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Total vertex irregularity strength of wheel related graphs*

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Abstract

For a simple graph G with vertex set $V(G)$ and edge set $E(G)$, a labeling $\phi : V(G) \cup E(G) \longrightarrow \{1, 2, \dots, k\}$ is called a *vertex irregular total k-labeling* of G if for any two different vertices x and y, their weights $wt(x)$

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and $wt(y)$ are distinct. The weight $wt(x)$ of a vertex x in G is the sum of its label and the labels of all edges incident with the given vertex x. The *total vertex irregularity strength* of G, denoted by $\mathrm{tvs}(G)$, is the smallest positive integer k for which G has a vertex irregular total k-labeling. In this paper, we study the total vertex irregularity strength of flower, helm, generalized friendship and web graphs.

1 Introduction

Let us consider a simple (without loops and multiple edges) undirected graph $G = (V, E)$. For a graph G we define a labeling $\phi : V \cup E \to \{1, 2, \ldots, k\}$ to be a *vertex irregular total k-labeling* of the graph G if for every two different vertices x and y of G, one has $wt(x) \neq wt(y)$ where the weight of a vertex x in the labeling ϕ is

$$wt(x) = \phi(x) + \sum_{y \in N(x)} \phi(xy),$$

where $N(x)$ is the set of neighbors of x. In [3] Bača, Jendroľ, Miller and Ryan defined a new graph invariant, called the *total vertex irregularity strength* of G, $\mathrm{tvs}(G)$, which is the minimum k for which the graph G has a vertex irregular total k-labeling.

The original motivation for the definition of the total vertex irregularity strength came from irregular assignments and the irregularity strength of graphs introduced in [5] by Chartrand et al., and studied by numerous authors [4, 6, 7, 8, 9].

An *irregular assignment* is a k-labeling of the edges

$$f : E \to \{1, 2, \ldots, k\}$$

such that the vertex weights

$$w(x) = \sum_{y \in N(x)} f(xy)$$

are different for all vertices of G, and the smallest k for which there is an irregular assignment is the *irregularity strength*, $s(G)$.

The lower bound on $s(G)$ is given by the inequality

$$s(G) \geq \max_{1 \leq i \leq \Delta} \frac{n_i + i - 1}{i}.$$

The first upper bounds including the vertex degrees in the denominator were given in [7]. The best upper bound known so far can be found in [10]. Namely, the authors have proved that

$$s(G) \leq \left\lceil \frac{6n}{\delta} \right\rceil.$$