11.

Corollary 11. Let $m \geq 4$ be an integer. Then the following 19 graphs are in $\mathcal{R}(mK_2, P_4)$.

1. $C_{4m-6}^{3,4}[(1,4),(3,6)]$,
2. $C_{4m-6}^{4m-9,4m-8}[(1,4m-8),(4m-9,4m-6)]$,
3. $C_{4m-6}^{3,4m-8}[(1,4m-8),(3,4m-6)]$,
4. $C_{4m-6}[(1,4m-8),(4m-10,4m-7),$ $(4m-10,4m-6),(4m-9,4m-7)]$,
5. $C_{4m-6}[(1,4m-8),(2,4m-7),(2,4m-6),$ $(4m-9,4m-7)]$,
6. $C_{4m-6}[(1,4),(2,5),(2,6),(3,5)]$,
7. $C_{4m-5}^{7}[(2,6),(3,7)]$,
8. $C_{4m-5}^{4m-5}[(2,4m-6),(4m-9,4m-5)]$,
9. $C_{4m-5}^{4m-5}[(2,4m-6),(3,4m-5)]$,
10. $C_{4m-5}^{4m-5}[(2,6),(3,4m-5)]$
11. $C_{4m-5}^{5}[(1,3),(2,6),(5,7)]$,
12. $C_{4m-5}^{4m-7}[(1,4m-9),(4m-10,4m-6),(4m-7,4m-5)]$,
13. $C_{4m-5}^{4m-7}[(1,4m-9),(2,4m-6),(4m-7,4m-5)]$,
14. $C_{4m-5}^{4m-7}[(1,3),(2,4m-6),(4m-7,4m-5)]$,
15. $C_{4m-5}^{5}[(1,3),(2,4m-6),(5,4m-5)]$,
16. $C_{4m-5}^{5}[(1,3),(2,6),(5,4m-5)]$,
17. $C_{4(m-1)}[(2,7),(4,7),(6,8)]$,
18. $C_{4(m-1)}[(2,4m-5),(4m-8,4m-5),$ $(4m-6,4(m-1))]$,
19. $C_{4(m-1)}[(2,7),(4,7),(6,4(m-1))]$.

4. CONCLUSION

In this paper, we discuss the construction of a disconnected Ramsey minimal graph in $\mathcal{R}(3K_2, P_4)$ from Ramsey minimal graph ini $\mathcal{R}(2K_2, P_4)$. We show that all disconnected graphs in $\mathcal{R}(3K_2, P_4)$ are $C_5 \cup P_4$, $C_6 \cup P_4$, $C_7 \cup P_4$, $C_4^+ \cup P_4$, and $3P_4$. In addition, we give some connected graphs in $\mathcal{R}(3K_2, P_4)$, namely, F_1, $C_6[(1,4),(2,5),(2,6),(3,5)]$, $C_7^5[(1,3),(2,6),(5,7)]$ $C_7[(1,5),(3,7)]$, $C_7^7[(2,6),(3,7)]$, $C_8[(2,7),(4,7),(6,8)]$ $C_6^{3,4}[(1,4),(3,6)]$ as depicted in Fig. 4. Furthermore, we also construct nineteen new families of Ramsey (mK_2, P_4) minimal graphs for $m \geq 4$.

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AUTHORS' CONTRIBUTIONS

Asep Iqbal Taufik: Conceived and designed experiments; Conducted experiments; Wrote the paper - original draft preparation.

Denny Riama Silaban: Supervision and validation; Wrote the paper - review and editing.

Kristiana Wijaya: Supervision and validation; Wrote the paper - review and editing.

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