Series: Advances in Computer Science Research

Proceedings of the International Conference on Mathematics, Geometry, Statistics, and Computation (IC-MaGeStiC 2021)

Search

+ Advanced search

SEARCH

45 articles

Proceedings Article

On Ramsey Minimal Graphs for a 3-Matching Versus a Path on Five Vertices

Series: Advances in Computer Science Research

Proceedings of the International Conference on Mathematics, Geometry, Statistics, and Computation (IC-MaGeStiC 2021)

< PREVIOUS VOLUME IN SERIES

The International Conference on Mathematics, Geometry, Statistics, and Computation (IC-MaGeStiC) was held on November 27th, 2021, Jember, East Java, Indonesia. This conference is an excellent forum for the researchers, the lecturers, and the practitioners in Mathematics, to exchange findings and research ideas on mathematics and science education and to build networks for further collaboration.

Please click here for the conference website.

MaGeStiC 2021 Committee are willing to express our sincere thanks to all the powerful supports from all the contributors and the reviewers and the kind publications of Atlantis Press for all accepted papers. Sincerely hope for more academic cooperation with all scholars and scientists of the related fields in the future.

MaGeStiC 2021 International Committee

Bibliographic information:

Title	Proceedings of the International Conference on Mathematics, Geometry, Statistics, and Computation (IC-MaGeStiC 2021)
Editors	Kristiana Wijaya, Universitas Jember, Indonesia
Part of series	ACSR
Volume	96
ISSN	2352-538X
ISBN	978-94-6239-529-9

+ Advanced search

SEARCH

45 articles

Proceedings Article

On Ramsey Minimal Graphs for a 3-Matching Versus a Path on Five Vertices

Kristiana Wijaya, Edy Tri Baskoro, Asep Iqbal Taufik, Denny Riama Silaban

Let G, H, and F be simple graphs. The notation F ⟶ (G, H) means that any red-blue coloring of all edges of F contains a red copy of G or a blue copy of H. The graph F satisfying this property is called a Ramsey (G, H)-graph. A Ramsey (G, H)-graph is called minimal if for each edge e ∈ E(F), there exists...

Article details

Download article (PDF)

Proceedings Article

Ramsey Graphs for a Star on Three Vertices Versus a Cycle

Maya Nabila, Edy Tri Baskoro, Hilda Assiyatun

Let G, A, and B be simple graphs. The notation G → (A, B) means that for any red-blue coloring of the edges of G, there is a red copy of A or a blue copy of B in G. A graph G is called a Ramsey graph for (A, B) if G → (A, B). Additionally, if the graph G satisfies that G − e ↛ (A, B), for any e ∈ E(G),...

Article details

Download article (PDF)

Proceedings Article

On Ramsey (mK_2, P_4)-Minimal Graphs

Asep Iqbal Taufik, Denny Riama Silaban, Kristiana Wijaya

Let F, G, and H be simple graphs. The notation F → (G, H) means that any red-blue coloring of all edges of F will contain either a red copy of G or a blue copy of H. Graph F is a Ramsey (G, H)-minimal if F → (G, H) but for each $e \in$ E(F), $(F - e) \not\to$ (G, H). The set $\mathscr{R}$(G, H) consists of all Ramsey (G,...

Article details

Download article (PDF)

Proceedings Article

Spectrum of Unicyclic Graph

Budi Rahadjeng, Dwi Nur Yunianti, Raden Sulaiman, Agung Lukito

Let G be a simple graph with n vertices and let A(G) be the (0, 1)-adjacency matrix of G. The characteristic polynomial of the graph G with respect to the adjacency matrix A(G), denoted by $\chi(G, \lambda)$ is a determinant of $(\lambda I - A(G))$, where I is the identity matrix. Suppose that $\lambda 1 \geq \lambda 2 \geq \cdots \geq \lambda n$ are the adjacency...

Article details

Download article (PDF)

Proceedings Article

Distinguishing Number of the Generalized Theta Graph

Andi Pujo Rahadi, Edy Tri Baskoro, Suhadi Wido Saputro

A generalized theta graph is a graph constructed from two distinct vertices by joining them with l (>=3) internally disjoint paths of lengths greater than one. The distinguishing number D(G) of a graph G is the least integer d such that G has a vertex labelling with d labels that is preserved only...

Article details

Download article (PDF)

Proceedings Article

Edge Magic Total Labeling of (*n*, *t*)-Kites

Inne Singgih

An edge magic total (EMT) labeling of a graph G = (V, E) is a bijection from the set of vertices and edges to a set of numbers defined by $\lambda: V \cup E \to \{1,2, ..., |V| + |E|\}$ with the property that for every xy ∈ E, the weight of xy equals to a constant k, that is, $\lambda(x) + \lambda(y) + \lambda(xy) = k$ for some integer...

Article details

Download article (PDF)

Proceedings Article

Further Result of *H*-Supermagic Labeling for Comb Product of Graphs

Ganesha Lapenangga P., Aryanto, Meksianis Z. Ndii

Let G = (V, E) and H = (V', E') be a connected graph. H-magic labeling of graph G is a bijective function $f: V(G) \cup E(G) \to \{1,2, ..., |V(G)| + |E(G)|\}$ such that for every subgraph H'of G isomorphic to H, $\sum v \in V(H')\ f(v) + \sum e \in E(H')\ f(e) = k$. Moreover, it is H-supermagic labeling if f(V) = {1,2 ,..., |V|}....

Article details

Download article (PDF)

Proceedings Article

Labelling of Generalized Friendship, Windmill, and Torch Graphs with a Condition at Distance Two

Ikhsanul Halikin, Hafif Komarullah

A graph labelling with a condition at distance two was first introduced by Griggs and Robert. This labelling is also known as L(2,1)-labelling. Let G = (V, E) be a non-multiple graph, undirected, and connected. An L(2,1)-labelling on a graph is defined as a mapping from the vertex set V(G) to the set...

- Article details
- Download article (PDF)

Proceedings Article

On the Minimum Span of Cone, Tadpole, and Barbell Graphs

Hafif Komarullah, Ikhsanul Halikin, Kiswara Agung Santoso

Let G be a simple and connected graph with p vertices and q edges. An L(2,1)-labelling on the graph G is a function f: V(G) → {0,1, ..., k} such that every two vertices with a distance one receive labels that differ by at least two, and every two vertices at distance two receive labels that differ by at...

- Article details
- Download article (PDF)

Proceedings Article

L(2,1) Labeling of Lollipop and Pendulum Graphs

Kusbudiono, Irham Af'idatul Umam, Ikhsanul Halikin, Mohamat Fatekurohman

One of the topics in graph labeling is L(2,1) labeling which is an extension of graph labeling. Definition of L(2,1) labeling is a function that maps the set of vertices in the graph to non-negative integers such that every two vertices u, v that have a distance one must have a label with a difference...

- Article details
- Download article (PDF)

Proceedings Article

Magic and Antimagic Decomposition of Amalgamation of Cycles

Sigit Pancahayani, Annisa Rahmita Soemarsono, Dieky Adzkiya, Musyarofah

Consider G = (V, E) as a finite, simple, connected graph with vertex set V and edge set E. G is said to be a decomposable graph if there exists a collection of subgraphs of G, say $\mathscr{H}$ = {Hi|1 ≤ i ≤n} such that for every i ≠ j, Hi is isomorphic to Hj, ∪ i=1n Hi = G and should satisfy that E(Hi) ∩ E(Hj)...

- Article details
- Download article (PDF)

Proceedings Article

A Minimum Coprime Number *for Amalgamation of Wheel*

Hafif Komarullah, Slamin, Kristiana Wijaya

Let G be a simple graph of order n. A coprime labeling of a graph G is a vertex labeling of G with distinct positive integers from the set {1, 2, ..., k} for some k ≥ n such that any adjacent labels are relatively prime. The minimum value of k for which G has a coprime labelling, denoted as pr(G), is...

- Article details
- Download article (PDF)

Proceedings Article

Rainbow Connection Number of Shackle Graphs

M. Ali Hasan, Risma Yulina Wulandari, A.N.M. Salman

Let G be a simple, finite and connected graph. For a natural number k, we define an edge coloring c: E(G) → {1,2,..., k} where two adjacent edges can be colored the same. A u − v path (a path connecting two vertices u and v in V(G)) is called a rainbow path if no two edges of path receive the same color....

- Article details
- Download article (PDF)

Proceedings Article

Local Antimagic Vertex Coloring of Corona Product Graphs $P_n \circ P_k$

Setiawan, Kiki Ariyanti Sugeng

Let G = (V, E) be a graph with vertex set V and edge set E. A bijection map f : E → {1,2, ..., |E|} is called a local antimagic labeling if, for any two adjacent vertices u and v, they have different vertex sums, i.e. w(u) ≠ w(v), where the vertex sum w(u) = Σe ∈ E(u) f(e), and E(u) is the set of edges...

- Article details
- Download article (PDF)

Proceedings Article

Local Antimagic Vertex Coloring of Gear Graph

Masdaria Natalina Br Silitonga, Kiki Ariyanti Sugeng

Let G = (V, E) be a graph that consist of a vertex set V and an edge set E. The local antimagic labeling f of a graph G with edge-set E is a bijection map from E to {1, 2, ..., |E|} such that w(u) ≠ w(v), where w(u) = ∑e ∈ E(u) f(e) and E(u) is the set of edges incident to u. In this labeling, every vertex...

- Article details
- Download article (PDF)

Proceedings Article

Implementations of Dijkstra Algorithm for Searching the Shortest Route of Ojek Online and a Fuzzy Inference System for Setting the Fare Based on Distance and Difficulty of Terrain (Case Study: in Semarang City, Indonesia)

Vani Natali Christie Sebayang, Isnaini Rosyida

Ojek Online is a motorcycle taxi that is usually used by people that need a short time for traveling. It is one of the easiest forms of transportation, but there are some obstacles in hilly areas such as Semarang City. The fare produced by online motorcycle taxis is sometimes not in accordance with the...

- Article details
- Download article (PDF)

Honorary Chair

Kusbudiono

Universitas Jember, Indonesia

Conference General Chair

Kiswara Agung Santoso

Universitas Jember, Indonesia

Steering Committee

I Made Tirta

Universitas Jember, Indonesia

Moh. Hasan

Universitas Jember, Indonesia

Inne Singgih

University of Cincinnati, United States

Natanael Karjanto

Sungkyunkwan University, South Korea

Novi Herawati Bong

University of Delaware, United States

Technical Committee

Abduh Riski

Universitas Jember, Indonesia

Ikhsanul Halikin

Universitas Jember, Indonesia

Bagus Juliyanto

Universitas Jember, Indonesia

Millatuz Zahroh

Universitas Jember, Indonesia

Organizing Committtee

Kristiana Wijaya

Universitas Jember, Indonesia

Yuliani Setia Dewi

Universitas Jember, Indonesia

Alfian Futuhul Hadi

Universitas Jember, Indonesia

Mohamat Fatekurohman

Universitas Jember, Indonesia

Agustina Pradjaningsih

Universitas Jember, Indonesia

Firdaus Ubaidillah

Universitas Jember, Indonesia

Kosala Dwidja Purnomo

Universitas Jember, Indonesia

Treasurer

Ahmad Kamsyakawuni

Universitas Jember, Indonesia

Dian Anggraeni

Universitas Jember, Indonesia

Advances in Computer Science Research, volume 96
Proceedings of the International Conference on Mathematics, Geometry, Statistics, and Computation (IC-MaGeStiC 2021)

On Ramsey Minimal Graphs for a 3-Matching Versus a Path on Five Vertices

Kristiana Wijaya[1,*], Edy Tri Baskoro[2], Asep Iqbal Taufik[3], Denny Riama Silaban[3]

[1]Graph, Combinatorics, and Algebra Research Group, Department of Mathematics, FMIPA, Universitas Jember
[2]Combinatorial Mathematics Research Group, FMIPA, Institut Teknologi Bandung
[3]Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Indonesia, Depok 16424
[*]Corresponding author. Email: kristiana.fmipa@unej.ac.id

ABSTRACT
Let G, H, and F be simple graphs. The notation $F \longrightarrow (G,H)$ means that any red-blue coloring of all edges of F contains a red copy of G or a blue copy of H. The graph F satisfying this property is called a Ramsey (G,H)-graph. A Ramsey (G,H)-graph is called minimal if for each edge $e \in E(F)$, there exists a red-blue coloring of $F-e$ such that $F-e$ contains neither a red copy of G nor a blue copy of H. In this paper, we construct some Ramsey $(3K_2,P_5)$-minimal graphs by subdivision (5 times) of one cycle edge of a Ramsey $(2K_2,P_5)$-minimal graph. Next, we also prove that for any integer $m \geq 3$, the set $R(mK_2,P_5)$ contains no connected graphs with circumference 3.

Keywords: *Ramsey minimal graph,* 3*-matching, Path.*

1. INTRODUCTION

Given simple graphs G and H, any red-blue coloring of the edges of F is called a (G,H)*-coloring* if it has neither red copy of G nor blue copy of H. The notation $F \rightarrow (G,H)$ means that in any red-blue coloring of F there exists a red copy of G or a blue copy of H as a subgraph. A graph F is said to be a *Ramsey* (G,H)*-minimal* if $F \rightarrow (G,H)$ but for any $e \in E(F)$ there exists a (G,H)-coloring on graph $F-e$. The set of all Ramsey (G,H)-minimal graphs is denoted by $R(G,H)$. Burr, Erdős, Faudree, and Schelp [1] proved that if H is an arbitrary graph then $R(mK_2,H)$ is a finite set. One of challenging problems in Ramsey Theory is to characterize all graphs in the set $R(mK_2,H)$ for a given graph H. As usual, K_n, C_n, and P_n denote a complete graph, a cycle, and a path on n vertices, respectively. For any connected graph G, and $m \geq 2$, the notation mG means a disjoint union of m copies of a graph G. A t*-matching*, denoted by tK_2, is a graph with t components where every component is a graph K_2.

In general, it is difficult to characterize all graphs belonging to $R(mK_2,H)$. However, for some particular graph H, this set $R(mK_2,H)$ has been known. For instance, Burr, Erdős, Faudree, and Schelp [1] showed that $R(2K_2,2K_2)=\{C_5,3K_2\}$ and $R(2K_2,K_3)=\{K_5,2K_3,G_1\}$, where G_1 is a graph having the vertex-set $V(G_1)=\{c,u_i,v_i,w_i \mid i=1,2\}$ and the edge-set $E(G_1)=\{cu_i,cv_i,cw_i \mid i=1,2\} \cup \{u_1u_2,v_1v_2,w_1w_2\} \cup \{u_1v_1,u_1w_1,v_1w_1\}$. Burr *et al.* [2] showed that $R(2K_2,P_3)=\{C_4,C_5,2P_3\}$. Baskoro and Yulianti [3] proved that $R(2K_2,P_4)=\{C_5,C_6,C_7,2P_4,C_4^+\}$, where C_4^+ is a graph formed by a cycle on 4 vertices C_4 and two pendants vertices so that two vertices of degree 3 in the cycle C_4 are adjacent. Furthermore, they [3] also proved that $R(2K_2,P_5)=\{C_6,C_7,C_8,C_9,2P_5\} \cup \{A_i \mid i \in [1,7]\}$, where A_is are the graphs depicted in Figure 1. Wijaya, Baskoro, Assiyatun, and Suprijanto [4] showed that the cycle C_s belongs to $R(mK_2,P_n)$ if and only if $s \in [mn-n+1 \leq s \leq mn-1]$. Other results on characterizing all Ramsey minimal graphs for the pair of a matching versus a path can be seen in [5 – 8].

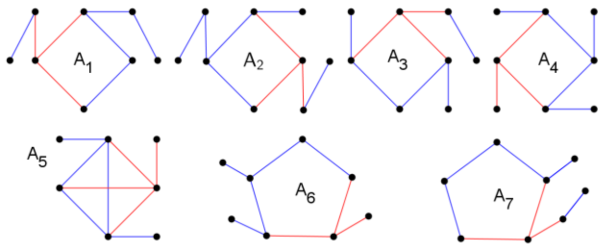

Figure 1 Some Ramsey $(2K_2,P_5)$-minimal graphs.

In [1], Burr, Erdős, Faudree, and Schelp gave a family of $\frac{(n+1)}{2}$ non-isomorphic graphs in $R(2K_2, K_n)$ for $n \geq 4$. These graphs are constructed from a complete graph K_{n+1}. In the same paper, Burr, Erdős, Faudree, and Schelp also gave a family of $(n-2)$ non-isomorphic graphs belonging to $R\big(2K_2, K_{1,n}\big)$. Motivated by them, Wijaya, Baskoro, Assiyatun, and Suprijanto [9] constructed some graphs in $R(mK_2, P_3)$ by subdivision (3 times) on any non-pendant edge of a connected graph in $R\big((m-1)K_2, P_3\big)$. Furthermore, Wijaya, Baskoro, Assiyatun, and Suprijanto [10] constructed a family of Ramsey (mK_2, P_4) minimal graphs from any Ramsey $((m-1)K_2, P_4)$ minimal graph by the subdivision process on any cycle-edge (4 times).

In this paper, we focus on constructing Ramsey $(3K_2, P_5)$ minimal graphs for 3-matching versus a path with five vertices. We also prove that there is no graph with circumference 3 belonging to $R(mK_2, P_5)$ for any integer $m \geq 3$. A *circumference* of a graph is the length of the longest cycle in that graph.

The following two lemmas provide the necessary and sufficient conditions for any graph in $R(3K_2, H)$ for any graph H.

Lemma 1.1 [9, 10] For any fixed graph H, the graph $F \longrightarrow (3K_2, H)$ holds if and only if the following four conditions are satisfied: (i) $F - \{u, v\} \supseteq H$ for each $u, v \in V(F)$, (ii) $F - \{u\} - E(K_3) \supseteq H$ for each $u \in V(F)$ and a triangle K_3 in F, (iii) $F - E(2K_3) \supseteq H$ for every two triangles in F, (iv) $F - E(S_5) \supseteq H$ for every induced subgraph with 5 vertices S in F. ■

Lemma 1.2 [9, 10] Let H be a simple graph. Suppose F is a Ramsey $(3K_2, H)$-graph. F is said to be *minimal* if for each $e \in E(F)$ satisfies $(F - e) \nrightarrow (3K_2, H)$, that is, (i) $(F - e) - \{u, v\} \not\supseteq H$ for each $u, v \in V(F)$, (ii) $F - \{u\} - E(K_3) \not\supseteq H$ for each $u \in V(F)$ and a triangle K_3 in F, (iii) $F - E(2K_3) \not\supseteq H$ for every two triangles in F, (iv) $F - E(S_5) \not\supseteq H$ for every induced subgraph with 5 vertices S in F. ■

Any graph satisfying all conditions stated in Lemmas 1 and 2 is a Ramsey $(3K_2, H)$ -minimal graph. The condition stated in Lemma 1.2 is called the *minimality property* of a graph in $R(3K_2, H)$.

Next theorem is one of the important properties of a Ramsey (mK_2, H)-minimal graph.

Theorem 1.3 [9] Let H be a graph and $m > 1$ be an integer. If $F \in (mK_2, H)$, then for any $v \in V(F)$ and $K_3 \subseteq F$, both graphs $F - \{v\}$ and $F - E(K_3)$ contain a Ramsey $\big((m-1)K_2, H\big)$-minimal graph. ■

2. MAIN RESULTS

In this section, we give some graphs belonging to $R(3K_2, P_5)$. We construct these graphs by the subdivision process on any cycle edge of a connected graph in $R(2K_2, P_5)$ depicted in Figure 1. Before doing this, first we show that a graph F_1, depicted in Figure 2, is a Ramsey $(3K_2, P_5)$-minimal graph. The vertex set of a graph F_1 is $V(F_1) = \{v_1, v_2, \ldots, v_{11}\}$ and the edge set of a graph F_1 is $E(F_1) = \{v_i v_{i+1} \mid i = 1,2, \ldots, 10\} \cup \{v_2 v_9, v_3 v_{10}\}$.

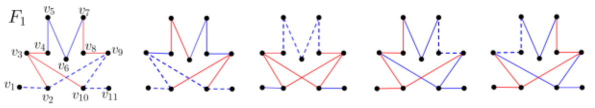

Figure 2 A graph F_1 and some red-blue colorings of F_1 so that F_1 contains no red $3K_2$ but it contains a blue P_5.

Proposition 2.1 Let F_1 be a graph on 11 vertices and 12 edges as depicted in Figure 2. The graph F_1 is a Ramsey $(3K_2, P_5)$-minimal graph.

Proof. First, we prove that for any red-blue coloring of F_1 there exists a red $3K_2$ or a blue P_5 in F_1. We can see that $F_1 - \{v_i, v_j\}$ always contains a path P_5 for any $1 \leq i, j \leq 11$. It can be verified that $F_1 - E(S_5) \supseteq H$ for every induced subgraph with 5 vertices S in F_1. Since F_1 has no triangle then by Lemma 1.1, $F_1 \rightarrow (3K_2, P_5)$. Next, we prove the minimality property of F_1. For any edge e we will show that $(F_1 - e) \nrightarrow (3K_2, P_5)$. If e is one of dashed edges in Figure 2, then each red-blue coloring in Figure 2 provides a $(3K_2, P_5)$ coloring on $F_1 - e$, namely a coloring that have neither red $3K_2$ nor blue P_5. Therefore $F_1 \in R(3K_2, P_5)$. ■

Next, we construct some Ramsey $(3K_2, P_5)$-minimal graphs from previous known Ramsey $(2K_2, P_5)$-minimal graphs by subdivision process. Consider each of Ramsey $(2K_2, P_5)$-minimal graphs in Figure 1. By the subdivision (5 times) on any of its cycle-edges we produce Ramsey $(3K_2, P_5)$-minimal graphs in Figure 3. In total, we obtain 12 non-isomorphic graphs belonging to $R(3K_2, P_5)$. Two non-isomorphic graphs F_2 and F_3 are obtained from the subdivision of A_1. Two non-isomorphic graphs F_4 and F_5 are formed from A_2. Two non-isomorphic graphs F_6 and F_7 are obtained from A_3. One graph called F_8 is obtained from the graph A_4. One graph F_9 is formed from A_5. Two non-isomorphic graphs F_{10} and F_{11} are obtained from the graph A_6. Last, two non-isomorphic graphs F_{12} and F_{13} are formed from A_7. In the following theorem, we will prove that these graphs are Ramsey $(3K_2, P_5)$-minimal graphs.

Theorem 2.2 All the graphs $F_2, F_3, \dots, F_{13}$ in Figure 3 are Ramsey $(3K_2, P_5)$-minimal graphs.

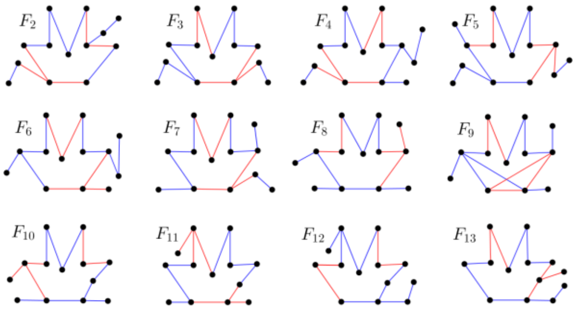

Figure 3 Some graphs belong to $R(3K_2, P_5)$.

Proof. Let F be any graph in Figure 3. It is easy to see that F satisfies all the conditions in Lemma 1.1. Then, $F \longrightarrow (3K_2, P_5)$ holds. Now, we will show the minimality property of F. Let e be any edge in F. If e is one of dashed edges, then a $(3K_2, P_5)$-coloring on $F - e$ is provided in Figures 4 and 5 for all cases of F and e. ■

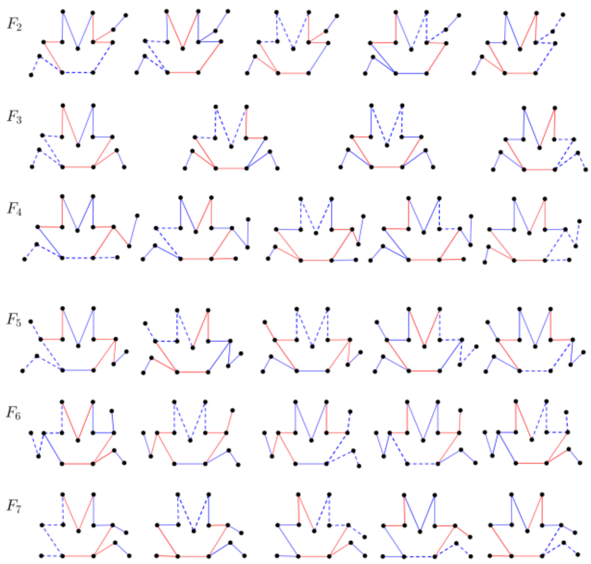

Figure 4 The $(3K_2, P_5)$-colorings on $F_i - e$ if e is one of dashed edges and for $i \in [2, 7]$.

Actually, there are two non-isomorphic graphs obtained by the subdivision (5 vertices) on any cycle edge of A_5 (see Figure 1). One of these two graphs is F_{10} and the other is obtained by subdivision (5 vertices) on the edge incident with a vertex of degree 4 and a vertex of degree 3. The last graph is a Ramsey-$(3K_2, P_5)$ graph but not minimal since it contains a graph $F_1 \in R(3K_2, P_5)$ (in Figure 2).

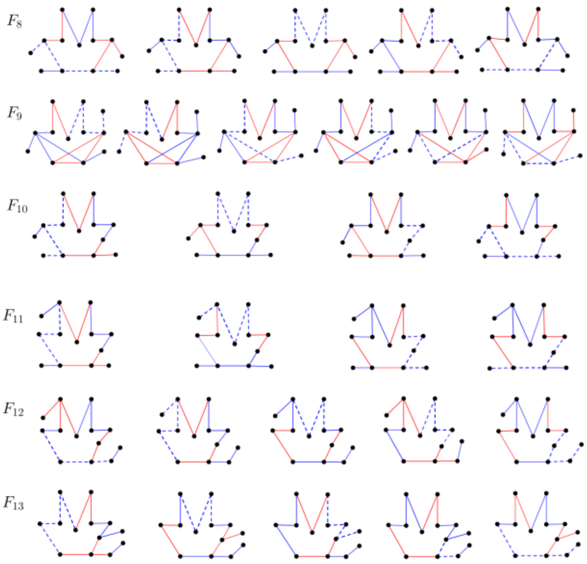

Figure 5 The $(3K_2, P_5)$-colorings on $F_i - e$ for $i \in [8, 13]$ if e is one of dashed edges.

In the following theorem, we will give a property of graphs belonging to $R(mK_2, P_5)$.

Theorem 2.3 There is no Ramsey (mK_2, P_5)-minimal graph with circumference 3 for any integer $m \geq 2$.

Proof. We will prove the theorem by induction on m. If $m = 2$ then it has been shown that there is no $(2K_2, P_5)$-minimal graph with circumference 3 (see [3]).

Assume that there is no (tK_2, P_5)-minimal graph with circumference 3 for any positive integer $t \leq m - 1$. We will show that there is no (mK_2, P_5)-minimal graph with circumference 3. Suppose to the contrary that there exists a graph F which is a Ramsey (mK_2, P_5)-minimal graph with circumference 3. Then, F must be a unicyclic graph. Let C be the cycle in F with $V(C) = \{u_1, u_2, u_3\}$. According to Theorem 1.3, $F - \{u_i\}$ for every $i \in [1, 3]$ contains a graph $G \in R\big((m-1)K_2, P_5\big)$. By assumption, the set $R\big((m-1)K_2, P_5\big)$ has no graph with circumference 3. So, G must be isomorphic to $(m - 1)P_5$. It forces that $F - E(C)$ is a graph $P_{n_1} \cup P_{n_2} \cup P_{n_3}$ where $n_1 + n_2 + n_3 \geq 5m = 15$. It implies that F contains a graph mP_5. Hence, F is not minimal. Otherwise, without loss of generality, we consider $n_1 + n_2 + n_3 = 5m - 1 \geq 14$ and assume $u_1 \in V(P_{n_1})$, $u_2 \in V(P_{n_2})$, and $u_3 \in V(P_{n_3})$. Suppose w.l.o.g. $n_1 \geq n_2 \geq n_3$ and $V(P_{n_1}) = \{u_1, v_{n_1-1}, v_{n_1-2}, \dots, v_2, v_1\}$ where v_1 is the pendant vertex of a path P_{n_1} and $E(P_{n_1}) =$

$\{u_1 v_{n_1-1}, v_i v_{i+1} | \ i \in [1, n_1 - 2]\}$. Clearly $n_1 \geq 5$. If $n_1 > 5$, we set the vertex $v_5 \in V(P_{n_1})$, then we obtain that $F - \{v_5\}$ does not contain a graph $(m-1)P_5$, which would contradict Theorem 1.3. In the case of $n_1 = 5$ we have $n_2 = 5$ and $n_3 = 4$. We obtain $F - \{u_1\} \not\supseteq 2P_5$, a contradiction with Theorem 1.3. Thus, the proof is complete. ■

3. CONCLUSION

In this paper, we discuss on the construction of Ramsey $(3K_2, P_5)$-minimal graphs. By the subdivision of any cycle edge of 7 Ramsey $(2K_2, P_5)$-minimal graphs (in Figure 1) we obtain 13 non-isomorphic Ramsey $(3K_2, P_5)$-minimal graphs. We also show that there is no Ramsey (mK_2, P_5)-minimal graph circumference 3 for any integer $m \geq 2$.

For a future work, we pose some open problems below.

Open Problem 1. Characterize all graphs belonging to $R(3K_2, P_5)$ by excluding all graphs resulted in this paper.

Open Problem 2. Are there any connected graphs with circumference 4 or 5 belonging to $R(3K_2, P_5)$?

Open Problem 3. Is it true that the subdivision (5 times) on any cycle-edge of a connected Ramsey $((m-1)K_2, P_5)$-minimal graph always produces a connected Ramsey (mK_2, P_5)- minimal graph?

ACKNOWLEDGMENTS

The second author thanks to the World Class Research (WCR) Program, Ministry of Research and Technology/National Research and Innovation Agency, Indonesia.

REFERENCES

[1] S.A. Burr, P. Erdős, R.J. Faudree, R.H. Schelp, A class of Ramsey-finite graphs, Proc. 9th Conf. Combinatorics, Graph Theory, and Computing, pp. 171–180.

[2] S.A. Burr, P. Erdős, R.J. Faudree, C.C. Rousseau, R.H. Schelp, Ramsey-minimal graphs for matchings, Proceedings-Fourth International Graph Theory Conference, pp. 159–168.

[3] E.T. Baskoro and L. Yulianti, Ramsey minimal graphs for $2K_2$ versus P_n, Advances and Applications in Discrete Mathematics 8, 2011, pp. 83-90. DOI: http://www.pphmj.com/journals/articles/880.htm

[4] K. Wijaya, E.T. Baskoro, H. Assiyatun, D. Suprijanto, On unicyclic Ramsey (mK_2, P_3) -minimal graphs, Procedia Computer Science 74, 2015, pp. 10–14. DOI: https://doi.org/10.1016/j.procs.2015.12.067

[5] I. Mengersen, J. Oeckermann, Ramsey sets for matchings, Ars Combinatoria, 56, 2000, pp. 33–42.

[6] D. Tatanto and E.T. Baskoro, On Ramsey $(2K_2, 2P_n)$ -minimal graphs, AIP Conference Proceedings 1450, 2012, 90. DOI: https://doi.org/10.1063/1.4724122

[7] H. Muhshi and E.T. Baskoro, On Ramsey $(3K_2, P_3)$ -minimal graphs, AIP Conference Proceedings 1450, 2012, 110. DOI: https://doi.org/10.1063/1.4724125

[8] K. Wijaya, E.T. Baskoro, H. Assiyatun, D. Suprijanto, On Ramsey $(4K_2, P_3)$-minimal graphs, AKCE International Journal of Graphs and Combinatorics 15, 2018, pp. 174–186. DOI: https://doi.org/10.1016/j.akcej.2017.08.003

[9] K. Wijaya, E.T. Baskoro, H. Assiyatun, D. Suprijanto, On Ramsey (mK_2, H)-minimal graphs, Graphs and Combinatorics 33, 2017, pp. 233–243. DOI: https://doi.org/10.1007/s00373-016-1748

[10] K. Wijaya, E.T. Baskoro, H. Assiyatun, D. Suprijanto, Subdivision of graphs in $R(mK_2, P_4)$, Heliyon 6, 2020, e03843. DOI: https://doi.org/10.1016/j.heliyon.2020.e03843